# Lightning Induced Voltages in Low-voltage Systems with Emphasis on Lossy Ground Effects

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*Abstract* —**A method of calculating the lightning induced voltages in power networks has been developed, taking the lossy ground effect on the electrical fields into account. The induced voltage in an overhead line terminated by impedances and it's dependency of stroke location and lossy ground effects is analysed using analytical equations.**

**A simple model for calculating induced voltages in larger power networks has been developed as well and implemented in ATP-EMTP using MODELS. The lossy ground dependent induced voltages are calculated and analysed for different network parameters and stroke location.**

**Loss effects on the induced voltage in an overhead line are seen to be important even for short distances (100-1000 m). The loss effect is mainly due to a reduction in the horizontal electrical field propagating over ground, so an overhead line shorter than 1000 m can in fact be assumed lossless. A line terminated by the characteristic impedance is very sensitive to loss effects. An arrester installed on the distribution transformer does not necessarily result in lower voltage at the other end of the overhead line.**

**Keywords** : Lightning induced voltage, loss effects, EMTP.

## I. INTRODUCTION

Induced overvoltage due to lightning is an important source of insulation failures in low-voltage systems. An optimal voltage protection of low-voltage systems is dependent on the ability to predict the overvoltages induced by lightning. The voltages are strongly influenced by ground losses, however, and the accuracy of the currently used methods for induced voltage calculations has been questioned, especially for nearby lightning which actually is of main interest.

Norton's method [1,2] is in this paper used to take lossy ground effects into account. This method has also been used in [3] to study loss effects on nearby lightning, showing large effect even for short distances.

An efficient vector potential formulation is used to obtain analytical expressions for induced voltages in an overhead line. This formulation results in an improved understanding of the influence of line terminations and stroke locations.

The models presented in e.g. [5, 6] do not take lossy ground effects into account, so a simple model has been developed in ATP [4] for studying more complex electrical networks. The line model used in this paper is simple compared to the more generalized model in [7], e.g. the overhead line is assumed to be lossless, which is a good approximation for short lines.

## II. VECTOR POTENTIAL FORMULATION

The vector potential $A$ will be used in this paper to calculate the induced voltage on an overhead line. The advantage with this formulation is that Norton's method for taking lossy ground effects into account, can be used directly. Besides, the vertical and horizontal electrical fields can both be expressed by the vector potential, making it easier to calculate the total line voltage and analyse it's dependency of the line terminations.

By choosing the *Lorentz gauge* for the vector potential divergence, the electrical fields can be expressed by the vector potential [2]:

$$E_x = \frac{c^2}{j\omega} \cdot \frac{\partial^2 A_\sigma}{\partial x \partial z} \quad (1)$$

$$E_z = \frac{c^2}{j\omega} \cdot \frac{\partial^2 A_\sigma}{\partial z^2} - j\omega \cdot A_\sigma \quad (2)$$

where $A_\sigma$ is the z-component of the vector potential.

### A. Basic configuration

The analysis made in this paper is based on the configuration shown in fig.1

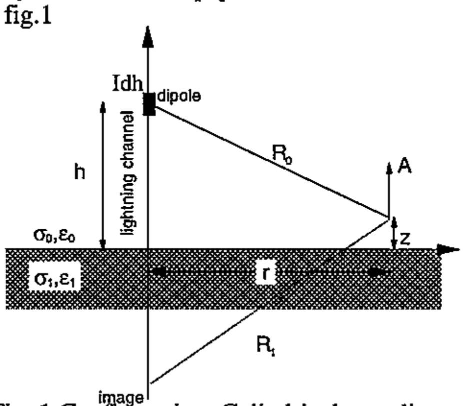


Fig. 1 Configuration. Cylindrical coordinates.

The lightning channel is assumed to be vertical, as shown in fig. 1. This configuration is used to calculate the vector potential caused by the current element $I \cdot dh$ at a height $h$. The observation point has a horizontal distance $r$ from the lightning channel and a height $z$. $R_0$ and $R_1$ are the distances from the dipole and its image respectively. The ground has a conductivity $\sigma_1$ and a permittivity $\epsilon_1 = \epsilon_0 \epsilon_r$.

