An Electromagnetic Transients Model of Multi-limb Transformers Using Normalized Core Concept

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Abstract: **This paper presents improvements to an existing multi-limb transformer model by applying *normalized core concept*. It shows that multi-limb transformers can be accurately modeled without having to specify actual core dimensions and winding turns number. The paper includes re-formulation of the unified magnetic equivalent circuit (*UMEC*) model to demonstrate the application of normalized core concept. The modified UMEC model in PSCAD/EMTDC is validated using experimental and field test data.**

Keywords: emtp, EMTDC, Modelling, Multi-limb, Core, Transformer, Magnetic Circuit, UMEC.

I. INTRODUCTION

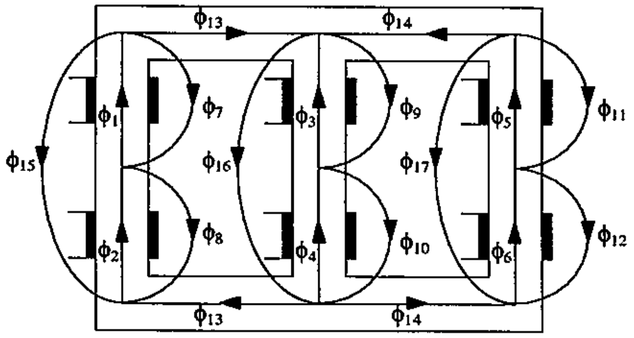
A transformer is represented in an electromagnetic transients program by its equivalent network consisting of resistors, inductors and capacitors. Most programs use transformer models based on Steinmetz equivalent circuit. This equivalent circuit can accurately represent single phase transformers. Its parameters can be easily obtained from standard transformer tests such as open circuit and short circuit tests. However, it is not adequate to represent multi-limb transformers which have non-uniform flux distribution and inter-phase coupling. Recently new methods for modeling multi-limb transformers (or deriving its equivalent circuits) have been proposed.

Using a state variable approach, and assuming a magneto-quasi-static condition, Chen has developed an inverse inductance matrix model for ATP. [1],[2] The principle of magnetic-electric circuit duality has been applied by Stuehm to create a multi-limb transformer model for EMTP. [3] A unified magnetic equivalent circuit (UMEC) approach [4],[5],[6] which directly converts any transformer magnetic circuit to an electric circuit equivalent, has been adopted by PSCAD/EMTDC.

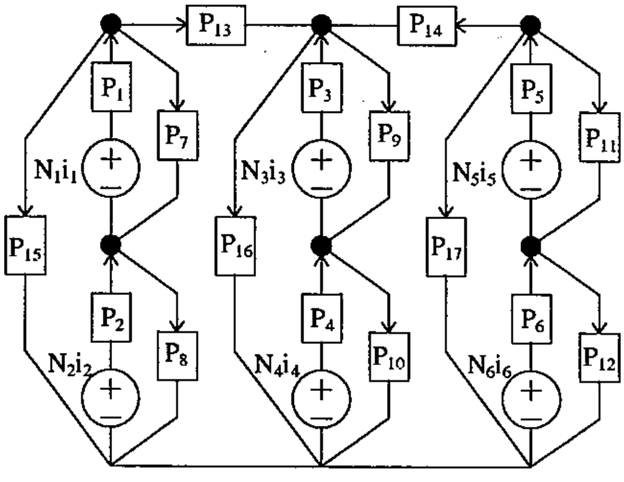
The UMEC transformer models require the user to input detailed core parameters. However magnetic core dimensions, winding turns number, and core B-H saturation characteristics are generally only available to the transformer manufacturer. Therefore, application of the existing UMEC models or, for that matter, many multi-limb transformer model has been limited.

This paper presents a further development to the UMEC transformer model that eliminates the necessity to input detailed core data. Now users can model a multi-limb transformer more accurately with the same data that they have been using with single phase banks. Exact aspect ratios (relative core dimensions), if specified, can improve accuracy but are not critical as shown by the sensitivity analysis in this paper.

The inductance matrix is computed using the concept of a normalized core. Results obtained with typical transformer data and detailed core parameter UMEC models are compared and verified with laboratory and factory test results.



