A Simulation of a Harmonic Current in a Building Distribution System

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Abstract The present paper proposes a simple rectifier model for a numerical analysis of harmonic currents in a building distribution system. The proposed model is stable and satisfactory accurate compared with conventional nonlinear model. Also an efficient method is proposed to analyze the harmonic characteristic by hands. It is demonstrated that the harmonic currents are reduced effectively by adopting a combination of delta-delta and delta-star transformers.

1. Introduction

A harmonic current in a distribution system shows a yearly increase with a spread of power electronic equipments. For an efficient reduction of the harmonic current, a numerical simulation is quite important. This paper investigates a diode model of Electro-Magnetic Transient Program for a harmonic analysis including a rectifier circuit and presents an approximate analysis method. Also the paper proposes an efficient and simple method for reducing the harmonic current on a distribution system in a building. A calculated result is compared with a field measurement.

2. Rectifier model for numerical simulation

Difficulty of a harmonic simulation mainly comes from a numerical instability caused by a diode model and a numerical integration rule. A diode is modeled by an ideal switch in the EMTP and its switching operation delays one time step. A small oscillation of the current or of the voltage across the diode leads to an unnecessary switching operation. It is well known that a true-nonlinear resistance model expressing a voltage-current characteristic of a diode reduces the instability. Though the diode is accurately represented by the nonlinear resistance, it requires a greater calculation time. Numerical oscillations are also caused by interruption of a current flowing through an inductor. This instability is improved using backward Euler rule instead of trapezoidal rule as in Ref. [1]. However, the improved integration scheme is not available in the ATP version of the EMTP. A simple solution for a stable analysis is an addition of a snubber circuit consisting of a series R-C circuit, to a diode in parallel. However, it is not clear how to determine the parameters of the snubber circuit. A scheme of synthesis of the circuit will be investigated in this section.

The numerical instability is observed when a diode turns off. Fig. 1 shows a schematic diagram of a diode current. At $t=t-2\delta t$, the current flows through the diode and the status of the diode is determined as "turn on" at the next time step. At $t=t-\delta t$, the reverse current is detected and the diode is treated as open circuit at the next time step ($t=t$). The voltage across the diode is determined by the outer circuit condition when the switch is opened. An inductor connected in series with the rectifier circuit causes a numerical oscillation by the current interruption. If a forward voltage is applied to the diode, the diode turns on at the next time step ($t=t+\delta t$). Thus, the parameters of the snubber circuit are determined as the negative voltage should be applied just after the diode is turned off.

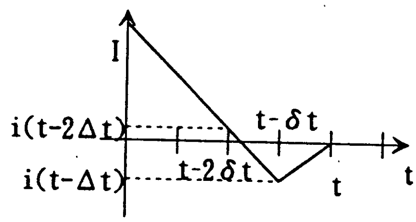
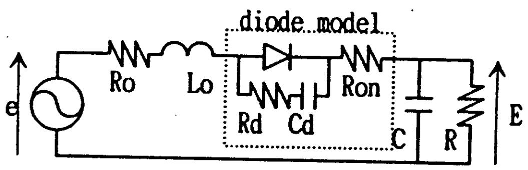
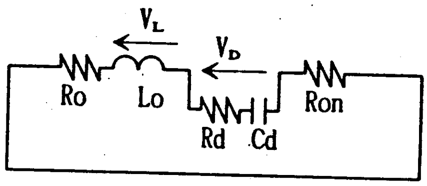


Fig. 1 Schematic diagram of diode current.



(a) Single phase half-bridge converter.
Ron:diode on-resistance.



(b) Equivalent circuit for analytical investigation.
Fig. 2 Circuit diagram.