### B. Lossy ground effect

Norton has formulated an approximation to the vector potential from a vertical current element over flat, homogenous, lossy ground, assuming a high ground conductivity [1, 2]. The vector potential, $dA_\sigma$, from a current element $I{\cdot}dh$ can be written as a sum of a lossless contribution $dA_0$ and a lossy ground contribution $dA_\Delta$:

$$dA_\sigma = dA_0 + dA_\Delta \qquad (3)$$

with

$$dA_0 = \frac{\mu_0 \cdot I \cdot dh}{4\pi} \cdot \left\{ \frac{e^{-jk_0R_0}}{R_0} + \frac{e^{-jk_0R_1}}{R_1} \right\} \qquad (4)$$

$$dA_\Delta = f_0 \cdot f_1 \cdot f_w \cdot I \cdot dh \qquad (5)$$

and where

$$f_0 = -\frac{\mu_0}{4\pi} \cdot 2u \sqrt{\frac{jk_0\pi}{2}} \qquad (6)$$

$$f_1 = \sqrt{R_1} \cdot \frac{e^{-jk_0R_1}}{r} \qquad (7)$$

$$f_w = e^{-w} erfc(j\sqrt{w}) \qquad (8)$$

$$w = \frac{-jk_0R_1^3}{2r^2} \cdot (\cos\theta_r + \Delta_0)^2 \qquad (9)$$

$$\Delta_0 = u \cdot \sqrt{1 - u^2 \sin^2\theta_r} \approx u \qquad (10)$$

$$u = \sqrt{\frac{j\omega\epsilon_0}{\sigma_1 + j\omega\epsilon_1}} \qquad (11)$$

$$\cos\theta_r = (z+h)/R_1,\ \sin\theta_r = r/R_1$$
$$R_1^2 = r^2 + (z+h)^2,\ R_0^2 = r^2 + (z-h)^2$$

and where $erfc$ is the complementary error function

### C. Lightning channel model

The vector potential from a current element ($I{\cdot}dh$) is given by (3). To calculate the total potential from a lightning channel, the corresponding current must be accounted for.

The simple Modified Transmission Line model (MTL) [8] has been chosen for this purpose. The MTL model has been evaluated [9] showing reasonable agreement with measurements for 5 km distances. The MTL model assumes that the current starts from ground and travels upward with a constant velocity $v$. The amplitude of the current is exponentially decaying with increasing height, with decaying constant $\lambda$. The current at the ground level is further assumed to be a step with amplitude $I_0$. In the frequency domain the current can be written:

$$I(h,j\omega) = \frac{I_0}{j\omega} \cdot e^{-h \cdot (j\omega/v + 1/\lambda)} \qquad (12)$$

An arbitrary current shape can be taken into account by a convolution based on the step response, since the system is assumed to be linear. If $g_0(t)$ is the step response of current $I_0$, then the response of a time varying current $i(t)$ reads

$$g(t) = \int_0^t \frac{g_0(t-\tau)}{I_0} \cdot \frac{\partial i(\tau)}{\partial t} \cdot d\tau \qquad (13)$$

The lightning current at ground can be approximated by e.g. the Heidler-model [10]:

$$i(0,t) = \frac{I_a}{\eta} \cdot \frac{(t/\tau_1)^m}{(t/\tau_1)^m + 1} \cdot e^{-t/\tau_2} \qquad (14)$$

where $I_a$ is the current amplitude, $\tau_1$ is the front time constant, $\tau_2$ is the half value time constant, m is an exponent {2..10} and $\eta$ is the amplitude correction factor:

$$\eta = \exp[-(\tau_1/\tau_2) \cdot (m \cdot \tau_2/\tau_1)^{1/m}] \qquad (15)$$

### D. Total vector potential

The total vector potential from a vertical lightning channel equals

$$A_\sigma = \int_0^H dA_\sigma = \int_0^H dA_0 + \int_0^H dA_\Delta = A_0 + A_\Delta \qquad (16)$$

where $H$ is the total height of the lightning channel.

Assuming a current according to (12) and using the Norton's approximation, this gives

$$A_0 = \frac{\mu_0}{4\pi} \cdot \frac{I_0}{j\omega} \cdot \int_0^H \left( \frac{e^{-jk_0R_0}}{R_0} + \frac{e^{-jk_0R_1}}{R_1} \right) \cdot e^{-h \cdot (j\omega/v + 1/\lambda)} dh \qquad (17)$$

$$A_\Delta = \frac{I_0 \cdot f_0}{j\omega} \cdot \int_0^H f_1 \cdot f_w \cdot e^{-h \cdot (j\omega/v + 1/\lambda)} \cdot dh \qquad (18)$$

## III. INDUCED VOLTAGE CALCULATION

In this chapter analytical equations for the frequency domain induced voltage components in an overhead line are deduced. Equations for the total voltage induced in a line terminated by impedances are presented and the voltage's dependency on lossy ground effects is analysed. Only the voltage at the line's terminals can be calculated by the outlined method, but a line can be split in several segments to calculate the voltage along the line.

