DYNAMIC EQUIVALENTS FOR ELECTROMAGNETIC TRANSIENT ANALYSIS INCLUDING FREQUENCY-DEPENDENT TRANSMISSION LINE PARAMETERS

Leonardo T. G. Lima
COPPE/UFRJ e UFF
R. Passo da Pátria 156
24.210-240 Niterói, RJ
e-mail: llima@caa.uff.br

Nelson Martins
CEPEL
C.P. 2754
20.001 Rio de Janeiro, RJ
e-mail: pacdyn@fund.cepel.br

Sandoval Carneiro Jr.
COPPE/EE/UFRJ
C.P. 68504
21.945-504 Rio de Janeiro, RJ
e-mail: sandoval@coep.ufrj.br

Abstract - **This paper describes a method to obtain dynamic equivalents for electromagnetic transient analysis including frequency-dependent transmission line parameters. The method uses the set of transfer function dominant poles and associated residues. This technique can be directly applied to both state equations or descriptor systems models and is highly suited to sparsity oriented applications. This feature obviates concerns with the electrical network size and topology.**

Keywords: Electromagnetic Transients Analysis, Modeling, Reduced Order Dynamic Equivalents, Dominant Poles, Frequency Dependent Parameters, Transfer Functions

1. INTRODUCTION

Reduced order network equivalents have been used to cope with otherwise prohibitive computational costs in electromagnetic transient analysis. The development and assessment of these equivalents have been a continuous research topic over the last decades [1, 2, 3, 4].

This paper models lumped parameters (RLC) networks of any order and topology through an augmented set of equations (descriptor systems) [5]. The resulting system matrices are highly sparse. The concept of transfer function dominant pole spectrum [6, 7] is exploited to obtain reduced order dynamic equivalents.

The most accepted models of transmission lines with frequency-dependent parameters are based on rational function approximations of the propagation and characteristic admittance transfer functions for each mode of propagation [1, 2]. These rational approximations can be readily implemented as state equations.

The method deals with state variable redundancies and can be efficiently applied to large scale networks. The complete network dynamic model is very sparse and several linear systems techniques (frequency response, numerical integration, modal analysis, etc.) can be directly and efficiently applied.

The dominant pole spectrum algorithm [6, 7] allows the selective computation of the reduced set of the eigenvalues which dominate the system response. The partial fraction expansion of the transfer function is truncated when the residue magnitude drops below a certain threshold. The dimension of the reduced model is then adjusted for a better fitting of both the frequency response and the time response of the system.

The time and frequency responses of the linear reduced order model will be compared with those of a full size test system obtained with EMTP simulations.

2. PROBLEM FORMULATION

The frequency-dependent transmission line model is based on the equivalent circuit [8, 9] shown in Figure 1.

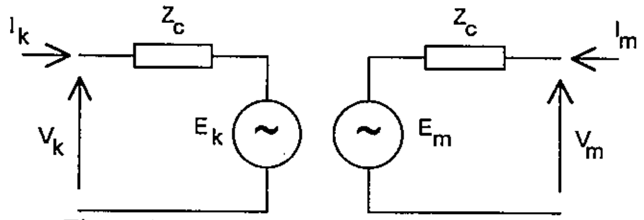
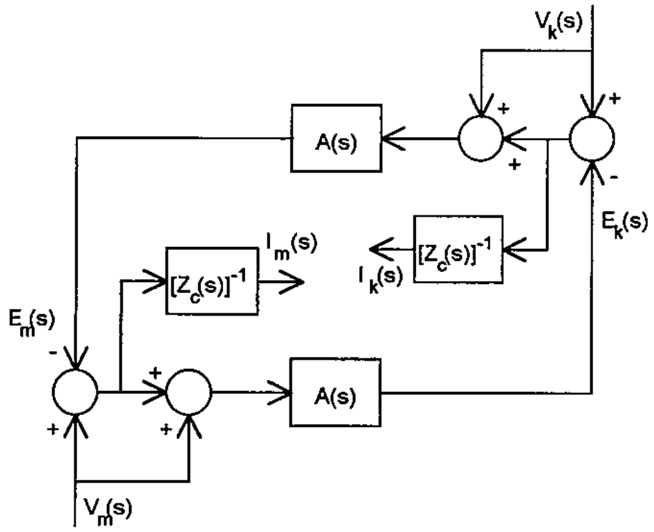


