Comparison Measures for Benchmarking Time Domain Simulations

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Abstract — **Benchmark comparison of EMTP simulations to actual waveforms is often done by presenting two separate plots or by overplotting the two waveforms. To improve on this approach for cases of periodic and steady-state nonperiodic (chaotic) responses, a standard set of analytical comparison measures is identified. These include correlation coefficient, mean square error, RMS and average values, DFT, phase plane trajectories, and invariant measure. A software tool is developed. Measured and simulated waveforms from actual ferro-resonance cases are used to illustrate these comparisons.**

Key words: Benchmarking, waveform comparison, transient analysis, nonlinear responses, chaotic responses, EMTP.

I. INTRODUCTION

In the case of benchmarking EMTP simulations, little attention is paid to the methods with which simulated waveforms are compared to actual measurements. It seems typical to declare success when the simulated time domain waveform has "nearly" the same peak value and "looks like" the measured waveform. While this may be the only practicable method for transient responses, there are many possible analytical measures which can be used in the case of the steady-state and chaotic responses of a nonlinear system. A great need is therefore identified for some standard measures that can be used as a basis of comparison. Many different comparison methods are identified here. Their degree of usefulness is dependent on whether the waveforms in question are periodic or chaotic (nonperiodic). The work presented here was done in order to benchmark transformer models developed for ferroresonance simulations in a five-legged core distribution transformer [5,7,8,9].

For periodic waveforms, several measures may be utilized:

- Peak values.
- RMS values.
- Average values.
- DFTs (Discrete Fourier Transforms).
- Comparison of phase plane trajectories.
- Invariant measure.
- Calculation of a correlation coefficient.
- Mean square error.

Not all of these measures are useful when considering chaotic waveforms. Since sampled chaotic waveforms are of finite length and have no definite period, average values and RMS values are not as meaningful. Thus, a reduced set of conventional comparison measures is available, but measures from nonlinear dynamics and chaos theory might be applied [1,2,6,10]. The following approaches are identified as being useful for chaotic waveforms:

- Peak values.
- DFTs (distributed and discrete frequency components).
- Invariant measure.
- Visual inspection of Poincaré sections.
- Fractal dimension of Poincaré sections.

The comparison measures mentioned here will be defined or described in the next section. Most of these measures are then incorporated in a software package, illustrating their usage in the process.

II. DEVELOPMENT OF COMPARISON MEASURES

Comparison of peak magnitudes is most valuable for sharp-peaked waveforms such as ferroresonant voltages and currents or transformer exciting current at high levels of applied voltage. Peak values can be obtained by simply scanning the waveform data for minima and maxima.

The average value of a waveform is important when dc offsets are present. Even when offsets are not present, a dc value might be artificially introduced into measured waveforms if an oscilloscope is not properly zeroed. Calculating the average therefore provides a means of spotting erroneous data. For voltage and current waveforms that more closely resemble a distorted sinusoid, RMS values are a useful basis of comparison. The average value F_{AVG} and RMS value F_{RMS} of a function f(t) over a period of T seconds are given by the expressions

$$F_{AVG} = \frac{1}{T}\int_0^T f(t)\,dt \qquad (1)$$

and

$$F_{RMS} = \sqrt{\frac{1}{T}\int_0^T f^2(t)\,dt} \quad . \qquad (2)$$

DFTs show the spectrum (sinusoidal frequency components) of the waveform. There are a plethora of software packages that provide Fast Fourier Transform (FFT) capabilities for spectral estimation of uniformly-sampled signals. The number of points, NPTS, that the commonly used "radix 2" FFT algorithm operates on is a power of 2. The uniform time base Δt of the waveform and the frequency spacing Δf of the FFT are related by

$$\Delta f = \frac{1}{NPTS \times \Delta t} \quad . \tag{3}$$

Artifacts (side-lobe distortions in the shape and errors in the magnitude of the FFT-derived spectrum) may appear due to automatic "features" of the FFT software, such as zero padding and windowing [3]. In addition, "spreading" due to the choice of a time step incommensurate with the period of the waveform will occur, and the frequency magnitude will be in error. These effects were investigated in [7], and are all related to the relationship given by (3).

DFTs are a suitable comparison for both periodic and chaotic waveforms. With proper implementation, a discrete "line spectrum" can be obtained for periodic waveforms. For chaotic waveforms, a distributed spectrum will be observed.