### A. Line configuration

The configuration used when calculating the induced voltage in an overhead line is shown in fig. 2. The overhead line has two terminals A and B with x-coordinates $x_A$ and $x_B$ respectively ($x_A > x_B$). The length of the overhead line is $L = x_A - x_B$.

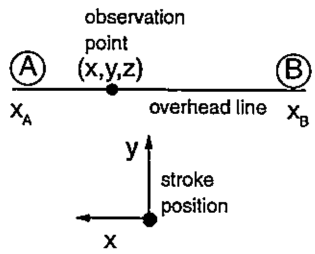


Fig. 2 Induced voltage configuration

The coupling between the induced fields and the overhead line is modeled by the Agrawal model [11]. The total line voltage is in this model a sum of the scattered voltage $U^S$ and the incident voltage $U_Z$.

$$U(j\omega) = U^S(j\omega) + U_z(j\omega) \tag{19}$$

where

$$U_z(j\omega) = -\int_0^z E_z(j\omega, z)\cdot dz \tag{20}$$

The following equations are valid for $U^S$:

$$\frac{\partial U^S}{\partial x} + Z\cdot I = E_x \tag{21}$$

$$\frac{\partial I}{\partial x} + Y\cdot U^S = 0 \tag{22}$$

The terminations of the overhead line can be included as shown in fig. 3 when the line is terminated by impedances.

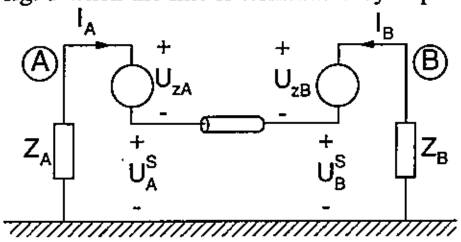


Fig. 3 Overhead line terminations.

*B. Total voltage*

The scattered voltage at terminal $A$, $U_A^S$, can be expressed as a sum of an incoming voltage wave $U_{inA}$ and a reflected wave $U_{refA}$ dependent on the termination:

$$U_A^S = U_{refA} + U_{inA} \tag{23}$$

The incoming voltage wave to terminal A consists of the time delayed, reflected wave from the other terminal and a contribution, $U_{xA}$, from the horizontal electrical field [5]:

$$U_{inA}(j\omega) = U_{xA}(j\omega) + U_{refB}(j\omega)\cdot e^{-\gamma\cdot L} \tag{24}$$

where

$$\gamma = \sqrt{Z\cdot Y} \tag{25}$$

is the transmission coefficient of the overhead line and where the incoming wave to an overhead line terminal ($x_A$) due to the horizontal electrical field $E_x$ is given by [5]:

$$U_{xA} = \frac{1}{2}\cdot\int_{x_B}^{x_A} E_x(j\omega, x)\cdot e^{-\gamma\cdot(x_A - x)}\cdot dx \tag{26}$$

The incoming voltage wave to the other terminal B, reads:

$$U_{xB} = \frac{1}{2}\cdot\int_{x_A}^{x_B} E_x(j\omega, x)\cdot e^{\gamma\cdot(x_B - x)}\cdot dx \tag{27}$$

The current into terminal A can be expressed as:

$$I_A = \frac{1}{Z'}\cdot\left(U_{refA} - U_{inA}\right) \tag{28}$$

where

$$Z' = \sqrt{Z/Y} \tag{29}$$

Assuming a terminating load $Z_A$, the current into the overhead line can be expressed as

$$I_A = -\frac{U_A^S + U_{zA}}{Z_A} \tag{30}$$

The same equations (as 23-30) can be developed for line termination B, by just changing index A to B.

