AN EFFICIENT METHOD TO DEAL WITH BOUNDARY CONDITIONS IN AN ELECTROMAGNETIC TRANSIENT ANALYSIS

A.Ametani
Doshisha University, Tanabe-cho, Kyoto 610-03, Japan

Abstract

The paper proposes an efficient method to deal with a number of boundary conditions along a distributed-parameter line. The boundary conditions are expressed as admittances between the line and the earth, and are combined with the well-known shunt admittance of the line. Thus, a nonhomogenous line composed of a number of boundary conditions is represented as an equivalent homogenous line. The proposed method greatly simplifies a transient analysis on transmission or communication lines, and is very stable and highly efficient in a computer analysis.

The model is applied to an overhead line with a grounded earth wire, a cleated overhead cable and an underground cable with sheath grounding. Calculated examples show a satisfactory accuracy.

Key words : distributed parameter, grounding, cable, boundary condition, transient

1. INTRODUCTION

An actual distributed-parameter line involves a number of boundary conditions which causes a difficulty and complication in an analysis of steady-state and transient characteristics of the line. For examples, an overhead line is composed of a number of towers or poles through which earth wires are grounded. In the steady-state and switching surge analysis, the towers are neglected by assuming that the earth wire voltage is the same as the earth voltage. It, however, can not be neglected in the analysis of a power line carrier and a lightning surge, and requires a tedious calculation/1/. Cable sheaths are grounded at every insulated junction. Also, a cable is fixed quite often by cleats for every few meters. The analysis of such the cable is troublesome and only few works have been carried out/2,3/.

The paper presents a simple and efficient model to deal with a number of boundary conditions which have a distributed nature. The basic idea of the model is that the boundary conditions can be represented as a part of a shunt admittance of a distributed-parameter line, when the number of the boundary conditions is large and each has the same linear characteristic as the other.

The proposed model is applied to an overhead line with a grounded earth wire, a cleated cable and an underground cable with a sheath grounding. Frequency responses of the input impedances are evaluated by the model and compared with the accurate results. Also, an electromagnetic transients are calculated using the proposed model, and its accuracy and efficiency are investigated. Transients calculations are carried out by the EMTP/4/.

2. THEORY OF DISTRIBUTED ADMITTANCE MODEL

Let's consider a distributed-parameter line with a number of boundary conditions illustrated in Fig.1(a). A circuit between nodes 'K-1' and 'K' is rewritten in Fig.1(b), when the distributed line is represented by a PI equivalent of lumped parameters. The total admittance Y between the nodes 'K-1' and 'K' is given by:

$$Y = Y' + Yb : \text{total admittance} \quad (1)$$

where Y' : shunt admittance of distributed line
Yb = Gb+jωCb : admittance representing a boundary condition

If a T equivalent of lumped parameters is adopted, the above equation is given at the node K.

The series impedance Z is not changed by the boundary condition in this case. Then, the circuit of Fig.1(b) involving the boundary conditions is represented by an equivalent distributed line with the series impedance Z and the shunt admittance Y given in eq.(1), but with no boundary condition. In other words, the boundary condition Yb is included into the shunt admittance Y of the equivalent homogeneous line as a distributed admittance. The approach may be called 'distributed admittance model'.

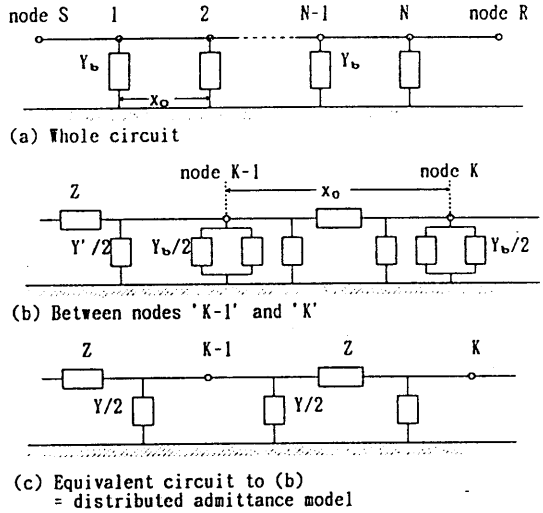


Fig. 1 A distributed-parameter line with N boundary conditions and an equivalent line with distributed admittance

The proposed model is based on the well-known PI equivalent which is an approximation of a distributed-parameter line. A more rigorous and accurate approach of dealing with a number of the boundary conditions has been developed by Wedepohl in ref.(1). It, however, is far more complicated and requires a tedious procedure to obtain an equivalent homogeneous line.The proposed model is quite simple and can be easily implemented into any computer programs of line/cable parameters and transients calculations.

