

# AGGREGATED MODEL OF DISTRIBUTION POWER NETWORKS DURING TRANSIENTS

I. MOCANU, M. MEUNIER

Ecole Supérieure d'Electricité

Service EEI

Plateau de Moulon

91192 Gif sur Yvette Cedex

France

E-mail : mocanu@supelec.fr

P. LASBLEIZ, A. GIARD

Electricité de France

Service Etudes de Réseaux

1 avenue du Général de Gaulle

92141 Clamart Cedex

France

## ABSTRACT

The distribution networks, overhead and underground, urban or rural, are exploited in sufficiently simple radial structures armed with a great number of different elements; for this reason it is difficult to modelise them. The objective of this paper is to present a systematic method (aggregation) which permits to obtain simple models for complex distribution networks during transients. This model may be characterised by a small number of parameters correlated to the physical nature of the main elements of the network. Also, this model should reproduce the transients in a frequency range of 0 to 1000 Hz.

**Keywords:** Distribution Network, Simplified Model, Electromagnetic Transient, Aggregation, Single-phase Fault.

## 1. INTRODUCTION

Some protective relays of distribution power networks operate during transients. These protection particularly detect self-extinguishing faults in the resonant grounding distribution networks. But, it is difficult to reproduce the transients in distribution networks. We are faced with two questions :

1) how to modelise as simple as possible these networks?

2) which are the important parameters to take into account and what are the range of their variations?

This paper presents a method which permits to obtain a small sized model of distribution networks. This model must correctly reproduce the phenomena up to 1000 Hz for different possible transients (connections, faults, ...).

Electromagnetic transients have been precisely analyzed in the transmission networks [2, 9, 10]. However, these works are not directly applicable to distribution networks [9], because they possess very short lines and have very complex geometry.

Distribution networks are represented by equivalents, whose parameters are chosen by 50 Hz analysis. On the other hand, no method exists which allows to comfortably obtain a simple model for a complex

distribution network, in order to represent their performances during transients. Indeed, the great number of equations to resolve, does not lead to a simple system.

The case of distribution networks was studied in following way :

- the distribution networks are represented in detail ;
- the aggregation method is applied;
- correlations between the model parameters and the physical nature of the main elements of the network are established.

In this way, the initial network is replaced by a small sized model, which preserves the most important characteristics of the network, while representing their performances during steady state and transients.

## 2. DISTRIBUTION NETWORK

### 2.1. General remarks

Distribution networks are exploited in sufficiently simple radial structures. Any HV/MV station possesses a radial configuration presented in fig. 1 :

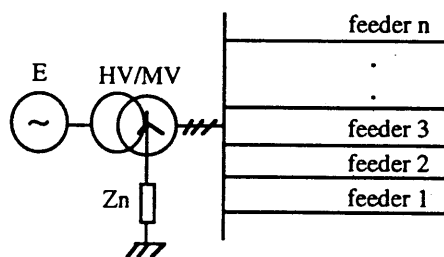


fig. 1

Generally, a feeder has the configuration of fig. 2 :

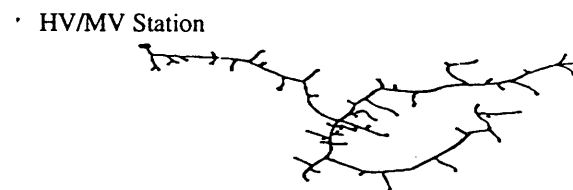


fig. 2

These feeders are made up of a great number of different various elements like : overhead lines and underground cables, transformers, compensation capacitors, loads, etc...

In the distribution networks there are several grounding systems, which results in an arrangement between the facility to detect swiftly a fault and the objective to reduce fault current amplitudes [5]. The neutral earthing of distribution networks can be directly linked to earth ( $Z_n = 0$ ), isolated ( $Z_n = \infty$ ) or connected to earth by a limitation impedance or a Petersen coil ( $Z_n$ ).

## 2.2. Simplifying assumptions

If a distribution network is considered in detail it is very difficult to write its equation. For this reason, simplifying assumptions are adopted :

- 1) An equilibrated, threephase infinite bus bar is supposed as voltage source;
- 2) The network is considered passive and symmetrical.
- 3) The saturation of the transformers is neglected. With this assumption the network is linear and the superposition theorem can be used;
- 4) The magnetisation currents of the transformers are neglected;
- 5) The loads are considered passive loads, due to the demand in MV/LV stations.

## 3. AGGREGATION METHOD

Among the various simplification methods [3, 8], the aggregation method was retained, due to its advantage in offering a simplified model which well represents the initial system performances during steady state and transients. Particularly, this method preserves the main modes of the initial system and gives a relation between the initial system and the simplified model where the stability is not affected [3, 8].