Equations (19, 23, 24, 28, 30) can be used to express the total line voltage $U$ as a function of $U_X$ and $U_Z$ and the parameters of the line and it's terminations:

$$U_A = \frac{\alpha_A}{(1 - e^{-2\gamma L}\cdot\rho_A\cdot\rho_B)}\cdot \left[U_{xA} + U_{xB}\cdot e^{-\gamma L}\cdot\rho_B - U_{zA}\cdot e^{-2\gamma L}\cdot\rho_B\cdot\alpha_A\cdot\frac{Z'}{2\cdot Z_A} - U_{zB}\cdot e^{-\gamma L}\cdot\alpha_B\cdot\frac{Z'}{2\cdot Z_B}\right] + U_{zA}\cdot\frac{\alpha_A}{2} \tag{31}$$

Changing index A to B and vice versa gives the line voltage at terminal B.

$\rho$ is the reflection and $\alpha$ is the transmission coefficients of the termination:

$$\rho_A = \frac{Z_A - Z'}{Z_A + Z'},\quad \alpha_A = \frac{2\cdot Z_A}{Z_A + Z'} \tag{32}$$

*C. Horizontal and vertical voltage components*

Replacing the horizontal electrical fields in (20, 26) by the vector potential formulation in (1, 2) gives:

$$\begin{aligned} U_{xA} &= \frac{c^2}{2j\omega}\cdot\int_{x_B}^{x_A}\frac{\partial^2 A_\sigma}{\partial x\partial z}\cdot e^{-\gamma\cdot(x_A - x)}\cdot dx \\ &= \frac{c^2}{2j\omega}\cdot\left(\left.\frac{\partial A_\sigma}{\partial z}\right|_{x_A} - \left.\frac{\partial A_\sigma}{\partial z}\right|_{x_B}\cdot e^{-\gamma\cdot L} - \gamma\cdot\int_{x_B}^{x_A}\frac{\partial A_\sigma}{\partial z}\cdot e^{-\gamma\cdot(x_A - x)}\cdot dx\right) \end{aligned} \tag{33}$$

The incident voltage can also be expressed by the vector potential:

$$
\begin{aligned}
U_{zA} &= -\int_0^z E_z(z,x_A)\cdot dz \\
&= -\int_0^z \left( \frac{c^2}{j\omega}\cdot\frac{\partial^2 A_\sigma}{\partial z^2} - j\omega\cdot A_\sigma \right)\cdot dz \qquad (34)\\
&\approx -\frac{c^2}{j\omega}\cdot\left.\frac{\partial A_\sigma(x_A)}{\partial z}\right|_0^z + j\omega\cdot z\cdot A_\sigma(x_A)_{z=0}
\end{aligned}
$$

The first two $\partial A_\sigma/\partial z$ terms in (33) are dominant. They will, however, often be balanced out by the $\partial A_\sigma/\partial z$ term in (34) (and the incident contribution from terminal B), evaluated at height z. The $\partial A_\sigma/\partial z$ term in (34) evaluated at height 0 is zero in a lossless situation, but becomes a very dominant contribution to the induced voltage when lossy ground effects are taken into account.

*D. Loss examples*

In this chapter, some calculation examples are performed to illustrate the influence of loss effects on induced voltages. The calculated induced voltages are shown in figures 5-9. Two stroke locations will be used as shown in fig. 4, and a step current is assumed. The voltage *UA* in figures 5- 9 is the voltage between terminal A and ground, taking loss effects into account, while *UA0* is the corresponding lossless voltage.

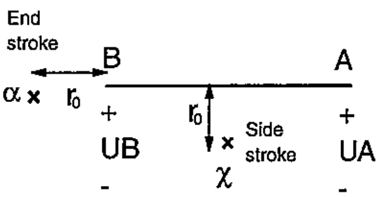


Fig. 4 Loss example configuration

- Lightning current parameters: $I_0 = 1$ A, $\lambda = 1500$ m and $v$=1,1·10$^8$ m/s. Lightning channel height $H$=4000 m.
- Line parameters: height $z$=5 m, diameter $d$=5 mm (Cu), length $L$=500 m.
- Ground characteristic: Uniform, single layer with $\sigma_1 = 0.01$ S/m and $\epsilon_r = 10$.

