Air–core Transformer: A Theoretical Analysis and Digital Simulation

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Abstract– **In this paper, a specially designed air–core transformer, to be employed in a small power tapping scheme from HVDC transmission systems, is studies. First, the transformer is analyzed by classical electromagnetic theory. Next, a small prototype transformer is built and studied in the laboratory. Experimental results, confirm the validity and accuracy of the proposed method for calculating the self– and mutual–inductances of the transformer. The results of the digital simulation of the transformer by the *EMTDC/PSCAD™* software package, too, are in good agreement with the lab observations.**

Keywords : Air–core Transformer, HVDC Transmission Systems, Small Power Tapping, Digital Simulation

I. INTRODUCTION

For the past few decades, High Voltage Direct Current (HVDC) transmission systems have served as efficient alternatives for transmitting bulk electric power over long distances, or as advantageous links between physically separated electric networks [1].

Although the HVDC transmission systems, if proved to be superior to AC transmission for a specific case, present notable economical, as well as technical advantages, these systems are usually designed on a point–to–point basis. As some of these HVDC transmission systems pass over relatively small communities with no connection to major power transmission systems, it is most desirable to find methods of economically connecting these communities to the HVDC system, to supply them with cheap, abundant electric energy [2].

For large loads, i.e. loads which are in the same order as the whole system's capacity, multiterminal schemes have been designed and commissioned. For small loads, however, the economical competitiveness compared with other alternatives (e.g. local generation, etc.), as well as technical reservations have to be fulfilled [2].

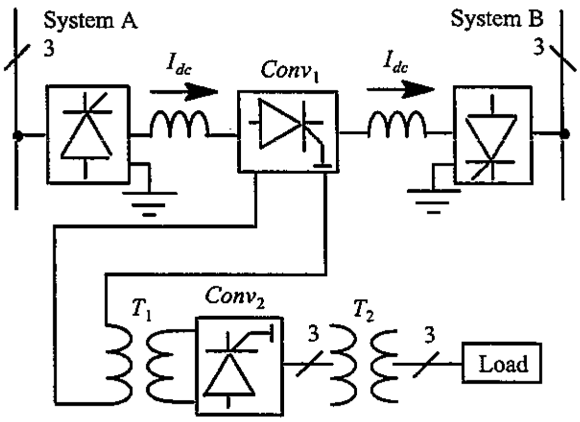
Based on the availability of powerful semiconductor devices with turn–off capabilities, i.e. GTO thyristors, a novel tapping scheme has been proposed by the authors. The proposed scheme is a series current–source tap which makes the power tapping possible by means of generating a small DC voltage and imposing it on the DC line. Fig. 1 shows the proposed scheme.

A significant advantage of the proposed scheme is the possibility of using a single–phase air–core transformer to achieve de–coupling between the high–potential HVDC line and the ground–potential local load to be supplied. Such a transformer, compared to the conventional iron–core transformers, will be less expensive, the losses are expected to be lower, and the weight of the transformer will be significantly lighter, which makes the tapping station easier to build.

Different aspects of the proposed scheme's operation, with a conventional single–phase iron–core transformer, have been studied by the authors and the results indicate its high degree of technical feasibility and satisfactory performance [3]. The focus of this paper is concentrated on the analysis of the air–core transformer.

II. THE AIR–CORE TRANSFORMER

The transformer which is going to be used in the proposed tapping scheme, must have the following specifications for the normal operating conditions:



$Conv_1$: *Single–phase series tap*

$Conv_2$: *single–phase to three–phase converter*

T_1 : *Single–phase air–core Transformer*

T_2 : *Three – phase Transformer*

Fig. 1 – The proposed tapping scheme

– It has to be able to de–couple DC voltages of up to 500 kV from the ground potential,

– It has to withstand peak AC voltages of up to 5 kV applied on its windings; and,

– It has to carry peak AC currents of up to 2 kA.

Considering the above requirements, the configuration shown in Fig. 2 has been proposed for the transformer. This specific configuration allows the transformer to isolate the HVDC line, connected to the upper winding, from the ground potential, and to withstand AC voltages applied on its primary and secondary windings by using ordinary insulation materials.