(a) Core flux paths



(b) Unified magnetic equivalent circuit

Fig. 1: A three-limb three-phase transformer

For simplicity and clarity, the normalized core concept is discussed using the linear circuit of a three-limb three-phase transformer as shown in Fig. 1. The final formulation is applicable to any configuration, for example, a five limb transformer including non-linearity.

II. THE UMEC TRANSFORMER MODEL

This section introduces the existing UMEC formulation. The flux paths of Fig. 1(a) correspond directly to the UMEC branches of Fig. 1(b). The transformer core is broken up into winding-limb (paths 1-6) and yoke (paths 13-14) branches. The leakage and zero sequence flux branches are paths 7-12, and 15-17 respectively. The UMEC formulation (Appendix A) creates the transformer inductance matrix **L** as given by Eqn. 1. [4],[5],[6]

$$\mathbf{L} = \mathbf{N}_{ss}\mathbf{M}_{ss}\mathbf{N}_{ss} \tag{1}$$

If the number of transformer windings is n, then $\mathbf{N}_{ss}$ is the $n*n$ diagonal matrix of winding turns number. $\mathbf{M}_{ss}$ is the $n*n$ upper left-hand partition of Eqn. 2 (see also Appendix A).

$$\mathbf{M} = \mathbf{P} - \mathbf{PA}(\mathbf{A}^T\mathbf{PA})^{-1}\mathbf{A}^T\mathbf{P} \tag{2}$$

If the number of UMEC branches is m, then **P** is the $m*m$ diagonal matrix of branch permeances. **A** is the $m*n$ rectangular branch connection matrix. Appendix B gives the branch connection matrix for the three-limb three-phase UMEC of Fig. 1(b).

III. DIFFICULTIES WITH UMEC MODEL

Eqns. 1 and 2 highlight the present difficulties of UMEC model application. Implementation requires knowledge of the primary and secondary winding turns number, and the branch permeance at each time step. Moreover, the permeance of the transformer core flux paths is a function of core dimensions as given by Eqn. 3.

$$P = \frac{\mu_O \mu_R A}{L} \tag{3}$$

where $\mu_O \mu_R$ is the core permeability taken from the steel B-H characteristic. A and L are the cross-sectional area and length of the magnetic circuit branch respectively. The requirements of winding turns number, steel B-H characteristic, and core cross-sectional area and length would limit the model usage to transformer manufacturers.

IV. NORMALIZED CORE CONCEPT

Consider the simple inductors shown in Fig. 2.

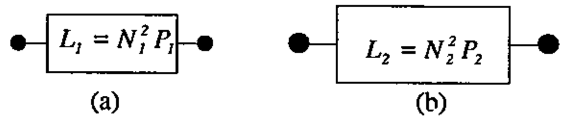


Fig. 2: Matched inductors

Inductance is given by Eqn. 4.

$$L = N^2 P \tag{4}$$

Even if the permeance P_1 of inductor (a) is different than permeance P_2 of inductor (b), the two devices can exhibit the same inductance if N_1 and N_2 are appropriately adjusted.

The inductor manufacturer must carefully select core dimensions, winding turns number, and steel characteristics based on practical constraints, present technologies and experience. However, to simulate these inductors in electrical domain we are not concerned with the differences in the design parameters such as core dimensions and turns numbers.

We can extend this philosophy to the UMEC model to eliminate its dependency on core geometry and turns number.

V. MODIFIED UMEC MODEL

The transformer inductance matrix can be calculated from the parameters: rating MVA, primary and secondary voltage V_1,V_2, leakage reactance X_l, rated angular frequency ω_0, magnetizing current I_m, and core aspect ratios r_A, r_L. All parameters, excluding the core aspect ratios, are transformer name-plate data. Moreover, in the transformer manual that accompanies purchase, it is reasonable to expect scale drawings of the transformer core. The core aspect ratios of Eqns. 5 and 6 can be obtained from such drawings.

$$r_A = \frac{A_y}{A_w} \tag{5}$$

$$r_L = \frac{L_y}{L_w} \tag{6}$$

where A_y and A_w are yoke and winding-limb cross-sectional areas respectively, L_y and L_w are yoke and winding-limb lengths respectively. The core dimension definitions are shown in Fig. 3.