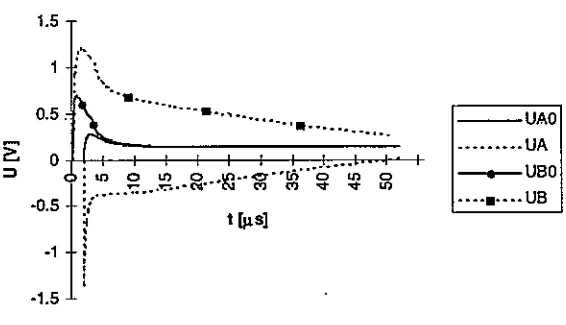


Fig. 5. $Z_A$=$Z_B$ = Z' End stroke (pos. α), $r_0$ =100m

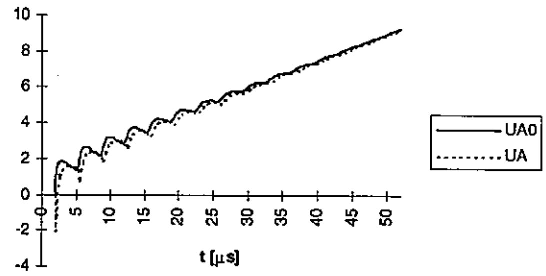


Fig. 6. $Z_A$=$Z_B$ = 1MΩ, End stroke (pos. α), $r_0$ =100m

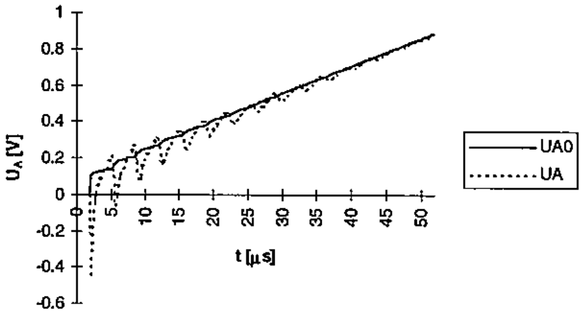


Fig. 7. $_{SA}$=$Z_B$ = 1 MΩ, End stroke (pos. α), $r_0$ =1000m

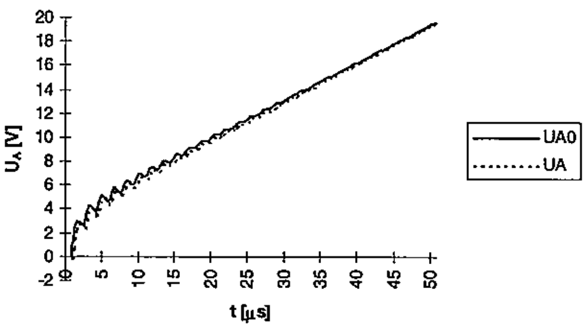


Fig. 8. $Z_A$=$Z_B$ = 1 MΩ, Side stroke (pos. χ), $r_0$ =100m

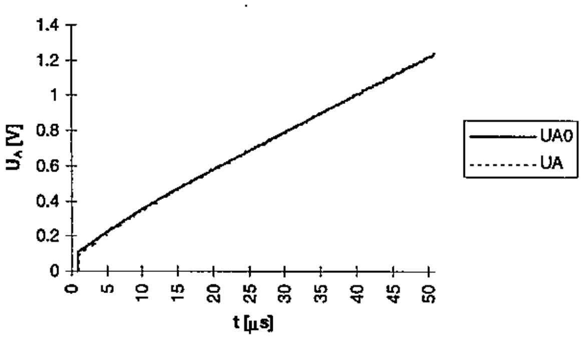


Fig. 9. $Z_A$=$Z_B$ = 1MΩ, Side stroke (pos. χ), $r_0$ =1000m

## IV. ATP-EMTP MODEL

Fig. 10 shows an equivalent circuit used to calculate induced voltages in an overhead line by ATP. This line is assumed lossless. The circuit in fig. 10 consists of two resistors, $Z'$, equal to the line's characteristic impedance and two type 60 sources calculated by MODELS.

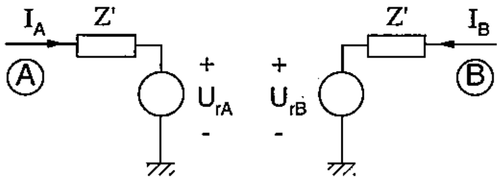


Fig. 10 EMTP model of overhead line.

where

$$U_{rA}(t) = U_{indA}(t) + U_B(t-\tau) + Z'\cdot I_B(t-\tau) \quad (35)$$

$$U_{rB}(t) = U_{indB}(t) + U_A(t-\tau) + Z'\cdot I_A(t-\tau) \quad (36)$$

and where

$$U_{indA}(j\omega) = 2\cdot U_{xA} + U_{zA} - U_{zB}\cdot e^{-\gamma L} \quad (37)$$

$$U_{indB}(j\omega) = 2\cdot U_{xB} + U_{zB} - U_{zA}\cdot e^{-\gamma L} \quad (38)$$

A simple model has been developed to calculate induced voltages in a lossless line. Fig. 10 is implemented in ATP and the voltages in (35-36) are calculated by MODELS (shown in app. B). The induced voltage terms in (37-38) are precalculated in the frequency domain and inverse Fourier transformed by a separate program, taking lossy ground effects on the electrical fields into account.