Fig. 2(a) shows a single-phase half-bridge rectifier circuit. The source voltage and dc output voltage slightly decrease after the diode turns off. If the time step is small and the smoothing capacitance is large enough, the voltage difference is negligible. The voltage across the diode just after the diode turns off is analytically obtained by a transient analysis of a series RLC circuit illustrated in Fig. 2(b) using a z-transform. Characteristics of voltage versus current of the inductor and the capacitor are formulated by a trapezoidal rule in the EMTP.

$$\{v_L(t) + v_L(t-\delta t)\}\delta t/2 = L\{i(t) - i(t-\delta t)\}$$
$$\{i(t) + i(t-\delta t)\}\delta t/2 = C\{v_C(t) - v_C(t-\delta t)\} \cdots (2.1)$$

where $v_L(t)$:voltage across inductor,
$v_C(t)$:voltage across a capacitor

The following equations are obtained by transforming eq. (2.1) into z-domain.

$$v_L(z) = R_L\{(1-z^{-1})/(1+z^{-1})\}i(z)$$
$$v_C(z) = R_C\{(1+z^{-1})/(1-z^{-1})\}i(z) \quad \cdots\cdots\cdots(2.2)$$
where $R_L = 2L/\delta t$, $R_C = \delta t/2C_d$

If Ro+Ron << Rd, the circuit equation in z-domain is expressed as follows.

$$v_L(z) + R_d i(z) + v_C(z) = 0 \quad \cdots\cdots\cdots(2.3)$$

The voltage across the diode $v_D(t)$ is obtained from eqs. (2.2) and (2.3).

$$v_D(t) = \{-R_d^2+(R_L-2R_C)R_d+R_C(3R_L-R_C)\}i(t-\delta t) + \{(R_d+R_C)(R_d-R_L-R_C)/(R_d+R_L+R_C)\}i(t-2\delta t) + v_D(t-\delta t) \quad \cdots\cdots\cdots(2.4)$$

For a stable simulation, the voltage across the diode $v_D(t)$ should be negative. The condition is satisfied by the relation expressed by eq. (2.5) which is obtained from eq. (2.4) using the relations $i(t-2\delta t)>0$, $i(t-\delta t)<0$ and $v_D(t-\delta t)=0$.

$$R_C < 3R_L$$
$$Ro + Ron << R_d < R_L + R_C \quad \cdots\cdots\cdots(2.5)$$

Thus, the parameters of the snubber circuit are obtained.

$$C_d > \delta t^2/12L$$
$$Ro + Ron << R_d < 2L/\delta t + \delta t/2C \quad \cdots\cdots\cdots(2.6)$$

When Lo is 0.4 mH and δt is 20 μs, Cd should be greater than 1/12 μF and Rd is less than 140 Ω (C=0.1 μF).

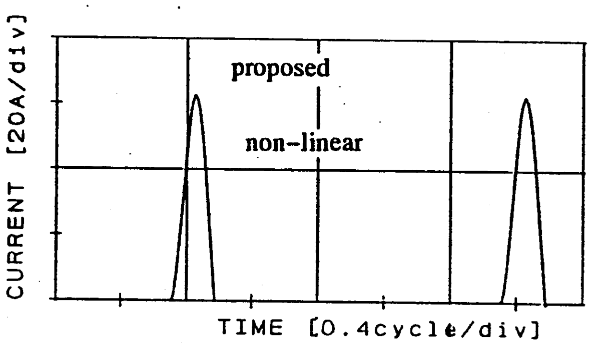


Fig. 3 Calculated results of diode current.

Fig. 3 shows calculated results by a type-92 nonlinear resistance model and by a simple diode model. From the figure, it is clear that the accuracy of the proposed simple model is satisfactory. A computation time required by the proposed model is about a half of that by the accurate model.

3. Approximate harmonic analysis method

3.1 Analytical Fourier expansion

A numerical simulation using a circuit analysis program, such as EMTP is quite valuable for an accurate investigation of a harmonic characteristic of a distribution system. Because the distribution system is complex even in a building, the simulation becomes complicated. In this section, an approximate method for a harmonic analysis will be proposed.

A three phase full-bridge rectifier circuit illustrated in Fig. 4 is typically used as a rectifier circuit in a general purpose inverter. A current waveform on the ac side is pulse-like as shown in Fig. 5(a), because a rectifier conducts in a period that a smoothing capacitor is charged. The wavefront is determined by a backward impedance (Zo) of a distribution system which is mainly determined by a leakage impedance of a transformer. Fig. 5 shows an approximation of a harmonic current composed by half sinusoidal waveforms.