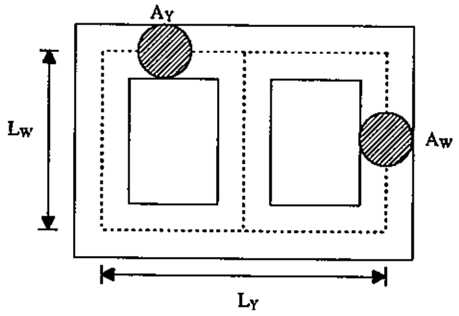


Fig. 3: Three-limb core dimension definitions

The three-limb three-phase transformer diagonal matrix of branch permeance **P** is comprised of winding-limb $P_{1\text{-}6}$, winding-leakage $P_{7\text{-}12}$, yoke-limb $P_{13,14}$, and zero-sequence $P_{15\text{-}17}$ elements. Eqns. 7 to 9 define these terms.

$$P_{1-6} = \frac{\mu_o \mu_r A_w}{L_w} = \frac{V_1\left(2 + \frac{r_L}{r_A}\right)}{\omega_o I_m I_{b1}} \frac{1}{N_1^2} = k_{1-6}\frac{1}{N_1^2} \tag{7}$$

I_{b1} is the current base on the transformer primary.

$$P_{7-12} = P_{15-17} = \frac{X_l Z_{b1}}{2\omega_o}\frac{1}{N_1^2} = k_{7-12}\frac{1}{N_1^2} = k_{15-17}\frac{1}{N_1^2} \tag{8}$$

Z_{b1} the impedance base on the transformer primary.

$$P_{13,14} = \frac{\mu_o \mu_r A_y}{L_y} = \frac{V_1\left(2\frac{r_A}{r_L} + 1\right)}{\omega_o I_m I_{b1}} \frac{1}{N_1^2} = k_{13,14}\frac{1}{N_1^2} \tag{9}$$

In Eqns. 7 and 9 the core permeability has been set according to Eqn. 10.

$$\mu_o \mu_r = \frac{V_1\left(2\frac{L}{A} + \frac{L_y}{A}\right)}{\omega_o I_m I_{b1}} \tag{10}$$

Eqn. 10 ensures that on open-circuit, at rated voltage, the rated magnetizing current flows in the outer transformer windings. In Eqn. 8 the permeance of the primary and secondary leakage branches are set equal, and in the absence of specified zero-sequence inductance, the permeances $P_{15\text{-}17}$ are set equal to the leakage. All equations can be redefined with variables referred to the secondary side (V_2, N_2, I_{b2}, Z_{b2}) rather than the primary side (V_1, N_1, I_{b1}, Z_{b1}).

The scalar $1/N^2$ is common to all elements of the matrix **P** and can be extracted.

$$\mathbf{P} = \frac{1}{N^2}\mathbf{K} \tag{11}$$

Substitution of Eqn. 11 into 2 gives

$$\mathbf{M} = \frac{1}{N_1}\left(\mathbf{K} - \mathbf{KA}\left(\mathbf{A}^T\mathbf{KA}\right)^{-1}\mathbf{A}^T\mathbf{K}\right)\frac{1}{N_1^2} = \frac{1}{N_1}\mathbf{M}^*\frac{1}{N_1} \tag{12}$$

and Eqn. 1 can be rewritten as

$$\mathbf{L} = \frac{1}{N_1}\mathbf{N}_{ss}\mathbf{M}^*_{ss}\mathbf{N}_{ss}\frac{1}{N_1} = \mathbf{N}^*_{ss}\mathbf{M}^*_{ss}\mathbf{N}^*_{ss} \tag{13}$$

The product $\frac{1}{N_1}\mathbf{N}_{ss} = \mathbf{N}^*_{ss}$ is a diagonal matrix with elements equal to unity or the transformer turns ratio. Thus,

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} \Rightarrow \mathbf{N}^*_{ss} = \mathbf{V}^*_{ss} \tag{14}$$

$$\mathbf{L} = \mathbf{V}^*_{ss}\mathbf{M}^*_{ss}\mathbf{V}^*_{ss} \tag{15}$$

Using Eqn. 15 the transformer inductance matrix is now defined from the UMEC using only the variables MVA, V_1, V_2, X_l, ω_0, I_m, r_A and r_L.