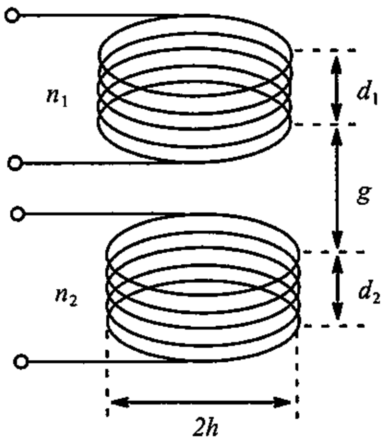


Fig. 2 – The air–core transformer

III. THEORETICAL ANALYSIS

In order to be able to study the performance of the transformer for various diameter and air–gap sizes, it is necessary to develop mathematical relationships for each coil's self–inductance, as well as the mutual inductance between the two coils. To achieve this, the classical electromagnetic theory was used [4].

Inductance is defined as the ratio of the total flux linkages to the current, I, which they link, or:

$$L = \frac{N \cdot \varphi}{I} \tag{1}$$

in which N is the number of turns of conductors and φ is the flux crossing the conductors' surface. Therefore, the flux which is created by a single current–carrying circular conductor at a similar circular surface placed at a vertical distance g from it, as shown in Fig.3, must be found.

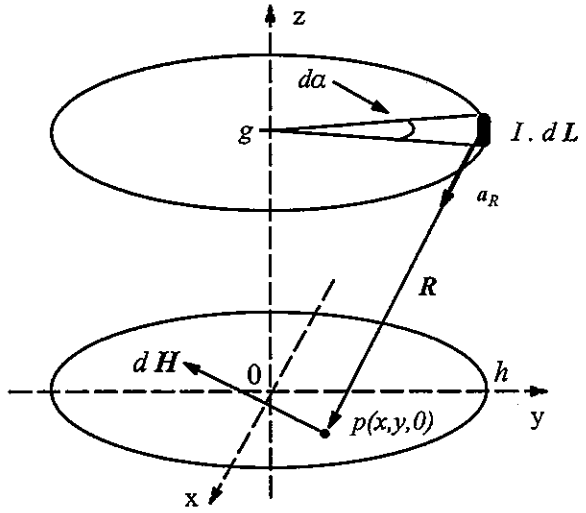


Fig. 3 – Two co–axial circular conductors

In order to calculate the total flux, due to the current I in the upper conductor, and crossing the area on the z=0 plane which is inside the circle r=h, the following relationship, known as Biot–Savart Law, is used to find the magnetic field intensity at a general point p(x,y,0):

$$d\,H = \frac{I \,.\, d\,L \times a_R}{4\,\pi\,R^2} \tag{2}$$

in which **dH** is the magnetic field intensity created by the differential current element I.dL.

Having the differential magnetic field intensity at the point p, and considering the fact that the magnetic medium in this particular case is air, the differential magnetic flux density is equal to:

$$d\,B = \mu_0 \,.\, d\,H \tag{3}$$

in which $\mu_0 = 4\,\pi * 10^{-7}$ is the permeability of the air. A linear integral of $\oint \mu_0 \, d\,H$ over the upper circle, gives the total magnetic flux density for the point p, i.e. **B**(x,y,0). Finally, a surface integral of $\oint B \,.\, d\,s$ over the surface z=0 and r=h, gives the total flux crossing the lower circle, due to the current in the upper circle.

In other words, if the coordinates in the x–y plane are represented by their polar equivalent (r,θ), and if the angle on the upper circle is denoted by a, then the total flux divided by current as a function of the conductor's radius, h, and the distance between the two circular planes, g, is found by Eq. (4).

$$\frac{\varphi(h,g)}{I} = \frac{h\,\mu_0}{4\,\pi} \int_0^h \int_0^{2\pi} \int_0^{2\pi} \frac{(\,rh - r^2 \cos a \,\cos\theta - r^2 \sin a \,\sin\theta\,)\;da\;d\theta\;dr}{[(h\,\cos a - r\,\cos\theta)^2 + (h\,\sin a - r\,\sin\theta\,)^2 + g^2\,]^{\frac{3}{2}}} \tag{4}$$

Note that in order to develop Eq. 4, the following equalities have been employed:

$$R^2 = (h \cos a - r \cos\theta)^2 + (h \sin a - r \sin\theta)^2 + g^2 ,$$

$$dL = h \, da \, \mathbf{a}_a ,$$

$$ds = r \, d\theta \, dr \, \mathbf{a}_z .$$

Also, note that because of the symmetry of the configuration, only vertical components of **dH** exist, and other components are cancelled out, thus simplifying the process of finding Eq. 4.