Phase plane trajectories can be obtained by plotting a waveform versus its derivative [1,10]. This method was clearly demonstrated in [8]. The trajectories for periodic waveforms repeat, providing a closed-path. Overplotting the phase plane trajectories of two waveforms provides a better comparison than overplotting just the actual waveforms. Both the magnitude and slope of the waveforms can thus be compared. Phase plane trajectories are not useful for chaotic waveforms, since there is no closed path.

Invariant measure is the two-dimensional distribution of the magnitudes and derivatives of the sampled points of the phase plane trajectory [10]. The distribution of amplitude values of a waveform can be considered a reduced-order invariant measure. One very important restriction on the use of invariant measure is that the points must be uniformly sampled in time.

Time cross-correlation [4] of two waveforms $g_1(t)$ and $g_2(t)$ having the same period T can be determined as

$$R_{12}(\tau) = \frac{1}{T} \int_{-T/2}^{+T/2} g_1(t)\, g_2(t+\tau)\, d\tau \quad . \tag{4}$$

Cross-correlation can be calculated for various shifts of τ, but to determine how alike two waveforms are, it is necessary to compare them when they are most nearly superimposed. Some way of synchronizing the waveform data must therefore be implemented. Matching of the positive-going zero crossings of the two waveforms seems be the most logical method of synchronization. A value of $\tau=0$ shall be defined to mean that the positive-going zero crossings of the two waveforms coincide. Positive values of τ mean that g_2 has been shifted backward relative to g_1.

Values of R_{12} must be normalized so that the cross-correlations for families of waveforms can be compared. The geometric mean of the autocorrelations $R_1(\tau)$ and $R_2(\tau)$ of the two waveforms evaluated at $\tau=0$ is used for this normalization. ($R_1(0)$ and $R_2(0)$ are the average powers of the signals $g_1(t)$ and $g_2(t)$, so normalizing in this way ensures that R_{12} will always be less than or equal to 1.0, with identical waveforms having a correlation coefficient of 1.0). R_{12} from the above equation is thus redefined as

$$R_{12}(\tau) = \frac{1}{T\sqrt{R_1(0)R_2(0)}} \int_{-T/2}^{+T/2} g_1(t)\, g_2(t+\tau)\, d\tau \quad . \tag{5}$$

Mean square error is defined as

$$MSE = \frac{\int_0^T \left[g_2(t+\tau) - g_1(t)\right]^2 dt}{\int_0^T g_1^2(t)\, dt} \tag{6}$$

where the square error between g_1 and g_2 integrated over the interval T is normalized to $g_1{}^2$ integrated over the same interval. In this case, g_1 is assumed to be the "true" value for the basis of comparison. g_2 is shifted by the same τ as described in (4) and (5).

A Poincaré section [1,10] is a set points synchronously sampled from the phase plane trajectory. Typically, the sampling is done once each period of the forcing function (for example, once each 60-Hz period). This is advantageous in the case of a chaotic waveform that has a blurred nonrepeating phase plane trajectory. Displaying the sampled points on the phase plane gives a convenient qualitative basis of comparison.

Since Poincaré sections are the synchronously sampled magnitude and derivative of the waveform, a great deal of information is at hand. The shape of the Poincaré section is highly sensitive to the behavior of the system being observed. Poincaré sections composed of points that are more uniformly distributed over the phase plane characterize a system having relatively little damping. Poincaré sections whose points are constrained to a smaller area of the phase plane are more highly "dissipative" [10].

Fractal dimension provides a numerical measure and basis for categorization and comparison of Poincaré sections. An algorithm to calculate fractal dimension was developed in [7] but was not implemented in the software to be discussed here.

III. PROGRAM IMPLEMENTATION

Many of the above comparisons were implemented in a program named "XCOMP," an acronym for "cross-comparison." The program allows comparison of both periodic and nonperiodic waveforms. All examples presented here are actual laboratory and simulation data [7,8].

