The Time Constant Method in Power System Dynamics Simulation

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Abstract - The objective of the paper is a presentation of a new approach in dynamic power system simulation. The time constant method (TCM) combines dynamic modeling with steady state analysis methods. Contrary to the EMTP solution system equations are transformed within the continuous time domain into a suitable form for simulation. So the TCM is not restricted to a certain numerical integration algorithm. The effort in the node voltage calculation is reduced, because the network branches are modeled not only by admittances but also by impedances.

Keywords: Dynamic Power System Simulation, Power System Transients, EMTP-method, Time Constant Method (TCM).

I. INTRODUCTION

An electrical power network is a dynamic continuous time system, described by differential and algebraic equations. For simulation these equations have to be converted into a suitable form, which allows a solution by numerical integration. The standard approach for this procedure is the state variable method, known from system theory. But the automatic generation of linearly independent differential equations is difficult to realize.

Therefore, the simulation of transients in electrical power systems is usually carried out by the difference admittance method [1], which is applied in well known programs like EMTP [2] and NETOMAC [3]. The differential equations of the network are transformed by means of the trapezoidal rule into difference equations. In the discrete time domain the system equations can be handled easier than in the original continuous time domain. All node voltages become calculable and all branches of the network can be independently integrated.

The difference admittance method, also called EMTP-method [4], produces many system variables, which have no physical meaning in the power systems. For that reason the signal flow of the continuous time network is often difficult to understand. This disadvantage can be avoided by means of the time constant method (TCM). The TCM imitates the EMTP method, but works in the continuous time domain [5, 6]. The differential equations of the network are converted into a suitable form for simulation without being linearly independent. In the continuous time domain all system variables can be physically interpreted. This paper presents theoretically the TCM, which is an alternative to the well known EMTP-method in power systems dynamics simulation.

II. BASICS OF THE TCM

A. Basic idea

The basic philosophy of the time constant method (TCM) is explained by means of a simple electrical network with two nodes (Fig. 1a). For dynamic simulation all RL-elements have the same time constant T_0 (Fig. 1b):

$$T_0 = \frac{L_{12}}{R_{12}^0} = \frac{L_{20}}{R_{20}^0} \tag{1}$$

The variable T_0 can be provided with any value greater than zero [5, 6]:

$$0 < T_0 \leq \infty \tag{2}$$

In Fig. 1b additional voltage source u_{12}^* and u_{20}^* are introduced into the network. These sources describe the voltage drops across the resistances R_{12}^* and R_{20}^*, which are necessary to meet the correct system parameters:

$$R_{12}^* = R_{12} - R_{12}^0 = R_{12} - \frac{L_{12}}{T_0} \tag{3}$$

$$R_{20}^* = R_{20} - R_{20}^0 = R_{20} - \frac{L_{20}}{T_0} \tag{4}$$

$$u_{12}^* = R_{12}^* \, i_{12} \tag{5}$$

$$u_{20}^* = R_{20}^* \, i_{20} \tag{6}$$

Due to their same time constants the RL-elements of Fig. 1b form a frequency independent voltage divider. Therefore, the input voltages of the RL-circuits u_{12}^0 and u_{20}^0 can be described as a function of the known node voltage u_1 and the auxiliary voltages u_{12}^* and u_{20}^*, depending on the reactor currents (see (5) and (6)):

$$\begin{bmatrix} u_{12}^0 \\ u_{20}^0 \end{bmatrix} = \frac{1}{L_{12}+L_{20}} \begin{bmatrix} L_{12}\left(u_1 - u_{12}^* - u_{20}^*\right) \\ L_{20}\left(u_1 - u_{12}^* - u_{20}^*\right) \end{bmatrix} \quad (7)$$