Let us consider a stationary linear system of order  $n$  :

$$(S_i) \begin{cases} \dot{x} = A \cdot x + B \cdot u \\ y = C \cdot x \end{cases} \text{ with: } x \in R^n, u \in R^r, y \in R^q$$

The general reduction problem consists of finding the best model which satisfies the equation :

$$(S_r) \begin{cases} \dot{z} = F \cdot z + G \cdot u \\ \hat{y} = H \cdot z \end{cases} \text{ with: } z \in R^m \text{ with : } m \ll n.$$

Then, we must deal with a parametrical optimisation problem in F, G and H.

The aggregation method consists of establishing a linear transformation from state  $x$  to state  $z$  so that :

$$z(t) = L \cdot x(t).$$

with the conditions :

$$\begin{cases} F \cdot L = L \cdot A \\ G = L \cdot B \\ z(0) = L \cdot x(0) \end{cases}$$

so that  $F$  preserves  $m$  eigenvalues of  $A$ .

The selection of the retained modes  $m$  in the model is done in two steps:

- first we compute the energy associated to each mode during the transient; we keep the modes whose energy is most important.
- then, we select the modes from their contribution to steady state.

This informations allow us to class the modes by their decreasing importance. Then, we can fix the appropriate dimensions of the model  $m$ , which depends on the number of the modes retained. The dominant modes are kept.

This aggregated model give us the best approximation if the modes are correctly selected.

So, we believe that aggregation proposes a general, systematic, powerfull method for the simplification of distribution networks.

## 4. STATE EQUATION OF THE NETWORK

State equations of the whole network is necessary to be able to apply aggregation. But it is necessary to chose transients models of each network element. In this way, the origin of each mode is known. Consequently, on diagonalizing the state equation we find that the modes are associated to the elements.

### 4.1. Network element models

Each element of the network is considered as a threephased quadripole [9]. The components of the network can be classed as follows :

- Overhead lines and underground cables reproduce transients if their representation is with distributed parameters. For this kind of representation, there are methods to generate equivalent lumped parameter models valid in a determined frequency range [9, 10]. For the lines shorter than 50 km encountered in distribution networks, and for frequencies less than 1000 Hz, these equivalents coincide with T lumped parameter circuit. For this reason we have adopted this representation.

- Linear branches, such as series elements (transformers, ...) and shunt elements (loads, ...) are represented by impedances or admittances [9].

- The dissymetries describe accidental situations in the networks. They are longitudinal dissymetries (phase interruptions, ...) and transversal dissymetries (earth faults, ...). Both are represented by series or shunt elements [1].

Symmetry among phases is assumed and the Karrenbauer transformation can be used to decompose the network. Current and voltage calculations will be done, after reconstitution of the three-phase system.

#### 4.2. State equation

A single phase formulation is used in order to simplify the presentation, even though our reasoning is based on the three-phase network.

An event (switching), which creates transients is equivalent with a second source in the network. The voltage of this source ( $V_{ev}$ ) is established by the supply source voltage ( $V_j$ ) and by the network conditions before the event. All the currents in the network are calculated by the superposition of the currents created by source  $V_j$  and by source  $V_{ev}$ . The currents produce by source  $V_j$  correspond to steady state. The currents produced by source  $V_{ev}$  characterise transients.

State equations were chosen as follows :

- input ( $u$ ) is the voltage before the event ( $V_{ev}$ );
- outputs ( $y$ ) are the currents in each feeder ( $i_{feederj}(t)$ , where  $j = 1, k$  and  $k$  is the feeder's number) and the bus bar voltage ( $v_{busbar}(t)$ ).

The diagonalized state equation ( $n$  order) is :

$$(Si) \begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \cdot \\ \cdot \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} -p_1 & 0 & \cdot & \cdot & 0 \\ 0 & -p_2 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & -p_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ 1 \end{bmatrix} |v_{ev}| \\ \\ \begin{bmatrix} v_{busbar} \\ i_{feeder1} \\ i_{feeder2} \\ \cdot \\ i_{feederk} \end{bmatrix} = \begin{bmatrix} r_{vbusbar1} & r_{vbusbar2} & \cdot & \cdot & r_{vbusbar n} \\ r_{ifeeder11} & r_{ifeeder12} & \cdot & \cdot & r_{ifeeder1n} \\ r_{ifeeder21} & r_{ifeeder22} & \cdot & \cdot & r_{ifeeder2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ r_{ifeederk1} & r_{ifeederk2} & \cdot & \cdot & r_{ifeederkn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \end{cases}$$

The real part of the eigenvalues of matrix A are negative ( $-p_i$ ), because the initial system is asymptotically stable.