3. FORMULATION OF LINE/CABLE PARAMETERS

The theory of the distributed admittance model is quite simple and is easily formulated in the overhead line case. But, its formulation for a cable is not straightforward, because Yb in Fig.1 appears not only to the earth but also between core and sheath conductors of the cable as illustrated in Fig.2. In the figure, Y1' and Y2' are the admittance of the cable itself, and Yb1 and Yb2 are the admittance per unit length representing the boundary conditions. Then, the total admittances are given by:

$$Y1 = Y1' + Yb1 : \text{core-to-sheath}$$
$$Y2 = Y2' + Yb2 : \text{sheath-to-earth} \quad (2)$$

In the case of an overhead line (=overhead cable only with a core) or no boundary condition between the core and the sheath, Y1 is deleted from eq.(2), and thus only the admittance to the earth is to be considered.

For the admittance Y1 and Y2 correspond to a modal component, i.e.Y1 is the coaxial (core-to-sheath) mode component and Y2 the earth-return (sheath-to-earth) mode component, those are to be converted to phasor components. The formulation of a phasor capacitance matrix [Ci] or corresponding potential coefficient matrix [Pi] has been explained in ref.(5). Let's assume that the admittances are composed only of capacitances. Then, the potential coefficient matrix is given in the following form for a single-phase cable illustrated in Fig.2.

$$[P_i] = [P'] + [P_b] = \begin{bmatrix} p1+p2 & p2 \\ & p2 \quad p2 \end{bmatrix} \qquad (3)$$

where $[P'] = \begin{bmatrix} p1'+p2' & p2' \\ p2' & p2' \end{bmatrix}$, $[P_b] = \begin{bmatrix} p1b+p2b & p2b \\ & p2b \quad p2b \end{bmatrix}$

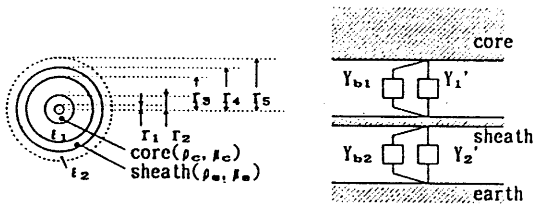
and $pm = 1/Cm$, $Ym = j\omega Cm$; $m = 1, 2, 1b$ and $2b$

subscript 'i' for phase 'i'

For an n-phase cable, the overall potential coefficient matrix [P] is given by:

$$[P] = \begin{bmatrix} [P1] & [0] & \cdots & [0] \\ [0] & [P2] & \cdots & [0] \\ \vdots & \vdots & \ddots & \vdots \\ [0] & [0] & \cdots & [Pn] \end{bmatrix} \qquad (4)$$

where submatrix [Pi] is given in eq.(3).



(a) Cross-section (b) An equivalent circuit of cable admittance

Fig.2 A cable with boundary conditions of core-to-sheath and sheath-to-earth

In the case of an overhead cable and an overhead line, the potential coefficient matrix of the cable system in the air has to be added to the above matrix as explained in ref.(5).

Having obtained the potential coefficient matrix, the capacitance matrix [C] is given in the following form.

$$[C] = [P]^{-1} \qquad (5)$$

When the admittances are composed of conductances and capacitances, the conductance matrix [G] has to be formulated in the same manner as the formulation of the capacitance matrix explained above. Finally, the overall admittance matrix [Y] is formulated in the following equation.

$$[Y] = [G] + j\omega[C] \qquad (6)$$

The above admittance matrix has taken into account the effect of the boundary conditions as an equivalent distributed admittance.

4. APPLICATION EXAMPLES

4.1 Overhead Line with Grounded Earth Wire

As the first example of the distributed admittance model, an input impedance and a transient voltage will be discussed for an overhead line with an earth wire(GW) grounded through resistance Rg=30[Ohm] illustrated in Fig.3.