Figure 1. Transmission Line Equivalent Circuit

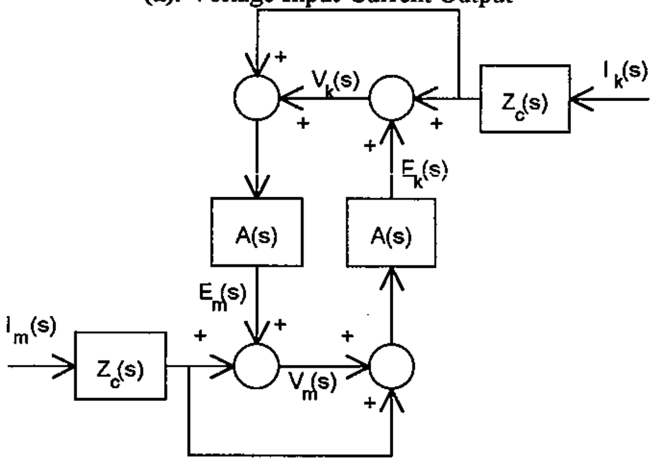
Since the characteristic impedance and the propagation function may be represented by rational functions approximations [1, 2], the equivalent circuit above yields the following equations:

$$\begin{aligned} E_k(s) &= [V_m(s) + Z_c(s)I_m(s)]A(s) \\ E_m(s) &= [V_k(s) + Z_c(s)I_k(s)]A(s) \\ V_k(s) &= Z_c(s)I_k(s) + E_k(s) \\ V_m(s) &= Z_c(s)I_m(s) + E_m(s) \end{aligned} \tag{1}$$

These equations may be represented in block diagram form, as shown in Figure 2.



(a). Voltage Input-Current Output



(b). Current Input-Voltage Output

Figure 2. Transmission Line Equivalent Block Diagrams

The block diagrams represent the transmission line as a two-input two-output transfer matrix in which the terminal voltages $V_k(s)$ and $V_m(s)$ and currents $I_k(s)$ and $I_m(s)$ are the variables of interest:

$$\begin{bmatrix} I_k(s) \\ I_m(s) \end{bmatrix} = [\mathbf{G}(s)]_{2x2} \begin{bmatrix} V_k(s) \\ V_m(s) \end{bmatrix} \qquad (2.a)$$

$$\begin{bmatrix} V_k(s) \\ V_m(s) \end{bmatrix} = [\mathbf{H}(s)]_{2x2} \begin{bmatrix} I_k(s) \\ I_m(s) \end{bmatrix} \qquad (2.b)$$

The frequency-dependent line model obtained with the J. Marti's setup of the EMTP (FDLINE) [1, 8] determines a rational approximation of the modal characteristic impedance $Z_c(s)$ and propagation function $A(s)$. These functions are expressed as transfer function poles and associated residues, which may therefore be written in a diagonal form state equation [10].

Note that the transfer function approximations of the FDLINE model have only real poles. The state equation can be written as

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u} \\ \mathbf{y} &= \mathbf{C}\,\mathbf{x} + \mathbf{D}\,\mathbf{u} \end{aligned} \qquad (3)$$

where

$\mathbf{A} = diag(\lambda_i)$

$\mathbf{B}$ = column vector with elements b_i

$\mathbf{C}$ = row vector with elements c_i

$\mathbf{D}$ = transfer function value when $s \to \infty$

where λ_i is the transfer function pole and the product $b_i.c_i$ is its associated residue.