VI. SINGLE PHASE EXCITATION TEST

A three-limb, three-phase laboratory transformer was selected to validate the normalized core concept. A single-phase excitation test was chosen to demonstrate the strong inter-phase magnetic coupling typical of this core type. The test was simulated using both the modified model and the old model. The modified model used the name plate data while the old model used the detailed core parameters given in Table 1.

Table 1: Laboratory transformer parameters

Name-plate data		Core parameters	
Rating	40kVA	A_w	0.0122 m^2
Configuration	star-star	L_w	0.175 m
Frequency	50Hz	A_y	0.0122 m^2
V_1	240V	L_y	0.180 m
V_2	70V	μ_r	2000
		N_1	108
		N_2	31

In Fig. 4 the transformer red-phase primary winding is energized. When the voltage V_{coup} is applied to only one winding, the magnetic flux from the energized limb returns via the remaining two limbs.

The return path reluctance for the red-phase flux Φ_r via the yellow winding-limb is less than that via the blue winding-limb. The yellow-phase flux Φ_y is thus slightly greater than $0.5\Phi_r$ and the blue-phase flux is slightly less than $0.5\Phi_r$. The magnitude of the primary yellow and blue phase voltages are slightly greater and less than $0.5V_{coup}$, respectively. The orientation of the yellow and blue phase windings phase shifts the primary and secondary voltages by 180 degrees.

If this multi-limb transformer was represented using 3 single phase transformers in the simulation, the voltages on the non-excited windings would be zero.

Experimental and simulated results for a 240V excitation of the laboratory transformer red-phase primary winding are shown in Fig. 5. The normalized core waveforms exactly match the detailed core model simulation and hence only one of these is shown. Simulated waveforms agree with the experimental data.

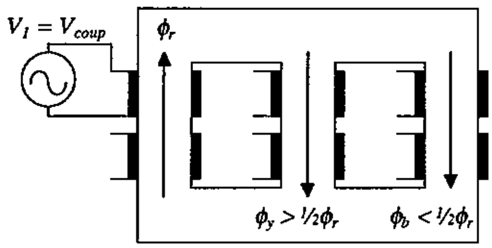


Fig. 4: Laboratory transformer single-phase excitation

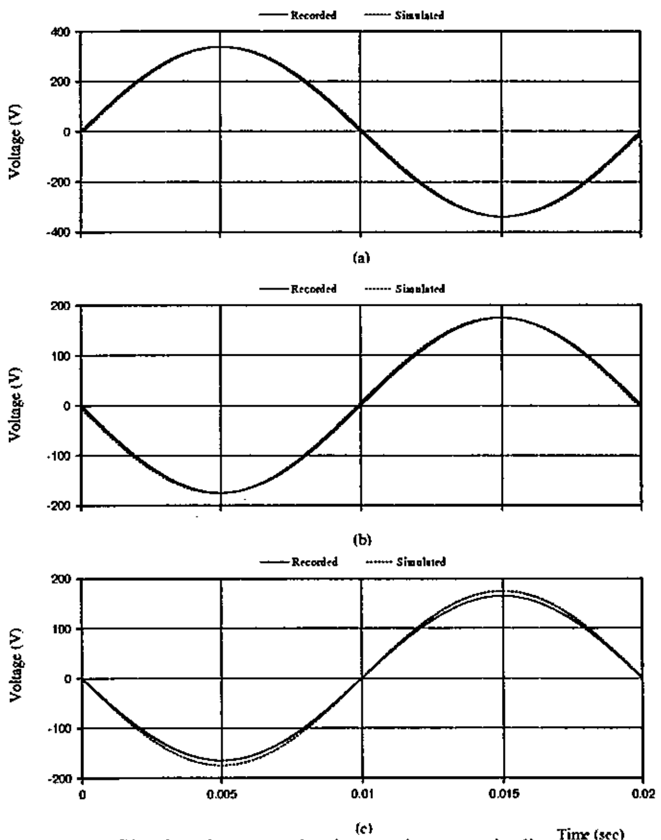


Fig. 5: Single-phase excitation, primary winding voltages; Solid = recorded, Dotted = simulated UMEC model: (a) Red-phase, (b) Yellow-phase, (c) Blue-phase