The induced voltage components formulated in (37-38) are very sensitive to lossy ground effects. They are twice the voltage induced in a matched termination line shown in fig. 5. Using the four induced voltage terms $U_{xA}$, $U_{xB}$, $U_{zA}$ and $U_{zB}$ directly in ATP will give numerical instability since they tend to cancel each other.

### *A. ATP Example*

The induced voltage in a low-voltage network, shown in fig. 11, has been calculated by usage of the model in fig. 10. The network consists of a 500 m long overhead line 5 m above ground. The line is assumed to be lossless with a characteristic impedance $Z' = 500\ \Omega$ and $v=c$. The overhead line is connected to an underground cable with length $L_C$ and characteristic impedance $Z_C = 40\ \Omega$ and wave velocity $v_C = 2\cdot 10^8$ m/s. It is assumed that no voltage is induced directly in the underground cable. The other end of the overhead line is connected to a transformer, modelled as a capacitance of 1 nF. The other end of the cable is connected to a domestic installation modelled as a capacitance of 20 nF. Four different stroke locations (α-δ), all with distance 100 m to the overhead line, have been analysed.

The lightning current at ground follows (14) with $I_a = 30$ kA, $\tau_1 = 1$ μs, $\tau_2 = 50$ μs and m=2.

The ground has conductivity $\sigma_1 = 0.01$ S/m and relative permittivity $\epsilon_r = 10$.

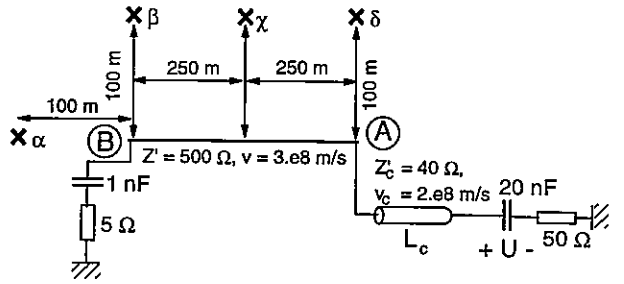


Fig. 11 Low-voltage network and stroke locations.

An arrester is also introduced, having the characteristic shown in tab. 1, with grounding resistance 5 Ω:

Tab. 1 Arrester characteristic

| I [A] | $10^{-4}$ | $10^{-2}$ | 1 | 10 | 100 | $10^3$ | $10^4$ |
|---|---|---|---|---|---|---|---|
| U [V] | 350 | 450 | 500 | 550 | 625 | 750 | $10^3$ |

Tab. 2 shows the maximum induced voltage across the installation without arrester (-), or the arrester installed in (A) or (B) with the four stroke locations (α-δ) and the cable length $L_c$ as parameters.

Tab. 2 Maximum induced voltage, $U_{max}$ [kV]

| $L_C$ | 50m | | | 100m | | | 200m | | |
|---|---|---|---|---|---|---|---|---|---|
| Arr. | A | B | - | A | B | - | A | B | - |
| α | 1.14 | 62.6 | 26.0 | 1.20 | 60.9 | 22.0 | 1.54 | 59.1 | 16.7 |
| β | 1.24 | 57.9 | 35.6 | 1.30 | 57.3 | 30.1 | 1.73 | 55.6 | 22.7 |
| χ | 1.44 | 6.94 | 53.2 | 1.49 | 6.26 | 44.7 | 1.75 | 5.45 | 34.1 |
| δ | 1.36 | 55.6 | 33.3 | 1.41 | 55.1 | 28.2 | 1.86 | 54.3 | 21.9 |