The characteristic impedance transfer function is proper [11], i.e. it has an equal number of poles and zeros. Thus, the inverse transfer function is also proper and can be represented as a state equation. Furthermore, the SISO inverse transfer function [11] can be directly obtained in its state equation form:

$$\begin{aligned} \overline{\mathbf{A}} &= \mathbf{A} - \mathbf{B}\cdot\mathbf{C}\cdot\mathbf{D}^{-1} \\ \overline{\mathbf{B}} &= \mathbf{B}\cdot\mathbf{D}^{-1} \\ \overline{\mathbf{C}} &= -\mathbf{D}^{-1}\cdot\mathbf{C} \\ \overline{\mathbf{D}} &= \mathbf{D}^{-1} \end{aligned} \qquad (4)$$

The propagation function can be expressed as the product of a rational transfer function and a pure time delay component [1]. The time delay can be exactly expressed, in the frequency domain, as the transcendental function below:

$$A_d(s) = e^{-s\tau} \qquad (5)$$

where τ is the time delay constant.

The method of this paper requires a state space realization for the system model. For the pure time delay, this realization can be based on the Padé approximation [12]:

$$e^{-s\tau} \cong \frac{2 - \tau s + \frac{(-\tau)^2}{2!}s^2 + \frac{(-\tau)^3}{3!}s^3 + \ldots}{2 + \tau s + \frac{(\tau)^2}{2!}s^2 + \frac{(\tau)^3}{3!}s^3 + \ldots} \qquad (6)$$

A Padé approximation of order 4 is quite adequate to this application, as the results will demonstrate.

The state space realization of the single-phase FDLINE model has an order equal to twice the order of $Z_c(s)$ plus twice the order of $A(s)$ plus twice the order of the Padé approximation.

3. RESULTS

3.1. Single-Phase Line Energization

The results presented in this section are related to a single-phase transmission line. The FDLINE model [1] yielded approximations of order 19 to the characteristic impedance and order 28 to the propagation function. In this case, with a 3rd order Padé approximation, the line model has 100 state equations.

A voltage unit step was applied at terminal k of the single-phase transmission line with terminal m grounded. Figure 3 shows the current at terminal k calculated from the block diagram of Figure 2.(a) for time delay Padé approximations of different order. These results are compared with those obtained with the EMTP.

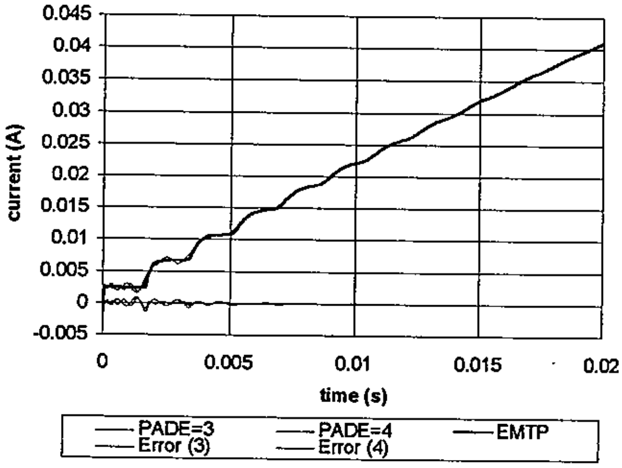


Figure 3. Step Response to Voltage Input

The curves labeled as "*Error*" are the differences between the EMTP response and the respective Padé approximation response. The error due to the Padé approximation is more noticeable in the very beginning of the simulation, where the time delay effect is more pronouced. The 4th order Padé approximation yielded slightly better results than the 3rd order one.

Figure 4 shows the current at terminal k when a sinusoidal voltage is applied at the terminal k while terminal m is grounded.

It is seen that the results are in very good agreement, with the maximum error being 0.0019 and 0.0029 for the 4th order and the 3rd order Padé approximations, respectively.

The block diagram of Figure 2 can be used as a fairly good frequency domain approximation of the FDLINE model. The order of the time delay Padé approximation can be adjusted to yield a better response.