The input impedance of the earth wire at node G0 in Fig.3(a) is first investigated. The surge impedances and propagation voltages of the earth wire itself and the equivalent line considering the grounding resistance obtained by the distributed admittance model are evaluated at 4MHz as:

earth wire :Zo'=553.9 Ohm, v'=298.5m/us
equivalent line:Zo =180.8 Ohm, v =165.2m/us
(Gb=1/Rg·xo=0.833E-3 S/m)

It is observed that the surge impedance and the propagation velocity are decreased by distributed conductance Gb representing the grounding resistance Rg. This is easily understood from the following formulas.

$$Zo = \sqrt{Z/Y}, \ \Gamma = a + jb = \sqrt{Z \cdot Y}, \ v = b/\omega \qquad (7)$$

where $Y = j\omega C$ for the case of no grounding resistance

$Y = Gb + j\omega C$, $Gb = 1/(Rg \cdot xo)$ for considering the grounding resistance as a distributed admittance

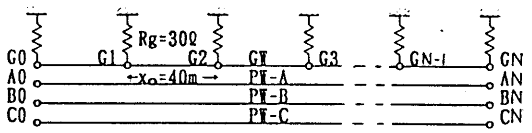
Fig.4(a) shows the accurate solution of the input impedance obtained from the GW circuit of Fig.3(a). Fig.4(b) is the calculated result in the simplified circuit illustrated in Fig.3(b), where the distributed admittance model is applied to the earth wire sections G1 to GN. No difference is apparent between the curves (a) and (b) in Fig.4. The result has proved the high accuracy of the distributed admittance model. In the figure, a sudden decrease of the input impedance is observed at every 3.75MHz. This corresponds to the frequency of the standing wave of which half the wave length equals to the span length xo of the earth wire, i.e.

$\lambda/2 = xo = 40m$, $f = co/\lambda = 3E8/80 = 3.75MHz (E8=10^8)$

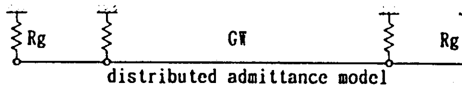
When the earth wire is represented by a PI equivalent of lumped parameters rather than a distributed parameter line, the input impedance becomes nearly constant as observed in the curve(c) in Fig.4. Thus, the distributed admittance model is said to be more accurate than the PI equivalent representation.

No difference is also observed between the accurate solution and calculated results by the distributed admittance model for the input impedance of a phase wire although the figures are not given in the paper.

Fig. 5 shows transient voltage at nodes AN and GN when the number of sections "N" is taken as 64 in Fig. 3. A unit step voltage is applied to node A0 through an inductance of 1mH representing a bus. The sections G0-G1 and GN-1-GN are represented by a PI equivalent circuit (simplified distributed admittance model). The simplified



(a) Original circuit



(b) Simplified model of earth wire with distributed admittance model

Fig.3 A 3-phase overhead line with a grounded earth wire

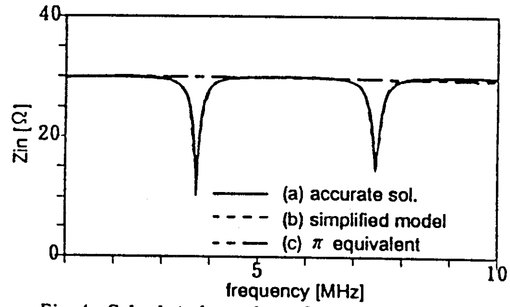
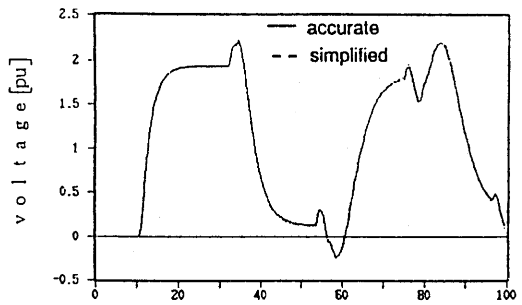


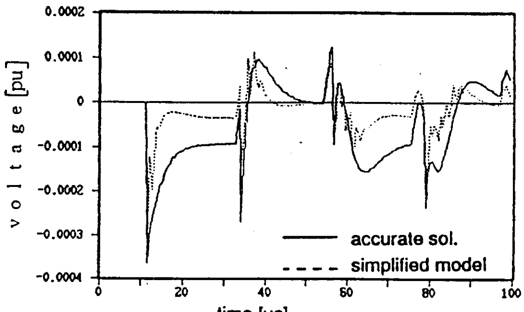
Fig.4 Calculated results of the input impedance at node G0 only with the earth wire

model shows a high accuracy compared with the accurate solution. The computation time was 258.52sec for the accurate solution and 18.57sec for the distributed admittance model with Δt=5ns and Tmax=100 μs by a COMPAQ-XE466 computer. Thus, with the same time step, the computation time is reduced to 1/14. The memory required for the computation is reduced in the same order as the computation time. The time step can be increased easily by 20times without a loss of the accuracy by the distributed admittance model because of no boundary condition. Then, the computation time was reduced to 1.26sec which was 1/200 of that required by the accurate solution.