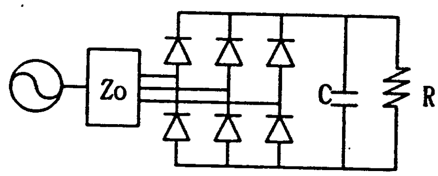


Fig. 4 Three-phase full-bridge rectifier circuit.

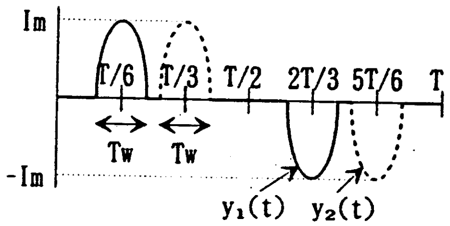


Fig. 5 A harmonic current waveform on ac side.

$$i(t) = i_1(t) + i_1(t)$$
$$i_1(t) = \text{Im}\sin(\pi t/T_w) \quad T/6-T_w/2 < t < T/6+T_w/2$$
$$i_1(t) = -\text{Im}\sin(\pi t/T_w) \quad 2T/3-T_w/2 < t < 2T/3+T_w/2$$
$$i_1(t) = 0 \quad t < T/6-T_w/2, \quad \cdots\cdots\cdots(3.1)$$
$$T/6+T_w/2 < t < 2T/3-T_w/2,\ t > 2T/3+T_w/2$$
$$i_2(t) = i_1(t-T/6)$$
where T:period, T_w:pulse width, Im:peak current

The above approximation gives analytical Fourier coefficients, and simplifies the harmonic analysis. Eq. (3.2) shows k-th Fourier sine (A_{1k}) and cosine (B_{1k}) coefficients of $i_1(t)$.

$$A_{1k} = 8\ \text{Im}\ x \sin(k\pi/3)\cos(k\pi x)/[\pi\{1-(2kx)^2\}]$$
$$B_{1k} = 8\ \text{Im}\ x \cos(k\pi/3)\cos(k\pi x)/[\pi\{1-(2kx)^2\}]$$
$$A_{1k} = B_{1k} = 0,\ k = \text{even} \quad \cdots\cdots\cdots(3.2)$$
where x=Tw/T

Fourier coefficients of $i_2(t)$ (A_{2k} and B_{2k}) are easily obtained from those of $i_1(t)$ by considering its phase difference from the $i_1(t)$.

$$A_{2k} = 8\ Im\ x \sin(2k\pi/3)\cos(k\pi x)/[\pi\{1-(2kx)^2\}]$$
$$B_{2k} = 8\ Im\ x \cos(2k\pi/3)\cos(k\pi x)/[\pi\{1-(2kx)^2\}] \quad \cdots\cdots(3.3)$$

From the above formulas, the Fourier coefficients of the current waveform i(t) are obtained.

$$A_{ak} = A_{1k} + A_{2k} = Im\ P_k\ \{\sin(k\pi/3) + \sin(2k\pi/3)\} = 2\ Im\ P_k \sin(k\pi/2)\cos(k\pi/6)$$
$$B_{ak} = B_{1k} + B_{2k} = Im\ P_k\ \{\cos(k\pi/3) + \cos(2k\pi/3)\} = 2\ Im\ P_k \cos(k\pi/2)\cos(k\pi/6) \quad \cdots\cdots(3.4)$$
$$P_k = 8\ x \cos(k\pi x)/[\pi\{1-(2kx)^2\}] \quad \cdots\cdots(3.5)$$

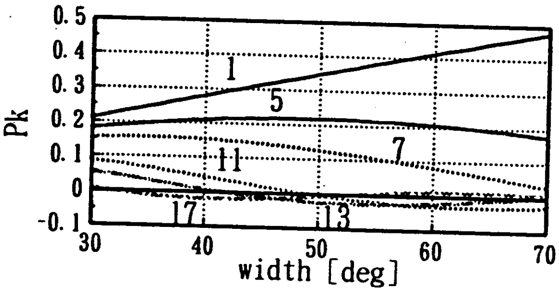
The above equation clearly shows that no third order harmonic exists.