VII. OPEN CIRCUIT TEST

Table 2 presents the name-plate data for the Manapouri three-limb, three-phase transformer units operated in the South Island, New Zealand ac system. The parameters required for the detailed core UMEC model of the Manapouri units are also given in Table 2.

Table 2: Manapouri transformer data

Name-plate data		Magnetic core parameters	
Rating	150MVA	A_w	0.5555 m^2
Configuration	delta-star	L_w	1.932 m
Frequency	50Hz	A_y	0.5635 m^2
V_1	13.8kV	L_y	4.000 m
V_2	220kV	B-H slope	7.33e-3
X_1	11.3%	N_1	78
		N_2	718
		$P_{7\text{-}12}$	1.67e-7
		$P_{15\text{-}17}$	1.67e-7

The open-circuit line currents of Fig. 6 correctly reflect the magnetic unbalance in the magnetizing current of the three-limb core type. The yellow and blue-phases carry the same current, whereas the red-phase current is greater. This is expected for a delta connected three-limb transformer. Both the modified UMEC and detailed core UMEC give the same result.

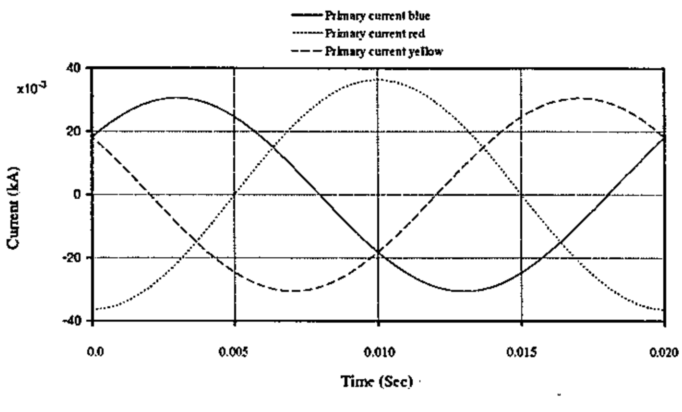


Fig. 6: Simulated no-load currents of Manapouri transformer

Finally, Table 3 compares the two models for open-circuit and short-circuit behavior with factory test results. The comparison shows a very good agreement between test results and simulation.

Table 3: Factory test data validation.

	Factory measurement	UMEC model simulation
No-load red-phase current	24.2 Amps	24.3 Amps
Short-circuit leakage reactance	11.3%	11.2%

VIII. SENSITIVITY ANALYSIS OF ASPECT RATIOS

To use the UMEC model, Users need core aspect ratio r_L and core cross-sectional area ratio r_A, in addition to the normal name-plate data. Users may have to estimate these ratios if they are not readily available. In most situations, r_A can be set to unity. From Eqn. 8 it is clear that the leakage paths are not affected by these ratios. They mainly affect the distribution of flux among the limbs.

Single phase excitaion test of Section VI was used to verify the effect of r_L on the electrical behaviour of the model. A 20% change in r_L produced only 0.5% change in the induced voltage. Similar results were obtained with other tests as well. Hence, it is clear that the electrical behaviour of the transformer is very insenitive to core aspect ratio and a default value can be used if the actual value is not available.

IX. CONCLUSIONS

Commonly available name-plate data is sufficient to accurately model multi-limb transformers in electromagnetic transient programs. The normalized core concept used in this paper to modify UMEC formulation can be applied to other multi-limb models [1] [2] and eliminate the need for detailed core geometry.