Due to (7) the RL-circuits can be integrated independently of each other:

$$\begin{bmatrix} \dot{i}_{12} \\ \dot{i}_{20} \end{bmatrix} = -\frac{1}{T_0} \begin{bmatrix} i_{12} \\ i_{20} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{12}} & 0 \\ 0 & \frac{1}{L_{20}} \end{bmatrix} \cdot \begin{bmatrix} u_{12}^0 \\ u_{20}^0 \end{bmatrix} \quad (8)$$

The equations (7) and (8) describe together with (5) and (6) the TCM-simulated network. The first order circuit of Fig. 1a is represented by a second order system with the eigenvalues

$$\lambda_1 = -\left(R_{12}+R_{20}\right)/\left(L_{12}+L_{20}\right) \quad (9)$$

$$\lambda_2 = -1/T_0 \quad (10)$$

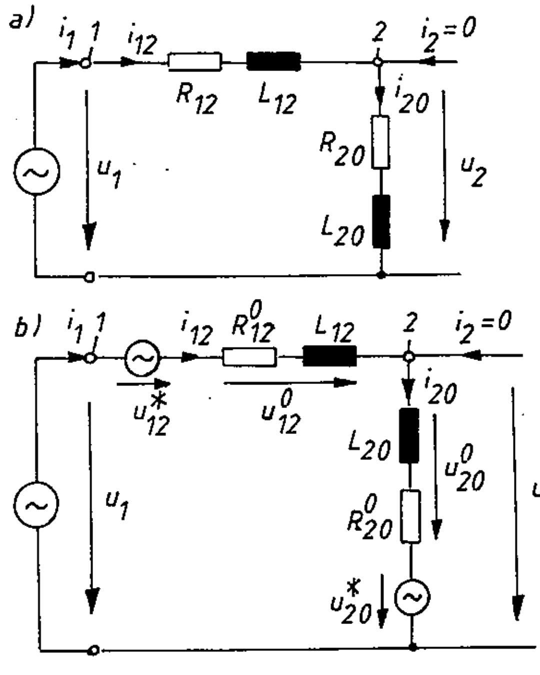


Fig. 1: Simple RL-network
a) circuit diagram
b) circuit diagram for TCM-calculation

The variable λ_1 is corresponding to the correct system eigenvalue, while variable λ_2 can be interpreted as a calculation variable, which does not really exist in the network. But the value of λ_2 influences the stability of the numerical integration. Therefore, a good choice of the time constant T_0 seems to be $T_0 \to \infty$.

B. Structure of system equations

In the following, for all RL-elements a time constant $T_0 = \infty$ is assumed. Then in Fig. 1b the resistors R_{12}^0 and R_{20}^0 are eliminated $\left(R_{12}^0 = R_{20}^0 = 0\right)$ and the auxiliary voltage sources are determined by

$$u_{12}^* = R_{12}\, i_{12} \quad (11)$$

$$u_{20}^* = R_{20}\, i_{20} \quad (12)$$

Under these conditions the network equations are given by [6]:

$$\begin{bmatrix} \dot{i}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} L_{12}^{-1} & -L_{12}^{-1} \\ -L_{12}^{-1} & L_{12}^{-1}+L_{20}^{-1} \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} -L_{12}^{-1} & 0 \\ L_{12}^{-1} & -L_{20}^{-1} \end{bmatrix} \cdot \begin{bmatrix} u_{12}^* \\ u_{20}^* \end{bmatrix} \quad (13)$$

or

$$\underline{\dot{i}}_n = \underline{L}_i\, \underline{u}_n + \underline{L}_i^*\, \underline{u}^* \quad (14)$$

$\underline{\dot{i}}_n$ differentiated node current vector
$\underline{u}_n$ node voltage vector
$\underline{u}^*$ auxiliary voltage vector
$\underline{L}_i$ matrix of inverse inductances
$\underline{L}_i^*$ auxiliary matrix of inverse inductances