The above observation has made it clear that the proposed model is very efficient and accurate enough.



(a) node AN



(b) node GN

Fig.5 Transient voltages at node AN and GN

4.2 Cleated Cable

A cable is quite often fixed by cleats at every few meters, and the cleats behaves electrically as capacitances to the earth. The effect of the cleats on a cable transient is, in general, neglected, because the number of the cleats is enormous and a transient calculation becomes too tedious and time consuming/2,3/. The distributed admittance model might be an efficient approach to take into account the effect of the cleats.

Fig.6 shows a model circuit of a gas-insulated cleated cable of which the surge impedance and propagation velocity of the coaxial mode at 10MHz are :

without cleat : Zo'=245.4 Ohm, v'=299.7 m/us
(C'=13.6pF/m)

with cleat : Zo =109.3 Ohm, v =133.4 m/us
(Cb=55pF/m)

The surge impedance is easily evaluated in the following equation.

$$Zo' = \mathrm{real}[\sqrt{Z/j\omega C'}]$$
$$Zo = \mathrm{real}[\sqrt{Z/j\omega(C'+Cb)}] = Zo'/\sqrt{1+Cb/C'} = Zo'/2.245 \qquad (8)$$

Eq.(8) agrees well with the values given above. The capacitance of the cleat decreases the surge impedance and the propagation velocity, and increases the attenuation constant. It is thus expected that wave distortion of a transient voltage is increased by the cleats of a cable.

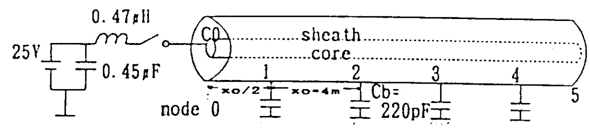


Fig.6 A cleated cable

Fig.7 shows the input impedance of the core at node C0 in Fig.6. (a) is the case of no cleat, and shows the peak of the input impedance at every frequency corresponding to half the wave length $\lambda'/4$ of the standing wave on the cable given by:

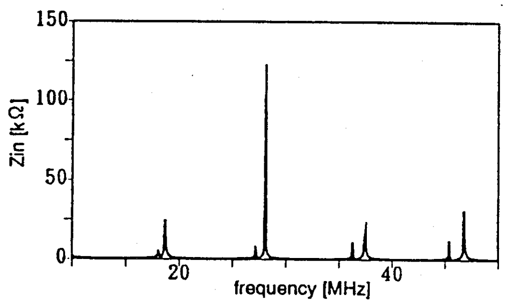
$\lambda'/2$ = 16m, fo' = v'/λ' = 9.4MHz

Fig.7(b) shows the accurate solution of the input impedance obtained from the circuit of Fig.6. Due to the cleat capacitance to the earth, the input impedance takes its peak only at the frequency fo corresponding to half the wave length.

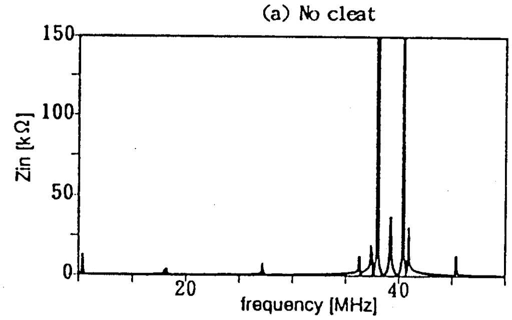
$\lambda/2$ = 4m, fo = v'/λ= 37.5MHz

Fig.7(c) is the calculated result by the distributed admittance model in which the first(node 1) and the last(node 4) cleat capacitances are installed as it is, and the cable from node 1 to 4 is represented by the distributed admittance model. The result in Fig.7(c) clearly shows the overlapped characteristic of Fig.7(a) and (b), i.e. it shows the impedance peaks at frequencies corresponding to fo' and fo given above. Therefore, it is concluded that the distributed admittance model can represent the boundary conditions at nodes 2 and 3 quite well.