Because of obvious difficulties in finding an analytical solution for the triple integral in Eq. 4, a commercial mathematical software package, Mathematica, was used to numerically find values of $\varphi(h,g)/I$ for different values of h and g. Figure 4 shows the $\varphi(h,g)/I$ curves for $1 < h < 20$, and g=0,1,2,3,4, for h and g in meters.

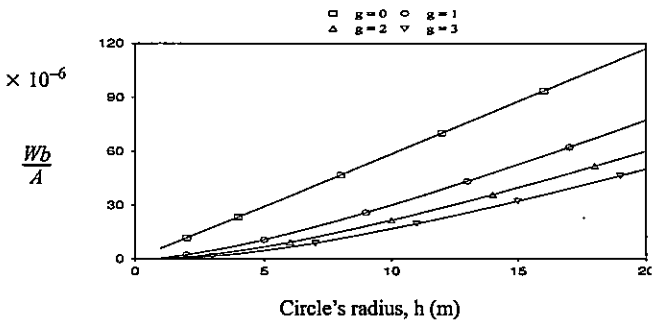


Fig. 4 – Flux per current

For g=0, the relationship is linear and can be represented by:

$$\frac{\varphi(h,0)}{I} = \frac{3 \pi \mu_0}{2} h . \qquad (5)$$

The self–inductance of a single circular conductor, therefore, is simply found from Eq. 5. Also, the mutual inductance between two single circular conductors with radius of h and separated by g, is equal to $\frac{\varphi(h,g)}{I}$ which can be found by numerical integration.

If coil number one is assumed to have N_1 concentrated turns, i.e. all turns can be considered with the same radius h, and at the same vertical position, its self–inductance is equal to:

$$L_{11} = N_1^2 \frac{\varphi(h,0)}{I} = \frac{3 N_1^2 \pi \mu_0}{2} h . \qquad (6)$$

Similarly, the self–inductance of the coil number two with N_2 concentrated turns is equal to:

$$L_{22} = \frac{3 N_2^2 \pi \mu_0}{2} h . \qquad (7)$$

The mutual inductance between coils number one and two, with the above conditions, is equal to:

$$M_{12} = N_1 N_2 \frac{\varphi(h,g)}{I} . \qquad (8)$$

For transformers, or in general, any two coupled coils, the coupling coefficient is defined as [5]:

$$k_{12} = \frac{M_{12}}{[L_{11} L_{22}]^{1/2}} . \qquad (9)$$

Using Eq. (6) to Eq. (8), the coupling coefficient is found to be equal to:

$$k_{12} = \frac{\varphi(h,g)/I}{\varphi(h,0)/I} . \qquad (10)$$

Fig. 5 shows the coupling coefficient of the transformer with concentrated turns in each coils, for g=1,2,3,4 m and $1 < h < 20$ m. Obviously, the coupling improves as the radius of the transformer increases, and/or as the two coils get closer to each other.

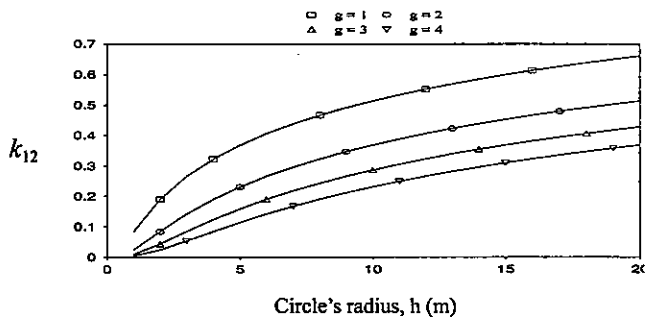


Fig. 5 – Coupling coefficient

IV. DISTRIBUTED COIL MODEL

The relationships developed in the previous section for the self–inductances and the mutual inductance of the two coaxial coils, were found based on the assumption of concentrated turns in each coil. The configuration proposed for the air–core transformer, however, employes two cylindrical one–layer coils, each consisting of a number of equi–radius turns, N, which are placed vertically on the top of each other. This distributed configuration directly affects the amount of flux linkage at any given position, and thus the inductances associated with the coils will be different than the ones given by Eq. 6 to Eq. 8. Various commercial field plotting computer programs can be found in the public domain to calculate this model, but since they were not readily available to the authors, a short computer program was used. The results of the program, to be shown in the following, were verified experimentally.