The matrices $\underline{L}_i$ and $\underline{L}_i^*$ can be determined in a systematical way using the knowledge of steady state network equations for constructing. The rules for building up the matrix $\underline{L}_i$ are identical with the rules for forming the nodal admittance matrix, when replacing admittances by inverse inductances:

$$\underline{L}_i = \underline{K}\, \underline{L}_b^{-1}\, \underline{K}^T \quad (15)$$

$$\underline{L}_i^* = -\underline{K} \cdot \underline{L}_b^{-1} \quad (16)$$

$\underline{K}^T$ bus incidence matrix
$\underline{L}_b$ branch inductances matrix

In the network of Fig. 1b the matrices $\underline{K}^T$ and $\underline{L}_b$ are given by

$$\underline{L}_b = \begin{bmatrix} L_{12} & 0 \\ 0 & L_{20} \end{bmatrix} \qquad (17)$$

$$\underline{K}^T = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \qquad (18)$$

Out of (13) the unknown node voltage u_2 can be determined:

$$u_2 = \left(L_{12}^{-1} + L_{20}^{-1}\right)^{-1} \cdot \left(L_{12}^{-1}\, u_1 - L_{12}^{-1}\, u_{12}^{*} + L_{20}^{-1}\, u_{20}^{*}\right) \qquad (19)$$

In the general case, the voltages in (19) are vectors and the inductances are matrices.

With the node voltages also the branch voltages are calculable:

$$\underline{u}_b = \underline{K}^T\, \underline{u}_n \qquad (20)$$

or

$$\begin{bmatrix} u_{12} \\ u_{20} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (21)$$

$\underline{u}_b$ branch voltage vector

Now, the RL-circuits are dynamically decoupled, because their input voltages are known. Fig. 2 shows the structure of the TCM-simulated system. The voltages of the RL-branches are calculated by a static network SN1. This block depends on network topology and system parameters and can simply be determined by matrix operations, known from steady state network analysis. The procedures, which have been carried out, are very similar to those of the EMTP-method [1].

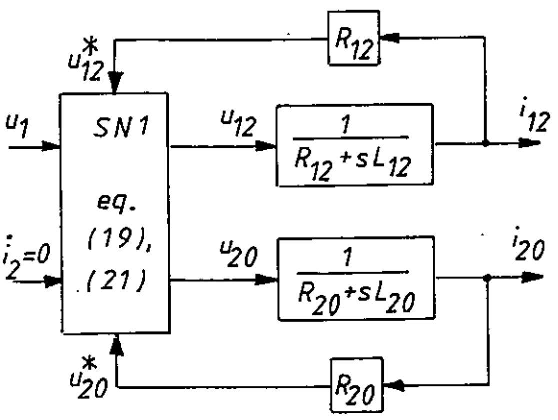


Fig. 2: TCM-simulated system of Fig. 1 ($T_0 = \infty$)

III. CHARACTERISTICS OF THE TCM

A. Linear RLC-branches

An electrical power system can be described by an equivalent circuit with ohmic resistors, capacitors, inductances, voltage and current sources. For simulation the RLC-network is decomposed into elementary branches of zero and first order. In Table 1 the different types of elementary branches are listed.

Using the TCM only the L- and series RL-circuits are represented by admittances. An admittance model is characterized by an input voltage and an output current. The other circuits (Table 1) use an impedance representation with an input current and an output voltage [6].

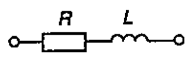
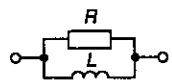
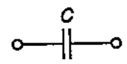
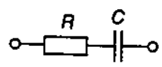
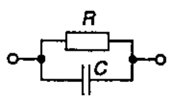
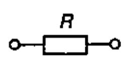
branch	model
	$Y = \frac{1}{sL}$
	$Y = \frac{1}{R + sL}$
	$Z = \frac{sRL}{R + sL}$
	$Z = \frac{1}{sC}$
	$Z = \frac{1 + sRC}{sC}$
	$Z = \frac{R}{1 + sRC}$
	$Z = R$