It is possible to simplify the whole state equation with the aggregation method. But the aggregation method is applied for each input-output pair, to furnish models for each feeder current  $i_{feederj}$  and bus bar voltage  $v_{busbar}$ . In this way, for each input-output pair, the dominant modes are retained.

#### 4.3. Transient voltages and currents

For each input-output pair a rational fraction can be write. If Laplace transform of  $v_{ev}$  is applied, the expressions of  $i_{feederj}$  and  $v_{busbar}$  are rational fractions.

For each rational fraction there are poles which correspond to the eigenvalues of matrix A and two complex conjugated poles of  $V_{ev}(p)$ .

It is possible to calculate the system response the sum of exponentially damped sinusoids.

In the distribution network, for an event,  $v_{busbar}(t)$  and each  $i_{feederj}(t)$  are represented by such sums, with the same number of terms.

In these expressions, there are : one pure sinusoid term (50 Hz), the source imposed steady state, and a variable number of exponentially damped sinusoids or dampings, function of the eigenvalues of matrix A, real or complex.

Between the dampings and the exponentially damped sinusoids only a small number have an important and significative value. These important terms correspond to dominant modes. For each input-output pair, these modes are furnished by the aggregation method and are retained in the expressions of currents and voltages.

After determination of the aggregated models of each input-output pair,  $\hat{v}_{busbar}(t)$  and  $\hat{i}_{feederj}(t)$  can be calculated as a sum of exponentially damped sinusoids.  $\hat{v}_{busbar}(t)$  and  $\hat{i}_{feederj}(t)$  give good transient approximations of the bus bar voltage and feeder's currents respectively.

### 5. NETWORK MODES

The aggregation method furnishes the state equation of the optimal model. It is interesting to establish physical relations between the parameters of the aggregated model and network elements, for the choice of a physical model. In the network state equation it is possible to observe :

- proper modes of elements;
- specific modes following events (switching).

If the network possesses many ramified feeders, it is difficult to identify the origin of each mode. Therefore, the aggregation method was first applied to a simple radial distribution network as depicted in figure 1. The studied network has three feeders. Each feeder is considered as a line with an end placed load. The neutral earthing is a resistance of 40  $\Omega$ .

Aggregated models are determined. The simplification is judged by a comparison between the results furnished by the model and the EMTP simulations [4], where the lines are represented by a lumped parameter model (T).

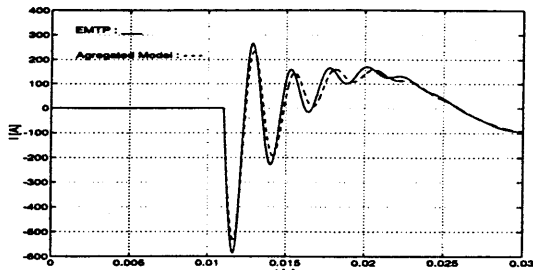
In this network, the possibility to use aggregation to determinate valid models of distribution networks was tested in two situations, connection of a feeder and single-phase fault.

#### 5.1. Connection of a feeder

Two feeders are supplied and the third is connected. Three dominant modes are retained in the aggregated model of

the third feeder current :

- two complex conjugated modes which correspond to the natural frequency of the line of the positive sequence;
- one real mode (negative) which corresponds to the load damping.



Current in a phase when feeder is connected  
fig. 3

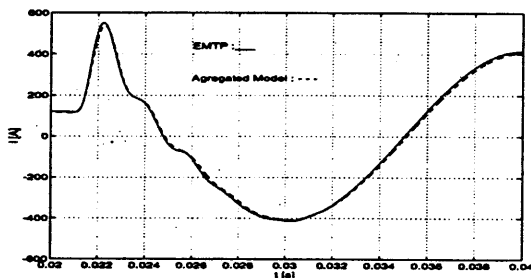
Therefore, when the network is loaded, the connection transient of a feeder depends on its own parameters, independent of the others.

## 5.2. Single-phase fault

In the considered network appears a single-phase resistive fault of  $10 \Omega$ , in the extremity of a feeder.

The aggregated model which represents the current in the short-circuited phase of the faulted feeder retains 5 modes:

- two complex conjugated modes which correspond to the natural frequency of the short circuited phase;
- two complex conjugated modes which correspond to the natural frequency of the positive sequence of the whole faulty network [6];
- one real mode (negative) which corresponds to a damping, introduced mainly by the fault and earthing resistances [5].



Current in faulty feeder (short-circuited phase)  
fig. 4

These modes, retained from the aggregated model, are specific of the single-phase faults in distribution networks.