$$A_{ak} = \pm\sqrt{3}\ P_k\ Im$$
$$B_{ak} = 0 \quad \cdots\cdots(3.6)$$
$$k = 6m\pm1 = 1, 5, 7, 11, 13, \cdots$$

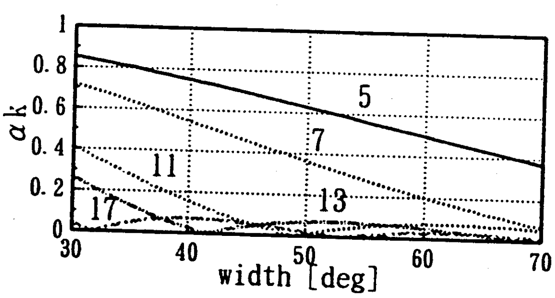
The above equation indicates that the characteristic of harmonic components generated by the rectifier circuit is determined only by the peak current Im and the function P_k, i.e., by a pulse height and a width of the current waveform. Normalized current harmonics are obtained from eq. (3.6) and are determined only by the pulse width.

$$\alpha_k = |A_{ak}/A_{a1}| = |P_k/P_1| \quad \cdots\cdots(3.7)$$

Fig. 6 shows the characteristic of P_k and α_k, by which the harmonic components can be evaluated at once.



(a) P_k

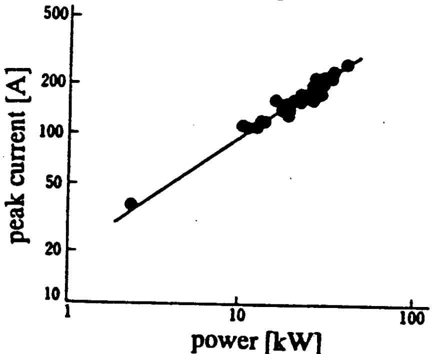


(b) α_k

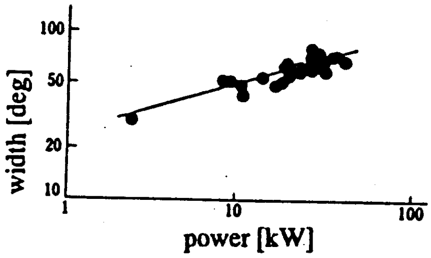
Fig. 6 Harmonic characteristics versus pulse width.

3.2 Current waveform versus power characteristic

If a current waveform of a rectifier circuit is known, a harmonic current in a distribution system can be estimated from the result of the previous chapter. It is, however, difficult to obtain the current waveforms of all the equipments installed in a building. The waveform parameters, i.e., the pulse height and width are experimentally obtained as a function of a load power in a building distribution system. Fig. 7 shows test results.



(a) Pulse-height versus power characteristic



(b) Pulse-width versus power characteristic

Fig. 7 Current waveform parameter versus power characteristic.

From the results, the following equations are obtained.

$$Im = 0.12\ P^{0.7}\ \ [A]$$
$$Tw = 2.6\ P^{0.3}\ \ [\text{deg at 60Hz}] \quad \cdots\cdots(3.8)$$

where P:load power

The above equation indicates that the product of the pulse height and the width ($Im \cdot T_w$) is proportional to load power P. The product is approximately proportional to an integral of current.. Because the source voltage waveform at the period of turn-on state is almost flat, the product expresses the load power.

Fig. 7 and eq. (3.8) show that the harmonic characteristic of any rectifier circuits installed in a building can be expressed by a function of load power. If the current waveform of a rectifier circuit is unknown, its harmonic characteristic is approximately represented by eqs. (3.6) and (3.8).

4. Parallel operation of delta-delta and delta-star three-phase transformers

The results in the previous chapter clarify that the proposed approximate method is useful for estimating a harmonic current generated by a three-phase full-bridge rectifier circuit. The low order harmonics are principal components as is well known.