Use of XCOMP is fairly simple. The user is prompted for the names of the two input data files. Binary data files having the EMTP .PL4 extension and the .ALL extension of the laboratory digital storage oscilloscope's waveform files are recognized. A preview display of the waveforms allows visual inspection prior to comparison, avoiding erroneous comparisons of unlike waveforms. Based on visual inspection, the user then specifies the period of the waveform (period 2 implies 30-Hz subharmonics, period 3 implies 20 Hz, etc.) and where along each waveform XCOMP is to begin searching for zero crossings to use for synchronization. This feature is especially useful for bypassing the transients occurring at the beginning of an EMTP simulation and for choosing a consistent starting point for waveforms which have subharmonics. The waveforms typically will have different time step sizes, requiring conversion of the data's time base. Linear interpolation is used. XCOMP then matches the waveforms' zero crossings, calculates required statistics, and plots the results on the screen, as shown in Figs. 2 and 3.

Fig. 2 shows output for the comparison of period one waveforms, demonstrating the capabilities of XCOMP. Waveforms are shown in upper left corner. Invariant measure is shown in lower right, DFT in lower left.

Equation (5) is implemented such that minor shifts in τ were made in the neighborhood of $\tau=0$, until a maximum value of R_{12} is found. Visual inspection of the two waveforms would indicate that they are nearly identical. The correlation coefficient of 0.9811 would also indicate close agreement. Mean square error has been found to be a much more sensitive and useful comparison. It has not yet been implemented, but will be added to XCOMP in the future.

The phase plane diagram is much better able to distinguish slight differences in the waveforms. Small differences in magnitude and slope are much more evident.

Invariant measure displays differences in distribution of the magnitudes of the data points. The magnitudes of all waveform points within the interval being analyzed are rounded to the nearest of 128 quantizing levels. The number of points at each level is plotted against the magnitudes of those levels. Note that the waveform traced by the solid line has data points whose amplitudes are more uniformly distributed than the other waveform. A triangle wave would have uniformly distributed amplitudes.

Mean values are both very small, as is expected. Extrema and RMS values are also given.

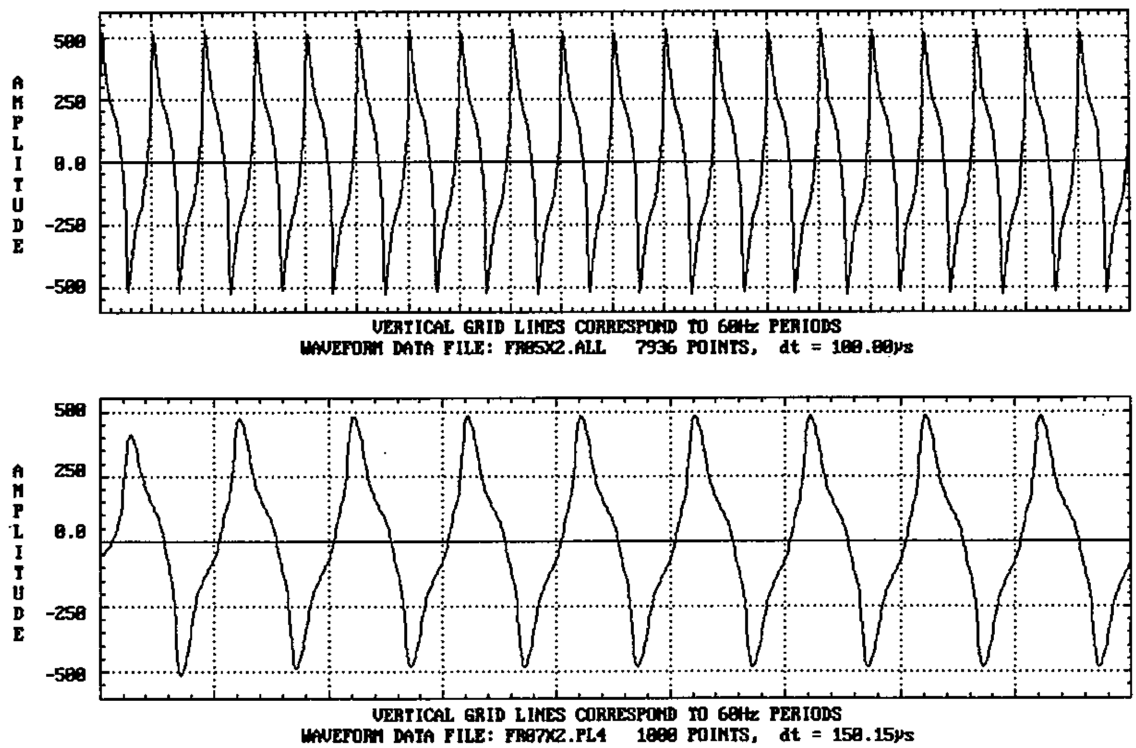

Fig. 1 - Demonstration of XCOMP's preview feature.