The aggregated model of the bus bar voltage has the same dominant modes.

These specific single-phase fault modes are also dominant in the models giving currents in sound feeders. But, in the models of the sound currents still two modes are dominant. These correspond to the natural frequency of the zero sequence of the sound feeder, simulated by the variations of bus bar voltages, caused by the dissymmetry.

## 5.3. Conclusion

This study for a simple case correlates the aggregated model parameters to the the global values of resistances, impedances and capacitances of each feeder.

The feeders are nearly independent for a given connection. This performance is caused by the weak source impedance, simply represented by the leakage impedance of the HV/MV transformer.

Also, it is possible to correctly represent dissymmetrical transients (single-phase fault) if the specific dissymmetrical modes are computed with the global data of each feeder. In this case, proper modes of each feeder and specific modes of the interactions between feeders are found.

This indicates that for complex distribution networks the problem can be decomposed. First, it is necessary to propose a simple model for each feeder. Then, for each type of events, the state equation is written and aggregation is applied.

## 6. SIMPLE MODEL OF FEEDER

### 6.1. General model of feeders

Any feeder can be represented by a "back-bone" scheme depicted in fig. 5 :

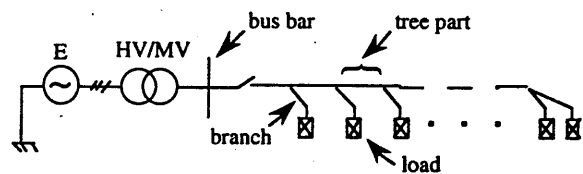


fig. 5

This model is very general because it represents urban networks (when branch length is very closed to zero) and rural networks (very ramified where each ramification can be represented in this manner). In this diagram, loads are distributed in each ramification.

For this generic feeder, aggregation was applied to determinate two kinds of models :

- an equivalent line to replace ramified feeders without loads;
- an equivalent line with an equivalent load to replace loaded ramified feeders.

The models must reproduce the transients of the initial ramified feeder.

The results obtained with these models are compared with two EMTP simulations. In the simulations the lines are represented with distributed parameters. The ramified feeder used for simulation is composed of 10 tree parts and 11 branches. The developed length of the ramified feeder is 40 km, where that of the branches represents 20 Km.

### 6.2. No-load ramified feeder

This study is interesting because, as the load neutrals are isolated, the zero sequence represents the no-load ramified

feeders.

Aggregation method is applied to a no-load generic "back-bone" feeder. As the aggregated model retained two dominant modes, we can consider that the physical model is an equivalent line where :

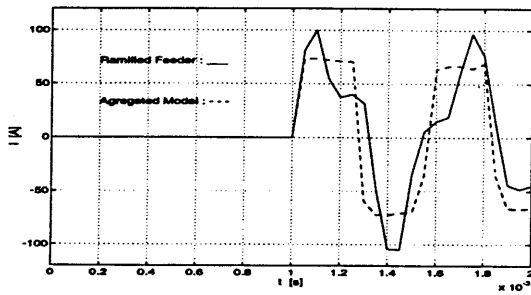
- the capacitance is equal to the sum of all capacitances of the feeder;

$$C_{eq} = \sum_{i=1}^n C_i, \text{ where } n \text{ is the number of line elements;}$$

elements;

- the impedance is equal to the sum of the series impedances of the longest branch.

$$Z_{eq} = \sum_{i=1}^l Z_{tree\ part\ i}$$



No-load feeder connection current  
fig. 6

The comparison between the ramified feeder composed of 21 line elements and one equivalent line is satisfactory.

### 6.3. Loaded ramified feeder

Generally, for a loaded ramified feeder, the aggregated model keeps three dominant modes. Hence, its physical representation is a line with a load. For this equivalent line :

- the capacitance is the sum of all capacitances of the feeder :

$$C_{eq} = \sum_{i=1}^n C_i$$

- the impedance is the impedance of the first part of the tree (next to switch), such that its characteristic impedance remains constant:

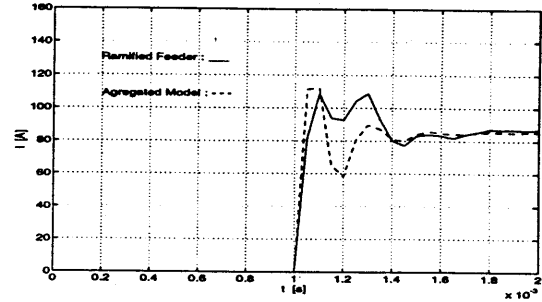
$$Z_{eq} = Z_{tree\ part\ 1}$$

The equivalent load is the sum of all feeder loads :

$$Y_{Leq} = \sum_j Y_{Lj}, \text{ where } j \text{ is the number of loads.}$$

The results (presented in the figure 7) are satisfactory, but their observation is difficult because the load damping reduces the time of the lines transients, in this case only

0.4 ms.