X. FURTHER WORK

Although only linear transformer model was discussed in this paper, the *normalized core* concept is equally applicable to non-linear part of the transformer. The non-linear behaviour of the core can be handled by repeated use of the linear model with permeances changing every time step or on a piece-wise linear basis. Modeling of saturation using these concepts along with results of other types of multi-limb transfomers will be the subject of a subsequent paper.

XI. REFERENCES

[1] X. S. Chen and P. Neudorfer, Liman Zhuang, "The Development of a Three-Phase Multi-Legged Transformer Model for Use With EMTP", *Department of Energy Contract #DE-AC79-92BP26702,* March 1993.

[2] Xusheng Chen, "A Three-Phase Three-Winding Core-Type Transformer Model for Low Frequency Transient Studies", IEEE/PES Summer Power Meeting, 96 SM 410-1 PWRD, July 1996.

[3] D. L. Stuehm, "Three-phase Transformer Core Modeling", Final Report to Bonneville Power Administration Award No. DE-BI79-92BP26700, February 28.

[4] W. Enright, J. Arrillaga, A. R. Wood, and N. R. Watson, "Modeling multi-limb transformers with an electromagnetic transient program", Paper presented at the *ELECTRIMACS 1996*, Saint Nazaire, France, September 1996

[5] W. Enright, J. Arrillaga, A.R. Wood, and N.R. Watson, "The use of a magnetic equivalent circuit in the current distribution between transformer windings", Power Systems World, pp.292-298, September 1996, Las Vegas.

W. Enright, *Transformer models for electromagnetic transient studies with particular reference to HVdc transmission*, Ph.D. thesis, University of Canterbury, Christchurch, New Zealand, 1996.

APPENDIX A

This formulation derives the transformer inductance matrix from the UMEC. As an example, refer to the three-limb three-phase UMEC shown in Fig. 1(b). The flux ϕ in each branch of the UMEC can be written in vector form

$$\tilde{\phi} = \mathbf{P}(\mathbf{N}\tilde{i} - \tilde{\theta}) \tag{16}$$

where $\tilde{i}$ is the vector of winding current (zero for branches with no winding) and $\tilde{\theta}$ is the vector of mmf across each branch.

At each UMEC node the flux must sum to zero, stated as

$$\mathbf{A}^T \tilde{\phi} = \tilde{0} \tag{17}$$

Application of the branch-node connection matrix **A** to the vector of nodal mmf *node* gives the branch mmf.

$$\mathbf{A}\,\tilde{\theta}_{node} = \tilde{\theta} \tag{18}$$

Multiplying Eqn. 16 by $\mathbf{A}^T$ and substituting in Eqns. 17 and 18 gives

$$\tilde{0} = \mathbf{A}^T\mathbf{P}\mathbf{N}\tilde{i} - \mathbf{A}^T\mathbf{P}\mathbf{A}\,\tilde{\theta}_{node} \tag{19}$$

Solving Eqn. 19 for *node* and multiplying both sides by **A** gives

$$\mathbf{A}\,\tilde{\theta}_{node} = \mathbf{A}(\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{P}\mathbf{N}\tilde{i} \tag{20}$$

Substitution of Eqn. 18 into Eqn. 20 allows Eqn. 16 to be written as

$$\tilde{\phi} = \mathbf{MN}\tilde{i} \qquad (21)$$

where

$$\mathbf{M} = \mathbf{P} - \mathbf{PA}(\mathbf{A}^T\mathbf{PA})^{-1}\mathbf{A}^T\mathbf{P} \qquad (22)$$

If the vector of branch flux is partitioned into the set that contains the branches associated with each transformer winding then Eqn. 21 becomes

$$\tilde{\phi}_s = \mathbf{M}_{ss}\mathbf{N}_{ss}\tilde{i}_s \qquad (23)$$

and the transformer inductance matrix can be written as

$$\mathbf{L} = \mathbf{N}_{ss}\mathbf{M}_{ss}\mathbf{N}_{ss} \qquad (24)$$

APPENDIX B

Following is the connection matrix A for the three-phase three-limb UMEC transformer shown in Fig. 1

$$[\mathbf{A}] = \begin{matrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{matrix}$$