In order to take into consideration, the effect of the distribution in the coils, the $\varphi(h,g)/I$ curve must be found for any specific constant radius, h, and with the air–gap, g, as the variable. Fig. 6 shows these curves for h=5,10,15,20 m, and 2 < g < 6 m.

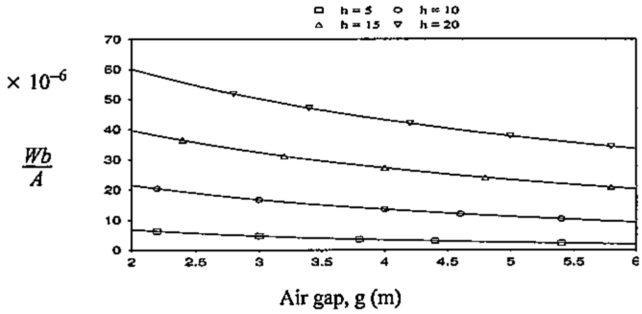


Fig. 6 – Flux per current

The computer program, then, is used to calculate the contribution of each turn to the total flux linking the same coil, as well as its contribution to the total flux which is linking the other coil, based on that turn's vertical position. Each turn is considered to be an independent single circular conductor. This assumption seems reasonably accurate, considering the large radius of the coil compared to the conductor's diameter. The program input includes the number of turns in each coil, the height and radius of each coil, and the air–gap distance. Using the specific $\varphi(h,g)/I$ curve numerically found for that specific radius, h, the self–inductances and the mutual inductance are found. The degree of accuracy of these quantities, obviously, depends on the number of points calculated for the flux/current curve. Fig. 7 shows the flowchart of the algorithm used for calculating the self inductances. A similar algorithm is used for mutual inductance.