A current waveform on the primary winding of a delta-delta transformer is similar to that on the secondary winding. Thus, the harmonic characteristic on the ac side is expressed as follows.

$$A'_{ak} = \pm\sqrt{3}\, n\, P_k\, Imd$$
$$B'_{ak} = 0 \qquad \cdots\cdots\cdots(4.1)$$
$$C'_{ak} = \sqrt{A'^{2}_{ak} + B'^{2}_{ak}} = \sqrt{3}\, n\, |P_k|\, Imd$$
$$k = 6m\pm1 = 1, 5, 7, 11, 13, \cdots$$

where Imd:peak current, n:voltage ratio

A current waveform on the primary winding of a delta-star transformer is determined by a difference between currents on phase-a and phase-c of the secondary winding. Considering phase angle difference between the primary and secondary windings, Fourier coefficients of the primary winding current (A''_{ak} and B''_{ak}) are obtained.

$$A_{ck} = A_{ak}\cos(2k\pi/3) - B_{ak}\sin(2k\pi/3)$$
$$= -(\pm\sqrt{3}/2)\, P_k\, Ims$$
$$B_{ck} = B_{ak}\cos(2k\pi/3) + A_{ak}\sin(2k\pi/3) \cdots\cdots(4.2)$$
$$= 3/2\, P_k\, Ims$$

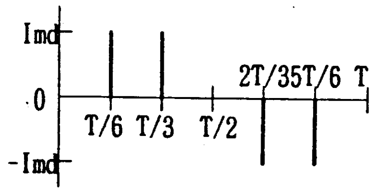
$$A''_{ak} = \{(A_{ak} - A_{ck})\cos(k\pi/6)$$
$$- (B_{ak} - B_{ck})\sin(k\pi/6)\}(n/\sqrt{3})$$
$$= \{(\pm 3/2)\cos(k\pi/6) + \sqrt{3}/2\sin(k\pi/6)\}\, n\, P_k\, Ims$$
$$= \pm\sqrt{3}\,(-1)^m n\, P_k\, Ims$$

$$B''_{ak} = \{(B_{ak} - B_{ck})\cos(k\pi/6) \qquad \cdots\cdots\cdots(4.3)$$
$$+ (A_{ak} - A_{ck})\sin(k\pi/6)\}(n/\sqrt{3})$$
$$= \{\sqrt{3}/2\cos(k\pi/6) - (\pm 3/2)\sin(k\pi/6)\}(P_k\, Ims\, n)$$
$$= 0$$
$$k = 6m\pm1 = 1, 5, 7, 11, 13, \cdots$$

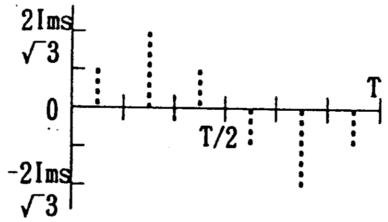
where A_{ck}, B_{ck}:phase-c Fourier sine and cosine coefficients of delta-star transformer on dc side; A''_{ak}, B''_{ak}:phase-a on ac side; Ims:peak current

Ac side currents of the delta-delta and of the delta-star transformers are schematically illustrated in Fig. 8(a) and (b), respectively. The bar in the figure expresses a pulse current waveform shown in Fig. 5. Eqs. (4.1) and (4.3) show each harmonic current is expressed only by the function P_k and the peak current Im. Also, there is no difference between their amplitudes ($|A'_{ak}|$, $|A''_{ak}|$) of the delta-delta and the delta-star transformers.

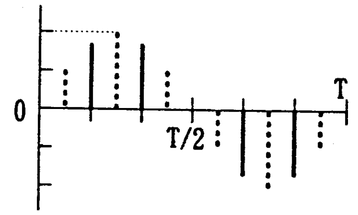
To reduce the harmonics by a filter circuit, quite large inductors and capacitors are necessary. This paper proposes a harmonic reduction method in a distribution system by a parallel operation of a delta-delta and delta-star winding transformers. This method has been commonly applied in a field of a dc transmission system which has no smoothing capacitor.