Table 1: Modeling of elementary branches

Fig. 3 shows the structure of a TCM-simulated linear RLC-network. The network is decomposed into the elementary branches, which are modeled by their transfer functions. According to the system topology the branches are connected via two static nets SN1 and SN2. The block SN1 was already discussed in section II. It calculates the input voltages $\underline{u}_{b1}$ and $\underline{u}_{b2}$ of the admittance branches depending on

- the known network voltages $\underline{u}$,
- the auxiliary voltages $\underline{u}^*$ and
- the output voltages $\underline{u}_{b3}$ to $\underline{u}_{b7}$ of the impedance branches.

The static network SN2 consists of an incidence matrix and determines the input currents $\underline{i}_{b3}$ to $\underline{i}_{b7}$ of all impedance elements. These variables depend on

- the output current $\underline{i}_{b1}$ and $\underline{i}_{b2}$ of the admittance branches and
- the known current sources $\underline{i}$ in the network.

The structure in Fig. 3 is characterized by a clear separation between a static part, which defines the topology of the power system, and a dynamic part, describing the transient behavior of the branches.

B. Branches with distributed parameters

In transient power system simulation lines are often described by distributed parameters [1]. Consider a lossless line with an inductance L' and a capacitance C' per length l (Fig. 4a). The dynamic behavior of the system is represented by two coupled difference equations:

$$i_1(t) = \frac{1}{Z_w} u_1(t) - \frac{1}{Z_w} u_2(t-\tau) - i_2(t-\tau) \quad (22)$$

$$i_2(t) = \frac{1}{Z_w} u_2(t) - \frac{1}{Z_w} u_1(t-\tau) - i_1(t-\tau) \quad (23)$$

with

$$\tau = l\sqrt{L'C'}, \qquad Z_w = \sqrt{L'/C'} \quad (24)$$

Fig. 4b shows the dynamic structure of the line with time delay blocks. According to (22) and (23) the terminal voltages $\underline{u} = [u_1\ u_2]^T$ are the input variables and the terminal currents $\underline{i} = [i_1\ i_2]^T$ are the output variables of the system. In TCM-simulation the lossless line behaves like a coupled admittance element (type YY in Table 2). If other terminal variables are defined as input variables, a distributed parameter line can also behave as an impedance element (type ZZ) or a combined admittance/impedance element (types YZ and ZY in Table 2). The correct model is determined by the electrical environment. Voltage sources and impedance elements at the terminals cause an admittance behavior, while current sources and admittance elements are responsible for an impedance behavior (Table 2).

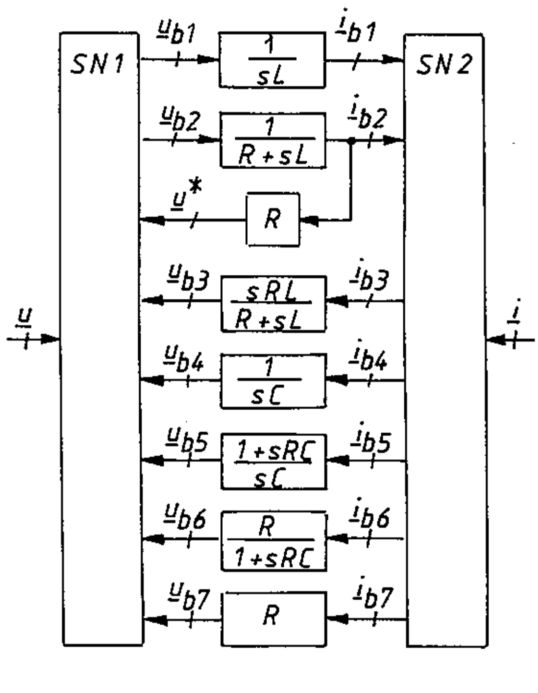


Fig. 3: TCM-simulated linear RLC-network ($T_0 = \infty$)