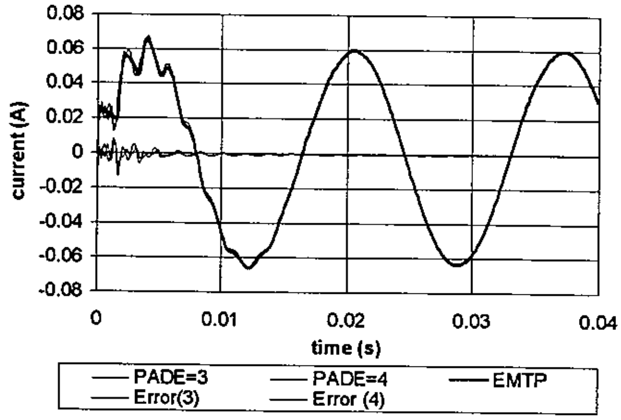


Figure 4. Response to a Sinusoidal Voltage Input

3.2. Low Order Dynamic Equivalents

A low-order dynamic equivalent can be used to reduce the complexity of a model and, therefore, the computational effort associated with its use.

Figure 5 presents the system configuration used to test the synthesis of the reduced-order dynamic equivalent. This system includes RLC lumped parameters network branches and a frequency-dependent transmission line with the same parameters as the one presented in the previous section.

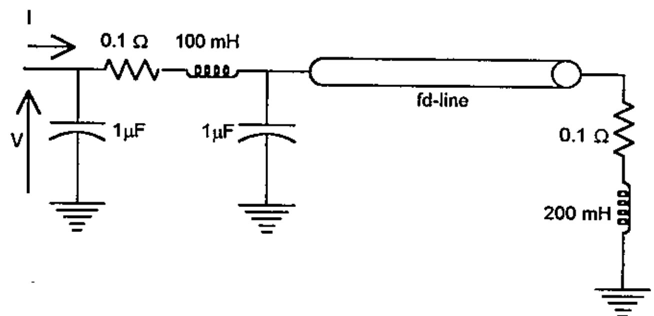


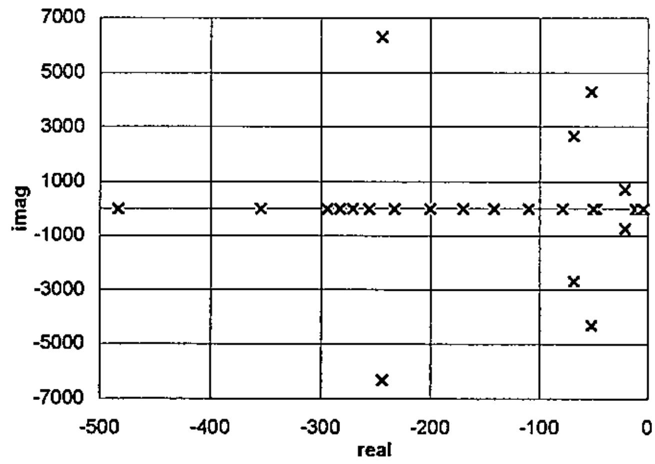
Figure 5. Test System Configuration

The complete dynamic model that describes this system has 111 equations. The current $I(s)$ and the voltage $V(s)$ will be considered as input and output variables respectively. Thus, the transfer function

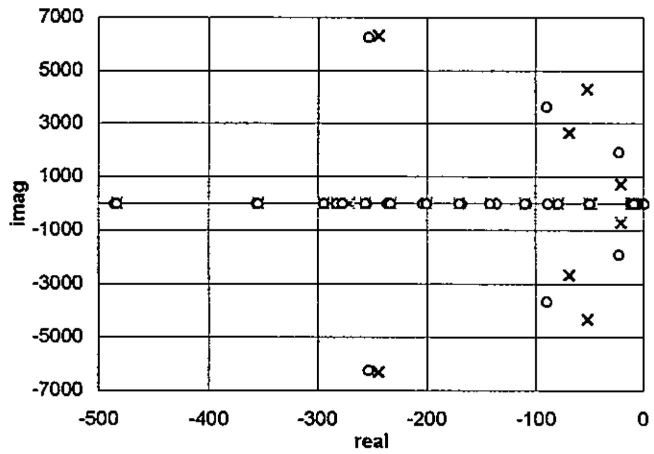
$$\frac{V(s)}{I(s)} = Z(s) \qquad (7)$$

is dimensionally equivalent to an impedance.