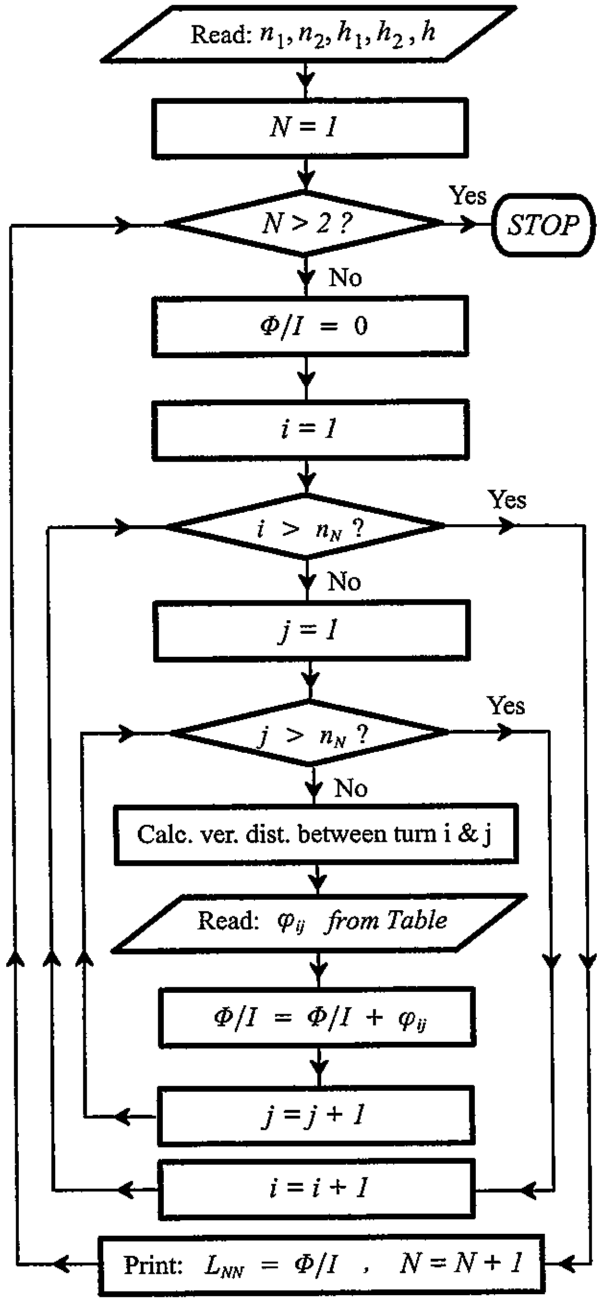


Fig. 7 – Flowchart for the self–inductances

V. EXPERIMENTAL ANALYSIS

In order to examine the accuracy of the theoretically found relationships, as well as the computer program which deals with the distribution in the coils, a prototype transformer with the same configuration as the proposed air–core transformer was built and studied. Table 1 shows the specifications of the prototype transformer.

Table 1 – Prototype transformer specifications

g = 20 mm	Coil #1	Coil # 2
N	*20*	*20*
Height, d_i (mm)	*39*	*39*
Radius, h (mm)	*175*	*175*

Using Eq. 4, the $\varphi(h,g)/I$ curve for 0 < g < 105 mm and h=175 mm was found, and is shown in Fig. 8.

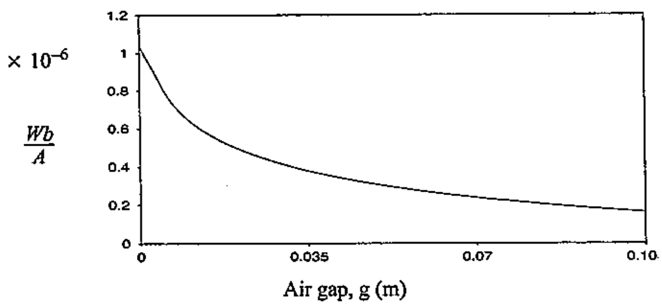


Fig. 8 – Flux per current for h=0.175 m

A. Laboratory Work

The self–inductance of each coil was directly measured by a digital inductancemeter, and was found to be equal to 0.255 mH for coil #1 and 0.258 mH for coil #2. To measure the mutual inductance between the two coils, the following configuration was used:

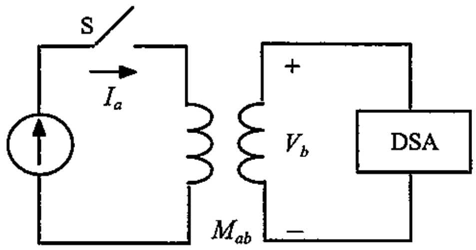


Fig. 9 – Measuring the mutual inductance

A DC current is injected into coil a. After the current reaches the steady–state level of I_a, the switch S is opened. The voltage induced in the coil b, then, is equal to:

$$V_b = M_{ab} \frac{di_a}{dt} . \qquad (11)$$

The digital signal analyzer, DSA, integrates the voltage across the coil b, and therefore:

$$M_{ab} = \frac{\int V_b \, dt}{\int di_a} = \frac{\int V_b \, dt}{I_a} . \qquad (12)$$

Table 2 shows the results of the measurement for the mutual inductance. The average measured mutual inductance is equal to 0.107 mH.

Table 2 – Measured mutual inductance

I_a (A)	$\int V_b \, dt$ (μH.A)	M_{ab} (μH)
1.97	211.0	107.0
2.10	237.0	113.0
2.21	226.5	103.0
2.30	249.0	108.0
2.39	249.0	104.0
2.49	272.0	109.0

The self–inductances and the mutual inductance between the two coils, were also calculated theoretically. Table 3 shows the results of the measurements, the calculated values, and the error for each case. The low level of error verifies the validity and the accuracy of the theoretical relationships.

Table 3 – Measured and calculated quantities

	Measured	calculated	Error
L_{11}	255 (μH)	257.93 (μH)	1.1%
L_{22}	258 (μH)	257.93 (μH)	0.03%
M_{12}	107 (μH)	112.06 (μH)	4.5%
k_{12}	41.7 %	43.4%	3.9%

B. Digital Simulation

In order to examine the degree of agreement between the prototype transformer's performance, and the transformer model in the digital simulation software package *EMTDC/PSCAD™* [5], an open–circuit (O.C.) test was performed on the transformer, and the same conditions were digitally simulated. Table 4 shows the results of the O.C. test, as well as the circuit quantities derived from them.