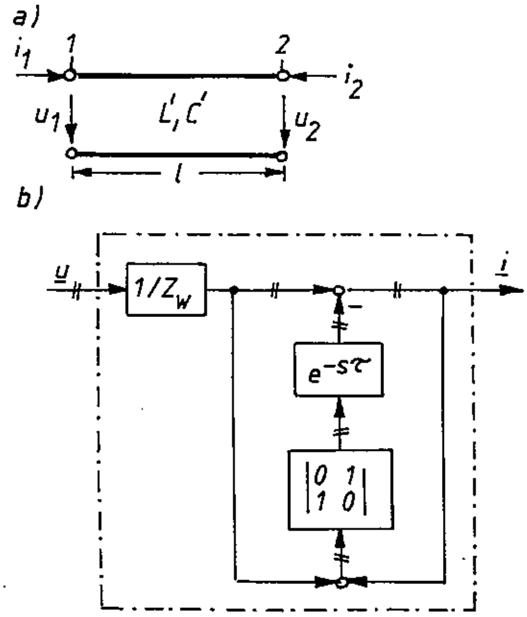


Fig. 4: Lossless line with distributed parameters
a) symbol
b) dynamic model with $\underline{u} = [u_1\ u_2]^T$ as input variables

C. Mutual couplings

In multiphase networks mutual couplings between the branches have to be considered. Fig. 5 shows a lumped parameter model of a symmetrical three phase line with neglected capacitors. Due to the mutual inductances M the matrix $\underline{L}_b$ of branch inductances becomes nondiagonal.

$$\underline{L}_b = \begin{bmatrix} L & M & M \\ M & L & M \\ M & M & L \end{bmatrix} \tag{25}$$

Therefore, an inversion of $\underline{L}_b$ (see (15) and (16)) is more difficult to carry out. But the operation principles of the TCM are not changed.

D. Nonlinearities

A TCM-simulation is also possible with nonlinear network parameters. But the nonlinear parameters have to be described in a suitable form, to avoid algebraic loops. In the following we consider the branches of Table 1 and assume nonlinear resistors and inductances. Usually, algebraic loops

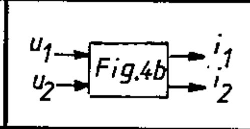
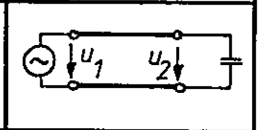
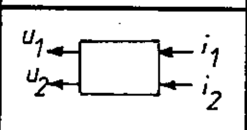
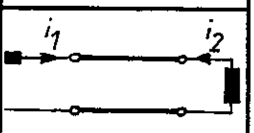
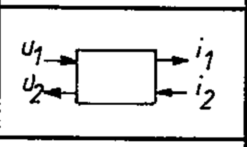
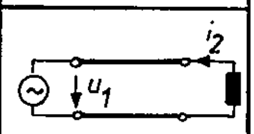
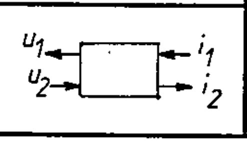
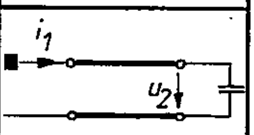
type	model	example
YY		
ZZ		
YZ		
ZY		

Table 2: TCM-simulation of lossless lines

do not occur, if the parameters R and L are defined as functions of the resistor and inductance current i:

$$R(i),\ L(i)$$

However, one exception exist. In a parallel RC-circuit (Table 1) the parameter R must be described in dependence on the resistor voltage u:

$$R(u)$$

Nonlinear inductances in admittance branches cause time varying parameters in the matrix $\underline{L}_b$. According to (15) and (16) the matrices $\underline{L}_i$ and $\underline{L}_i^*$ have to be calculated new at every time step. Therefore, the definition of the static network SN1 (Fig. 3) requires more computation time.