Figure 6.a shows the poles, while Figure 6.b shows both poles and the zeros of this transfer function. One must note that several poles are partially canceled by a nearby zero. A low-order approximation of this transfer function may therefore neglect these poles. The remaining poles are considered dominant poles of the transfer function.



(a). Poles



(b). Poles and Zeros

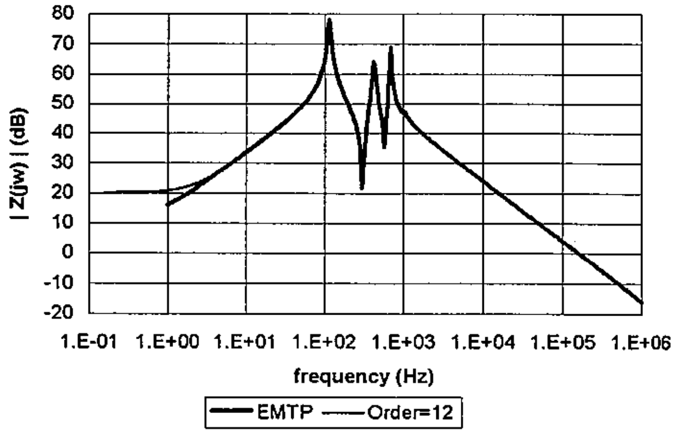
Figure 6. Transfer Function Pole and Zero Spectra

Table 1 presents the poles and associated residues of the reduced-order model considered in this paper. The reduced order model defined by these poles and residues has order 12. All the other poles of the system have residues with magnitude smaller than 0.3% of that of the greatest residue.

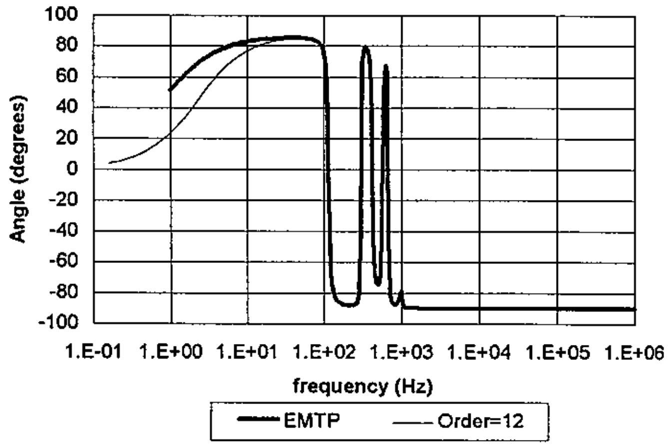
Table 1. Reduced Order Model Poles and Residues

POLE		RESIDUE	
real	imag	real	imag
-52.376	+ 4305.678	183102.907	-6742.172
-20.905	+ 722.580	169044.222	-1291.908
-69.236	+ 2666.716	139667.050	10278.789
-244.311	+ 6298.359	11196.520	-2795.859
-1462.140		-2632.120	
-1016.548		-2319.746	
-1480.442		-1417.564	
-270.856		-1029.676	

Figure 7 compares the frequency responses of the system, obtained through a frequency scan on the EMTP and on the 12th-order model. Note that the major difference lies in the lower frequency range (< 2 Hz).



(a). Magnitude Plot



(b). Phase Plot

Figure 7. Frequency Response of the Test System

Figure 8 shows the response (terminal voltage) of the test system depicted in Figure 5 to a step input (terminal current) obtained through an EMTP simulation.