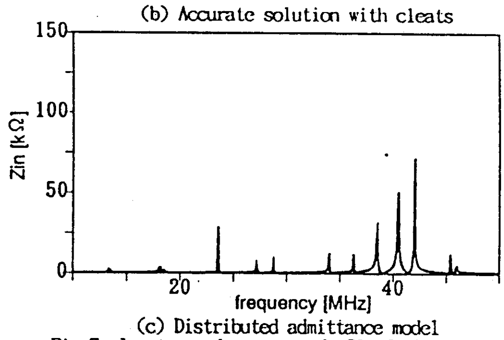
Fig.8 shows calculated results of a transient voltage on the sheath at the receiving end in Fig.6. (a) is the case of neglecting the cleats. (b) is the accurate solution on the cleated cable, and (c) the



(a) No cleat



(b) Accurate solution with cleats



(c) Distributed admittance model

Fig.7 Input empedance at node C0 of cleated cable

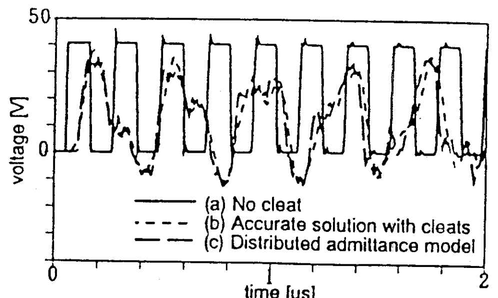


Fig. 8 Transient sheath voltage at the receiving end on the cleated cable

result by the distributed admittance model. The oscillating period of the transient voltage is about doubled and the maximum voltage is reduced by about 10% by the cleats. The calculated result of the sheath voltage by the distributed admittance model in Fig.8(c) shows a satisfactory agreement with the accurate solution of Fig.8(b). A better agreement is observed for a transient voltage on the core although the results are not given in the paper.

4.3 Underground Cable with Sheath Grounding

The sheath conductor of a coaxial cable is in most cases grounded through resistance Rg at every hundreds meter as illustrated in Fig.9. The distributed admittance model is applied to simplify a transient calculation on the cable, and calculated results are shown in Fig.10 when a unit step voltage is applied to the phase 'A' core at the sending end. No difference is observed for the core voltage at the receiving end between the accurate solution and the result obtained by the distributed admittance model, though a minor difference is observed for the sheath voltage. It is thus concluded that the distributed admittance model is effective to calculate a cable transient with sheath grounding.

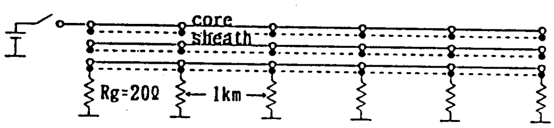


Fig. 9 A 3-phase underground cable

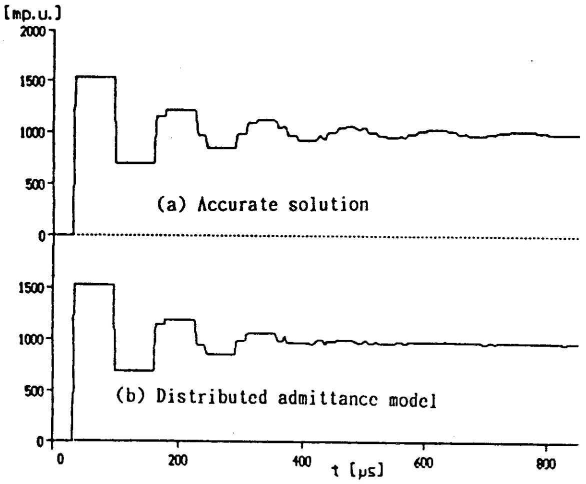


Fig. 10 Transient core voltavge at the receiving end on the undergroud cable

5. CONCLUSIONS

A distributed admittance model to deal with a number of boundary conditions have been proposed, and its application to an overhead line with a grounded earth wire, a cleated cable, and an undergrounded cable with grounded sheaths have been demonstrated. Application examples have proved a satisfactory accuracy of the proposed model provided that the first and/or the last boundary condition is considered as it is.

By approximating the first and/or last section of a line with a PI or T equivalent of lumped-parameter circuits, a time step of a transient calculation can be taken by about 'N' times greater in the circuit represented by the distributed admittance model than that determined by the line length between 2 adjacent boundaries in the original circuit with 'N' boundary conditions. Thus, a required memory and a CPU time are reduced to be about '1/N'. This makes the proposed model to be very efficient to deal with a number of boundary conditions in a transient calculation.

The present paper has given only a basic idea of the distributed admittance model which simplifies a transient calculation involving a number of boundary conditions, and thus requires a further investigation of its application, especially an application to a nonlinearity with a distributed nature. The model, however, is expected to be applicable to various boundary condition problems as an efficient approach, and has been implemented into the EMTP CABLE CONSTANTS. Also, it is noteworthy that a series-connected boundary condition can be handled in the same manner by representing the boundary condition with a distributed impedance rather than the distributed admittance.

6. ACKNOWLEDGEMENTS

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