There is also another possibility to take nonlinear branches into consideration. They can be coupled with the linear net by means of iterative procedures.

E. Switching operations

Circuit breakers in power systems can simply be represented by ideal switches [1]. A switching operation changes the topology of the power network. Then the static program part (blocks SN1 and SN2 in Fig. 3) must be adapted to the new conditions. The dynamic program part is not influenced by switching operations like in the EMTP-method.

In small RLC-networks with many switches (e.g. power electronic circuits) we can also use another strategy [5]. In this case, the static nets SN1 and SN2 are described by a constant structure but time varying coefficients of the internal matrices. The coefficients depend on a switching vector $\underline{v}$, which is introduced as an additional input variable into the system. The components of this vector have to be determined a priori for all relevant switching states of the power system. In larger networks this concept cannot be recommended, because the dimension of the vector $\underline{v}$ becomes too large.

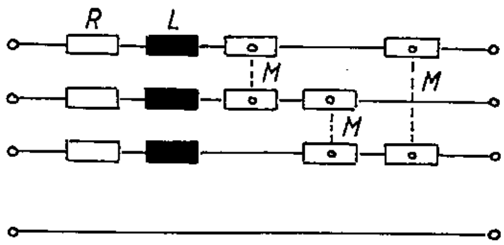


Fig. 5: Model of a three phase line with lumped parameters

F. Comparisation with the EMTP-method

The TCM- and EMTP-method are based on the same philosophy. A dynamic decoupling of the power network is achieved by the calculation of all node voltages. The TCM carries out this procedure in the continuous time domain. So, the result is independent of a special integration algorithm. The dynamic structure of the system equations can be modeled by a block diagram, which gives a good insight into the signal flow of the power system.

The EMTP-method is working in the discrete time domain using a fixed numerical integration algorithm (trapezoidal rule). Many system variables have no physical meaning.

Therefore, a continuous time interpretation of the network equations is problematic.

The main advantage of the TCM is the reduced effort in node voltage calculation. Consider the simple 4 node network of Fig. 6. Using the EMTP-method all branches are modeled by admittances. So, 3 node voltages (u_2, u_3 and u_4) must be calculated by inverting a (3 x 3)-matrix. From the TCM-point of view all node voltages are known due to the impedance branches (capacitors).

For numerical integration the TCM-converted continuous time network equations have to be transformed into the discrete time domain. This procedure is not necessary in the EMTP-method, because the system equations are already in the discrete time domain. A discretization of the TCM model consumes additional computation time [6] and compensates the advantages in the node voltage calculation. From the present point of view TCM and EMTP-method need the same effort in power system simulation.

IV. CONCLUSION

The time constant method (TCM) is a new approach for transforming differential equations of an electrical power network into a suitable form of simulation. It works in the continuous time domain and is not restricted to a certain numerical integration algorithm. The network branches are modeled by admittances and impedances. All admittance elements have the same time constant T_0 (preferably $T_0 = \infty$). Due to the impedance branches the effort in node voltage calculation is reduced. With the knowledge of all node voltages in the network a dynamic system decoupling can be achieved. The TCM is an alternative to the EMTP-method, which is used in the well known program packages as EMTP and NETOMAC.

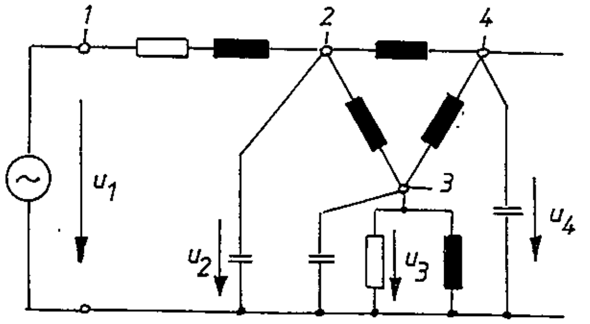


Fig. 6: Electrical network with 4 nodes

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