(a) Pulse current at delta-delta transformer.



(b) Pulse current at delta-star transformer.



(c) Total current.

Fig. 8 Schematic diagram of a current waveform in parallel operation.

Fig. 8(c) shows the current waveform flowing from a distribution line when the delta-delta and delta-star transformer is operated in parallel. The current is expressed as a sum of the primary currents of each three-phase transformer. It is apparent in Fig. 8 that the parallel operation of the three-phase transformers decreases the harmonics generated by the rectifier circuits with smoothing condensers.

A harmonic current in a distribution system is expressed by its Fourier coefficients A_k and B_k by the proposed method.

$$A_k = A'_{ak} + A''_{ak}$$
$$= \pm\sqrt{3}\, n\, \{P_k(T_{wd})\, Imd + (-1)^m P_k(T_{ws})\, Ims\}$$
$$B_k = (B'_{ak} + B''_{ak}) \qquad \cdots\cdots\cdots(4.4)$$
$$= 0$$
$$k = 6m\pm1 = 1, 5, 7, 11, 13, \cdots$$

If each load is balanced, i.e., Imd = Ims ≡ Im and $T_{wd} = T_{ws} \equiv T_w$, eq. (4.4) is simplified to eq. (4.5).

$$A_k = \pm 2\sqrt{3}\, n\, P_k(T_w)\, Im$$
$$B_k = 0 \qquad \cdots\cdots\cdots(4.5)$$
$$k = 12m\pm1 = 1, 11, 13, \cdots$$

The above equation indicates that 6m±1-th harmonics are eliminated by the parallel operation, if each load of the transformer is balanced.

A normalized harmonic current is easily obtained by eq. (4.5). Fig. 9 shows a harmonic characteristic versus a load ratio of each transformer. A comparison of Fig. 9

with Fig. 6(b) proves an effectiveness of the proposed harmonic elimination method. If the loads are balanced within 20%, 5th harmonic current is reduced from 60% to 10% in the case of T_w=60°.

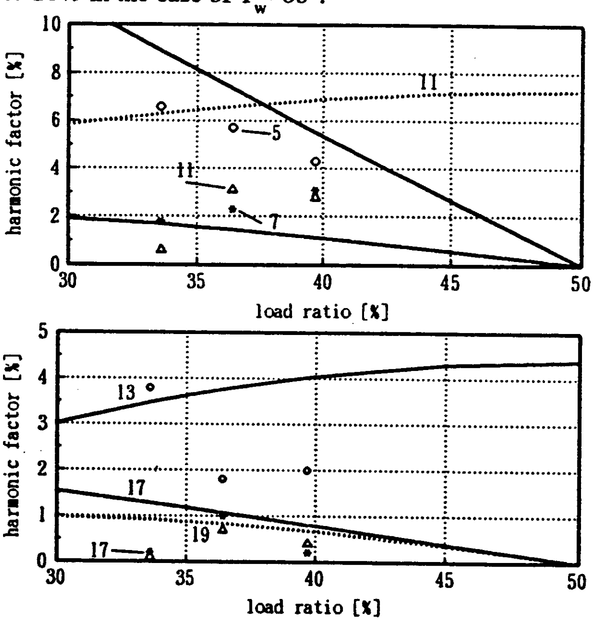


Fig. 9 Normalized harmonic current versus load ratio.

5. Harmonic current reduction in a building

This section explains practical application of the proposed harmonic reduction method using a delta-delta and a delta-star transformer in a distribution system of a building. The low order harmonics produced by a rectifier circuit are eliminated by an inherent characteristic of a six-phase circuit. Thus, a key of a designing a building distribution system is an equilibration of loads.