Load feeder connection current  
fig. 7

### 6.4. Conclusion

A general ramified feeder representing rural and urban network is proposed. Aggregation is applied to this ramified feeder and two models for no-load and load feeder are obtained.

### 7. VALIDATION

The presented method is applied to a resonant grounding distribution network located in the east of France, fig. 8 :

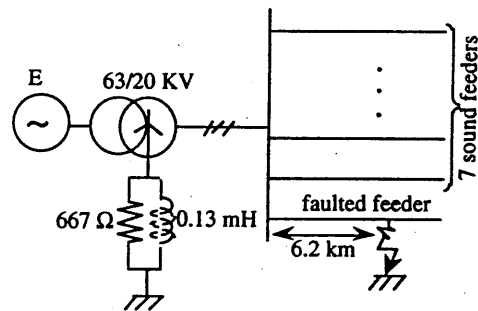
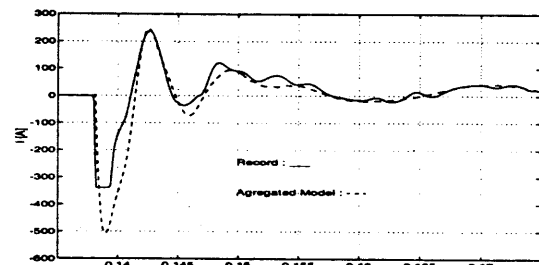


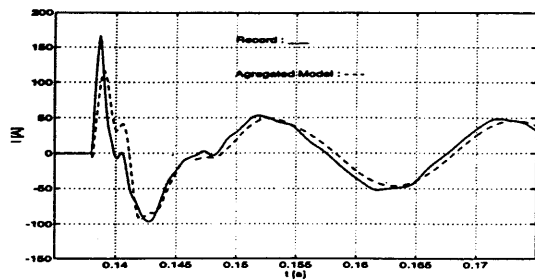
fig. 8

In the study network the ramified feeders were replaced by simple equivalent models. The aggregation was applied to each input-output pair of the state equation. Then, the models which furnish the residual current of the faulty feeder and the residual currents of each sound feeder, were determined. These currents are compared with the recorded currents \* :

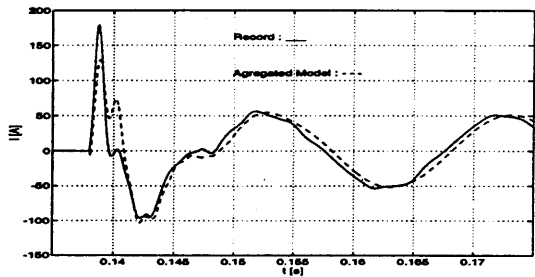


Residual current in the faulty feeder  
fig. 9

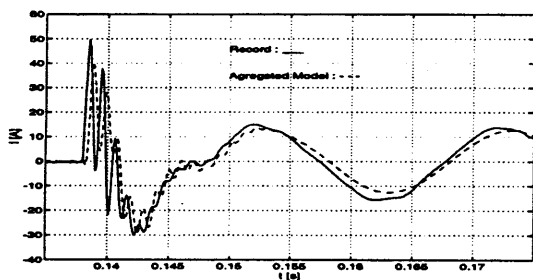
\* The saturation of the measure system introduces a non-linearity in the residual faulty feeder record.



a



b



c

Residual current in three sound feeders  
fig. 10.

## 8. CONCLUSIONS

This paper presents a method to obtain simple models that representing transients of distribution networks.

1) In complex distribution networks the problem can be decomposed as follows :

- proposition of simple models for each feeder;
- application of the aggregation method to the state equation of the network for each event.

2) Two simplified models for ramified feeders are proposed in this paper :

- a non-loaded ramified feeder can be replaced by an equivalent line. For this line the capacitance is equal to the sum of all capacitances of the feeder and the impedance is equal to the sum of the series impedances of the longest branch.
- a loaded ramified feeder can be replaced by an equivalent line with a load. For the equivalent line, the capacitance is the sum of all capacitances of the feeder and the impedance is the impedance of the first tree part. The equivalent load is then equal to the sum of all feeder

loads.

3) The comparison between aggregated models obtained in this manner with physical recordings, proves that this method is valid.

4) This method is general and can be used for any type of transient in distribution networks.

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