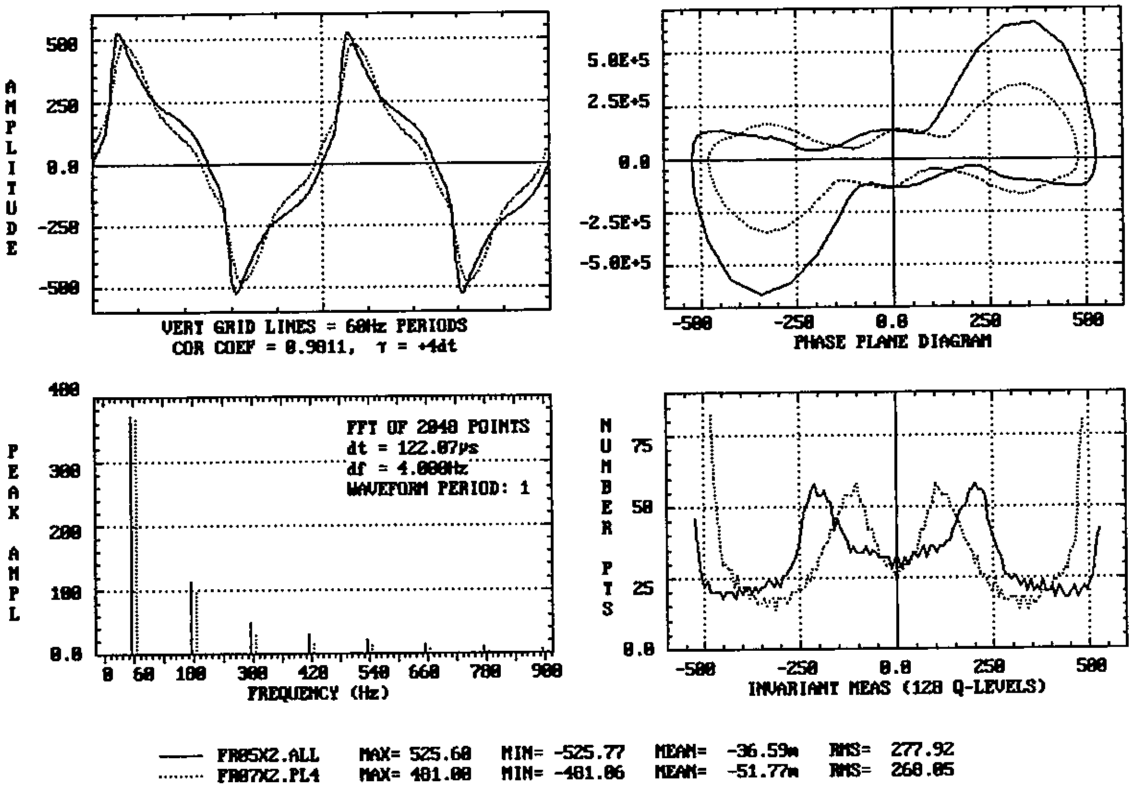

Fig. 2 - Example of graphical output from XCOMP program for comparison of periodic waveforms.