Fig. 10 illustrates the harmonic reduction method applied to an actual system. The building has a generator for feeding electric energy and heat because the electric fee of Japan is quite expensive (17 to 30 US cents/kWh). The base loads of the generator are pumps and fans for air-conditioning. These loads are driven by inverters for efficient air-conditioning and for saving energy. The amount of the rectifier loads among the total load of the generator reaches 50%. Thus, harmonic reduction becomes quite important for a countermeasure against a damage of the generator and the other equipments. Because the inverters for air-conditioning are dispersedly installed within a building, a grouping method is quite effective. A distribution system of a building is conventionally divided into several sections only from a view point of a construction designing. A customary zoning of a building which is divided into two sections by a passage way is illustrated in Fig. 11(a). It is easily estimated that the load pattern of the northern part of the building is far different from that of the southern part. More electric power and heat are consumed in the northern part in winter. If the power for fans or pumps is fed by two circuits into these zones, the peak power is quite different from each other. An energy consumption of an inverter controlled air conditioner installed in a southern area of a building is different from that in a northern area. Thus, the zoning method shown in Fig. 11(b) is efficient rather effective than the conventional zoning to balance the loads for efficient harmonic reduction.

Figs. 12, 13 and Table 1 show a field measurement and a simulated result in a building distribution system. It is clear that the proposed method is quite efficient for reducing 5-th and 7-th harmonics.

6. Conclusions

The proposed simple rectifier model gives a stable solution with half the calculation time of an accurate nonlinear model. Thus, a circuit which includes many converters such as a building distribution system can be easily simulated by the proposed model.

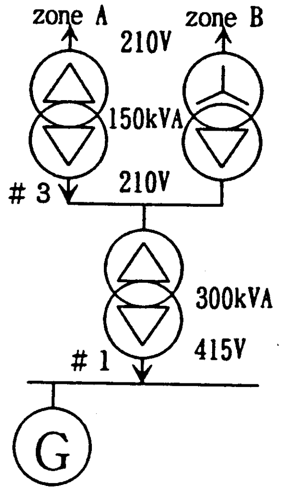
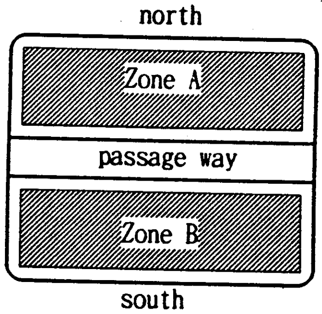
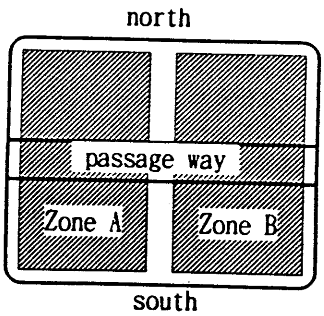


Fig. 10 A distribution system.

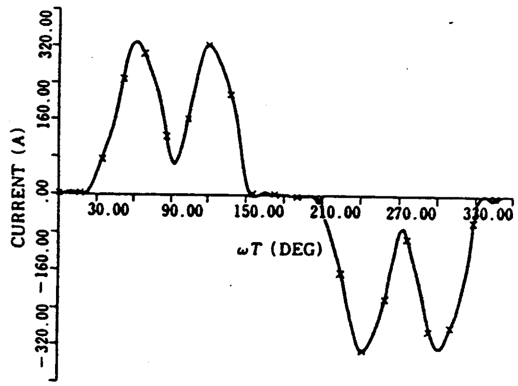


(a) Conventional

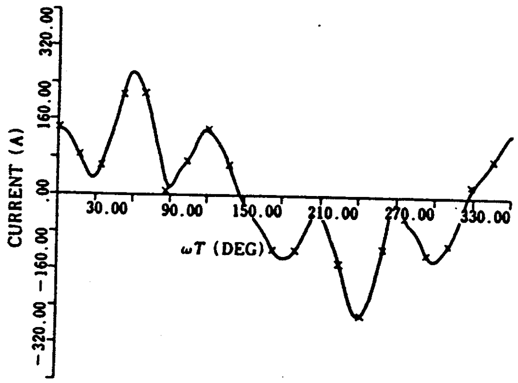


(b) Proposed zoning

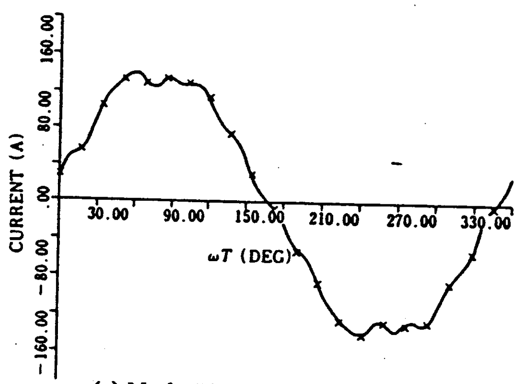
Fig. 11 Zoning method.