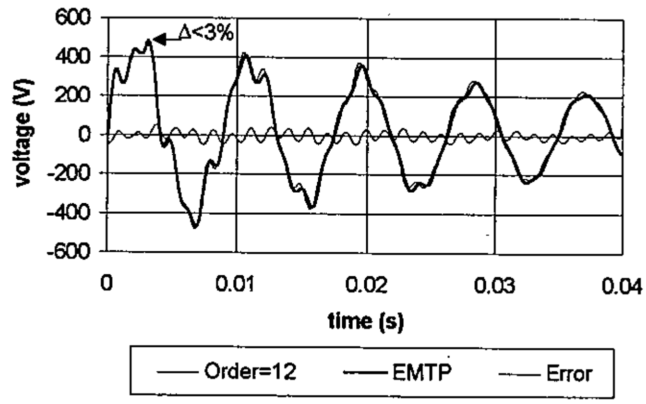


Figure 8. Step Response to Current Input

The response of the reduced-order model is also plotted for comparison and the peak value error is seen to be smaller than 3%.

4. CONCLUSIONS

The dominant pole spectrum algorithm [6, 7] was applied to obtain a reduced-order dynamic equivalent of a network containing a frequency-dependent transmission line modeled using the J. Marti's setup of the EMTP (FDLINE).

One of the contributions of this paper is to present a block diagram (Figure 2) for the 2x2 transfer function matrix representing the transmission line model. This block diagram clearly shows the interdependencies among the variables of interest and leads to a simple state space realization.

Frequency and time responses of a test system containing lumped RLC parameters and one transmission line with frequency-dependent parameters were obtained with the EMTP program and compared with those of the reduced-order model. The complete system model has 106 state variables, 102 of those due to the frequency-dependent line model. Note, however, that only 12 poles were maintained in the reduced-order model.

This 12^{th} order model is shown to be fairly accurate in both frequency and time domain and could be used as an equivalent of the system in electromagnetic transient analysis. The fitting of the reduced order model can be further improved by the inclusion of other sub-dominant transfer function poles.

Although the results presented were based on the FDLINE model [1, 8], the same technique can be applied to any line model based on rational approximations of the frequency-dependent parameters [4, 9, 14].

The technique described here can be readily extended for multi-phase systems. Work along this line is under way and will be reported in the near future.

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Leonardo T. G. Lima received his B.Sc. (1986) and M.Sc. (1991), both in electrical engineering, from Federal University of Rio de Janeiro, Brazil. Mr. Lima is a lecturer at the Department of Electrical Engineering, Universidade Federal Fluminense, and is presently working towards a Ph.D. degree at the Federal University of Rio de Janeiro. His current work and interests include power system dynamics, control theory applications and the development of large scale power system analysis tools.

Nelson Martins received the B.Sc. (1972) from the University of Brasilia, Brazil, the M.Sc. (1974) and Ph.D. (1978) degrees, from the University of Manchester Institute of Science and Technology (UMIST), UK, all in electrical engineering. Dr. Martins works at CEPEL, the Brazilian electrical energy research center, in the development of methods and computer tools for power system dynamics and control.

Sandoval Carneiro Jr. was born in Brasil in 1945. He received the degree of Electrical Engineer from the Catholic University of São Paulo (FEI), Brasil, in 1968, the M.Sc. from COPPE/UFRJ in 1971 and the Ph.D. degree in Electrical Engineering from the University of Nottingham, England, in 1976. From 1971 to date he has been a lecturer at the Federal University of Rio de Janeiro and was promoted to Full Professor in 1992. From 1982 to 1985 he was Director (Dean) of its Graduate School of Engineering (COPPE). From 1987 to 1988 and in 1994 he was a Visiting Professor with the University of British Columbia, Canada. Dr. Carneiro's research interests are on power system analysis and simulation of electromagnetic transients, including corona.