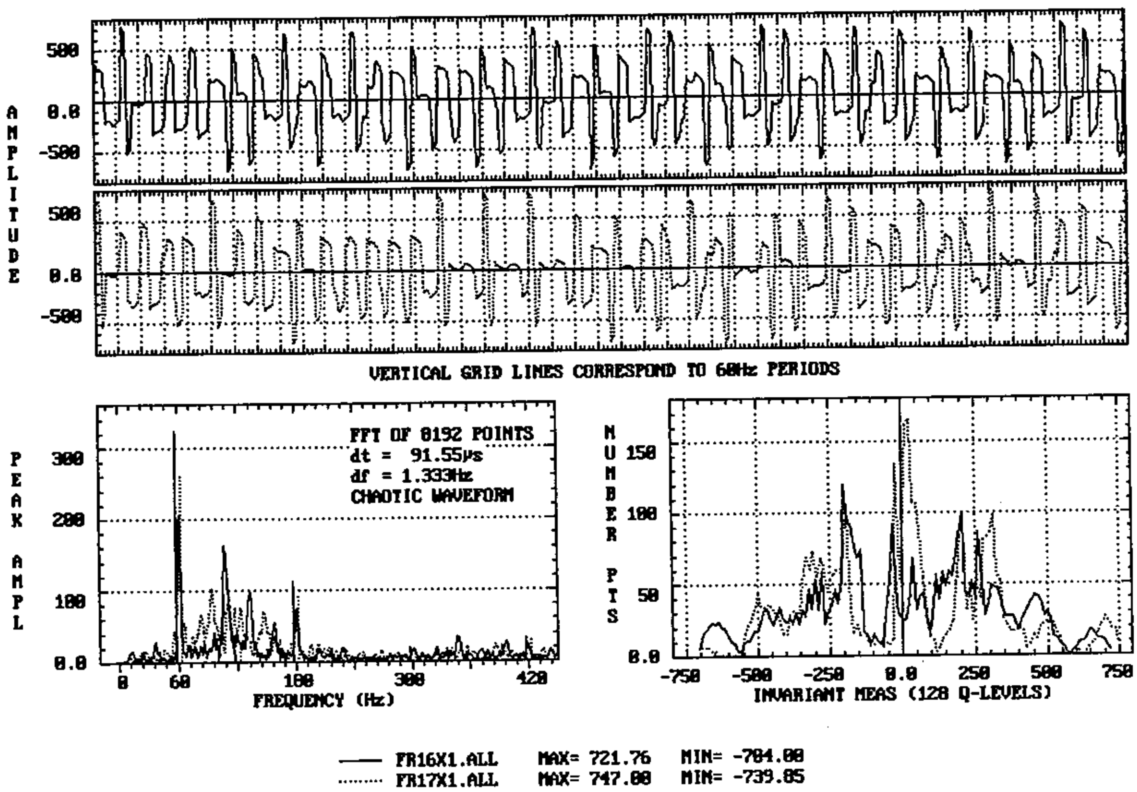

Fig. 3 - Comparison of chaotic waveforms using XCOMP program.

The DFTs are nearly in agreement. The implementation used here is based on relationships given by (3). When the time base of the waveforms is adjusted, a Δt is chosen such that NPTS is exactly a whole number of cycles, and Δf ensures that frequency components provided by the FFT fall exactly on the 60-Hz harmonics and subharmonics. No windowing is used. This results in line spectra. When a commercial software package was used to perform an FFT on the same waveforms, it automatically applied zero padding and windowing. It did not yield integer frequency spacing. Problems with side lobes, frequency spreading and incorrect magnitude scaling were evident. When performing an FFT on periodic waveforms, XCOMP never requires zero padding or windowing and will always provide a correctly scaled line spectrum. Selection of Δt is illustrated in Table 1.

Δt	NPTS	Δf, Hz	60-Hz Cycles
91.553μs	8192	1.333	45.00
100.00μs	2048 4096 8192	4.883 2.441 1.220	12.29 24.57 49.15
122.07μs	2048 4096 8192	4.0 2.0 1.0	15.00 30.00 60.00
244.14μs	1024 2048 4096	4.0 2.0 1.0	15.00 30.00 60.00

Table 1 - Selection of Δt when performing FFT.

Fig. 3 shows an example of XCOMP output when comparing two chaotic waveforms. Only the extrema, invariant measures and the DFTs can be shown for this type of waveform. Note the distributed frequency spectrum that is characteristic of a chaotic signal. Poincaré sections and fractal dimension were not implemented in XCOMP, but have been investigated [7,8]. Since only one point can be sampled per 60-Hz cycle, a very long waveform sample would be required to obtain the 3000 to 8000 points needed. The subject of fractal dimension and its usefulness as a comparison measure shall be addressed in a future paper.

IV. CONCLUSIONS

Many waveform comparison methods were identified and discussed. A normalized correlation measure was developed and implemented. Mean square error measure was identified as being a more sensitive comparison measure. Waveforms of standard Δt such as 100 μs must be resampled to avoid FFT-induced artifacts in the resulting spectrum. It may be useful to identify and evaluate more comparison measures for chaotic waveforms.

XCOMP is a useful tool which provides a meaningful set of analytic comparison measures for benchmarking. It will be ported over to more modern operating systems and extended to allow input of additional data file formats.

V. ACKNOWLEDGEMENTS

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