The results obtained when an arrester is installed at point B can be shown from (31) assuming terminal B to be grounded ($\rho_B$=-1, $\alpha_B$=0). Inserting the expressions for the incoming voltage wave (33) and the incident voltage (34) gives the following expression for the voltage at terminal A:

$$U_A = \frac{c^2}{j\omega}\cdot\frac{2\cdot Z_A}{Z_A+Z'}\cdot\frac{\frac{\partial A}{\partial z}(x_A,0)\cdot\cosh(\gamma\cdot L)-\frac{\partial A}{\partial z}(x_B,0)}{e^{\gamma\cdot L}+\rho_A\cdot e^{-\gamma\cdot L}}+R \quad (39)$$

where R is the sum of the integration terms from $U_x$ and the radiation terms from $U_z$ which are small compared to the ∂A/∂z terms. The terminating cable impedance $Z_A$ (40 Ω) results in $\rho_A$=-0.85. The ∂A/∂z terms evaluated at z=0 are zero in a lossless situation, but about equal to the ∂A/∂z terms evaluated at the line heigh z for a lossy ground. Eq. (39) thus shows that the configuration in fig. 11 is very influenced by a lossy ground. When the stroke location is on the mid-point of the line (χ) the two ∂A/∂z(z=0) terms in (39) will for moderate frequencies balance each other due to symmetry. In this situation the R term in (39) will be dominant. When no arrester is installed the voltage at A can be approximated by, assuming the transformer to be an open end:

$$U_A = \frac{c^2}{j\omega}\cdot\frac{2\cdot Z_A}{Z_A+Z'}\cdot\frac{\frac{\partial A}{\partial z}(x_A,0)\cdot\sinh(\gamma\cdot L)}{e^{\gamma\cdot L}-\rho_A\cdot e^{-\gamma\cdot L}}+R \tag{40}$$

which is less sensitive to lossy ground effects and not so significantly influenced by stroke locations.

## V. DISCUSSION

Norton's method has been evaluated in [12] showing a deviation < 10 % from the exact solution for distances 100 m - 10 km and resistivity 100 Ωm.

Losses in the overhead line itself is of minor importance compared to the losses in the electrical fields from the lightning channel, due to propagation over ground.

Fig. 5 shows that an overhead line terminated by the characteristic impedance is very sensitive to loss effects, especially for an end stroke. At the far end, the voltage even change polarity compared to the lossless situation. The induced voltages in a matched termination line are equivalent to the induced voltage terms formulated in (37-38).

Figures 6-7 show that the lossy ground effect increases with distance for an end stroke. The induced voltage in this configuration is strongly dependent on the horizontal electrical field which is highly affected by the lossy ground.

Figures 8-9 show a reduced lossy ground effect with increasing distance. This is also reasonable since the side stroke configuration becomes less dependent on the horizontal electrical field with increasing distance. If all points along an overhead line have the same distance to the lightning stroke, the induced voltage is then dependent on the vertical electrical field only. For large distances, the side stroke configuration approaches such situation. A longer overhead line will result in increased loss effect.

Tab.2 shows that the maximum induced voltage across the domestic installation often increases when an arrester is installed at the transformer (B), compared to a no-arrester situation. This phenomenon is caused by the lossy ground. If the ground is lossless, the column B voltages are reduced substantially compared to the values given in tab. 2

## VI. CONCLUSIONS

Lossy ground effects is important to consider for end stroke configurations. An overhead line terminated by it's characteristic impedance is sensitive to lossy ground effects.

Losses in an overhead line of length <1000 m is negligible compared to losses in the exciting electrical fields.

An arrester installed at the distribution transformer does not necessarily give a reduction in induced voltage at the other end of an overhead line. This theoretical result is rather surprising and needs to be verified by measurements before any final conclusions can be drawn.

## APPENDIX

### *A. Model for induced voltage calculations*

The model shown below is used in ATP-EMTP's MODELS to calculate induced voltage in an overhead line. The induced voltage sources `UINA` and `UINB` are calculated by a separate program and included as type 1-2 sources.

```
MODEL INDUS
CONST c     {VAL:3.e8}
INPUT IA,IB,UINA,UINB,UA,UB
DATA Z,L
OUTPUT UrA,UrB
VAR UrA,UrB,Tr,UrefA,UrefB
HISTORY
  UrefA {dflt:0}
  UrefB {dflt:0}
INIT
  Tr:=L/c
ENDINIT
EXEC
  UrefB:=(UB+Z*IB)
  UrefA:=(UA+Z*IA)
  UrA:=UINA+Delay(UrefB,Tr-timestep,1)
  UrB:=UINB+Delay(UrefA,Tr-timestep,1)
ENDEXEC
ENDMODEL
```

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