Table 4 – Open–circuit test results

Measured quantities	Calculated quantities
V_a = 2.43 V (rms)	Z_a = 243.7 mΩ
I_a = 9.97 A (rms)	R_a = 224.3 mΩ
P_a = 22.3 W	L_{11} = 252.8 μH

Fig. 10 shows the voltages observed in the lab, and the corresponding EMTDC simulation results.

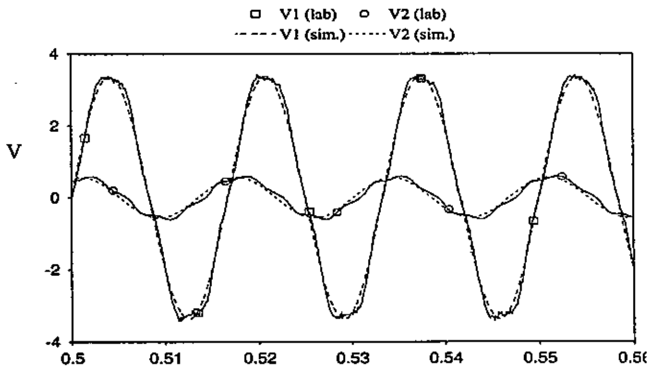


Fig. 10 – Comparison between lab and simulation results

The digital model accurately simulates the behavior of the transformer, and therefore, it can be used to simulate the performance of larger transformers, to be employed in the tapping scheme. It is noteworthy that the primary voltage used in the laboratory, because of the specific power supply used, is distorted due to harmonics, and the harmonics are reflected in the secondary voltage. The fundamental components of both the simulated and the measured voltages, however, are in good agreement.

VI. LARGER TRANSFORMERS

Having established the validity and the accuracy of the theoretical relationships, and because of the linear property of air, it is concluded that any air–core transformer with the proposed configuration, and with any size, can be analyzed and its inductances can be found very accurately. The EMTDC/PSCAD software package, then, can be used to predict the behavior of the whole tapping scheme.

Table 5 shows the results of calculations for a number of large size air–core transformers. As the length of the insulator strings in Manitoba's Nelson River HVDC scheme transmission lines is about 3.6 m, the air–gap, g, is assumed to be equal to this value. Also, all transformers have been assumed to have 30 turns in each coil, and each coil is assumed to have a height of 0.5 m.

Table 5 – Calculation results for large transformers

Radius (m)	$L_{11} = L_{22}(mH)$	M_{12} *(mH)*	k_{12} *(%)*
1.8	*5.70074*	*0.17383*	*3.0*
3.0	*10.90006*	*0.81207*	*7.5*
5.0	*20.51667*	*2.92318*	*14.2*
10.0	*46.26232*	*11.93017*	*25.8*
15.0	*73.94557*	*24.09618*	*32.6*
20.0	*102.90337*	*38.26720*	*37.2*

It is noteworthy that the authors' correspondence with a Canadian manufacturer of air–core reactors for high voltage applications, revealed that present manufacturing equipment limits the radius of the reactors to 1.8 m, for which the coupling coefficient will be around 3%. Calculations for h=1.8 m and g=3.6 m have resulted in the same value. It is, however, believed that manufacturing equipment might be adjusted for higher sizes, if there is enough demand for larger coils.

VII. CONCLUSION AND FUTURE WORK

In this paper, the idea of using a specially designed air–core transformer in small power tapping from HVDC transmission systems was presented and theoretical relationships for calculating the inductances of the transformer were developed.

A prototype transformer was built, and the laboratory results proved the validity and high accuracy of theoretical relationships. Also, the performance of the prototype transformer was compared to the results of the digital simulation performed by the EMTDC/PSCAD software package. Experimental and simulation results are in good agreement.

The performance of the tapping scheme with a large air–core transformer, with a radius of 5 m, is currently being undertaken by the authors, and appropriate means for compensating the high leakage reactance and high magnetizing current in the transformer are being evaluated.

VIII. ACKNOWLEDGEMENT

The idea of using the air–core transformer in the tapping scheme was proposed in an internal report for the Manitoba HVDC Research Centre.

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