(a) Node #3 current waveform

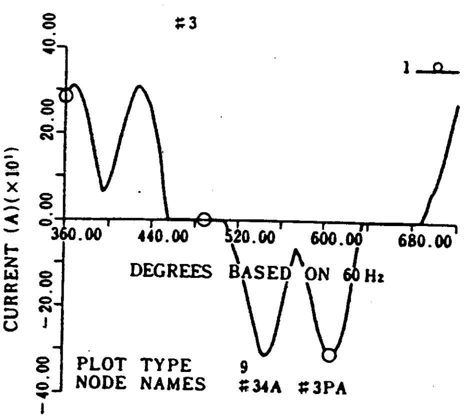


(b) Node #4 current waveform

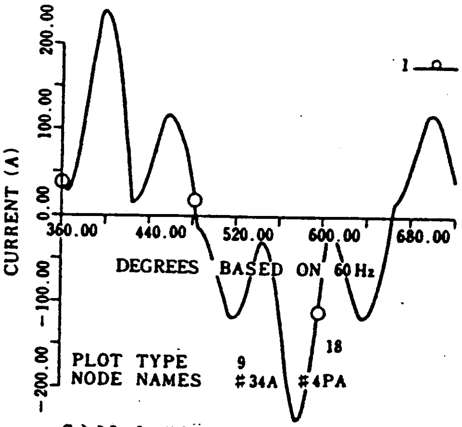


(c) Node #1 current waveform

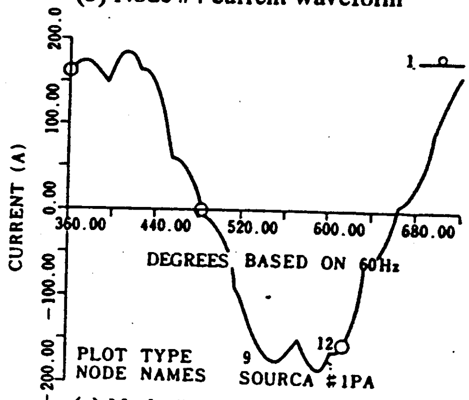
Fig. 12 Field test results.



(a) Node #3 current waveform



(b) Node #4 current waveform



(c) Node #1 current waveform

Fig. 13 Calculated results by EMTP.

Table 1 Harmonic current of a building distribution system

	#3 current[A]		#1 current[A]	
order	test	EMTP	test	EMTP
1	164.7	156.4	99.2	127.7
3	2.5	0.2	1.4	0.1
5	73.7	71.6	5.0	12.0
7	32.1	28.4	2.5	2.8
9	1.3	0.0	0.2	0.0
11	6.0	6.6	1.1	5.2
13	6.2	2.8	3.3	2.6
15	1.1	0.0	0.2	0.0
17	5.9	3.8	0.9	0.7

An approximate harmonic analysis method is quite useful for estimating a harmonic characteristic at a design stage of the building distribution system. Harmonics are satisfactorily reduced by the proposed method using delta-delta and delta-star transformers. The above is confirmed by a field measurement and a numerical simulation by the EMTP.

7. References

(1) J.R.Marti and J.Lin, "Suppression of numerical oscillations in the EMTP," IEEE Trans. on Power Systems, Vol. 4, No. 2, pp. 739-747, 1989

(2) N.Nagaoka and H.Imai, "Harmonic current simulation of a distribution System within a building," J. of Inst. Elect. Installation Eng. Japan, Vol. 11, No. 2, pp. 99-109, 1991