FREQUENCY-DEPENDENT TRANSMISSION LINE MODELLING FOR A REAL-TIME DIGITAL SIMULATOR

J. Sousa M.T. Correia de Barros
IST - Technical University of Lisbon
Instituto da Energia - INTERG
Av. Rovisco Pais, 1096 Lisboa Codex
PORTUGAL

Olivier Huet
EDF - Electricité de France
1, av. du General de Gaulle
92141 Clamart
FRANCE

Abstract - This paper presents a real-time transmission line model with frequency-dependent parameters developed and implemented in the digital transient network analyser ARENE, developed at EDF. The line model is derived by recursive convolution in modal domain, and real frequency-constant transformation matrices are considered. To obtain the frequency-domain rational approximation of the propagation function and characteristic admittance, an accurate model of reduced order is achieved by using an optimal fitting algorithm. The efficiency of the model is evaluated both by speed and accuracy criteria, comparisons being made with EMTP simulation using the frequency-dependent J. Marti model. In the test case presented a 37-µs computation time was achieved for a three-phase untransposed transmission line.

Keywords : Transmission Line Modelling, Real-Time Digital Simulation, Electromagnetic Transients.

I. INTRODUCTION

In result of the great improvements in computer science and technology, digital simulation of electromagnetic transients has increased its domain of application. Recently this domain has been extended to the testing of electrical equipment such as protection relays [1,2]. If this type of tests is to be done in closed loop, then the digital simulation should be performed in real-time.

Therefore, not only the accuracy of the simulation is restricted to the validity of the models employed, but also to the time constraints imposed by a limited calculation time. This is specially so in the case of transmission line modelling where the traditional digital models should be adapted to fulfil the requirements of accuracy and calculation speed.

Most of the real-time frequency-dependent transmission line models so far implemented in time-domain simulators [3,4] are based on the Marti's representation [5] where the line characteristic impedance and propagation function are expressed as a sum of exponential terms in time.

Nevertheless, the line model as described by Marti is not directly suited to real-time simulation, mainly because the order of the model should be reduced and defined beforehand in order to assure a constant and delimited time-step.

Moreover, the line representation should meet the formal requirements of the real-time simulator in which it is implemented.

The transmission line model presented in this work is implemented in the digital transient network analyser ARENE, developed at EDF. It is derived as a constant admittance in parallel with a current source that depends on the past values of the voltages and currents at the line terminals. At a given time-step, each line terminal is decoupled from the other, allowing the calculation to be simultaneously performed in parallel multi-processors. The number of terms for the characteristic admittance and propagation function are defined to be dependent on the calculation time and not on the accuracy of the fitting process.

II. TRANSMISSION LINE MODELLING

A. Frequency-Dependent Line Model

Let us consider the transmission line represented in Fig.1 and described by its per unit length parameters: resistance R, inductance L, conductance G and capacitance C. The frequency-domain equations describing the line behaviour are:

$$\frac{dV(z,\omega)}{dz} = -Z(\omega)I(z,\omega)$$
$$\frac{dI(z,\omega)}{dz} = -Y(\omega)V(z,\omega) \quad (1)$$

where:

$V(z,\omega)$	: line voltage at coordiante z
$I(z,\omega)$	: line current at coordinate z
$Z(\omega)=R(\omega)+j\omega L(\omega)$	: series impedance per unit length
$Y(\omega)=G+j\omega C$	: shunt admittance per unit length

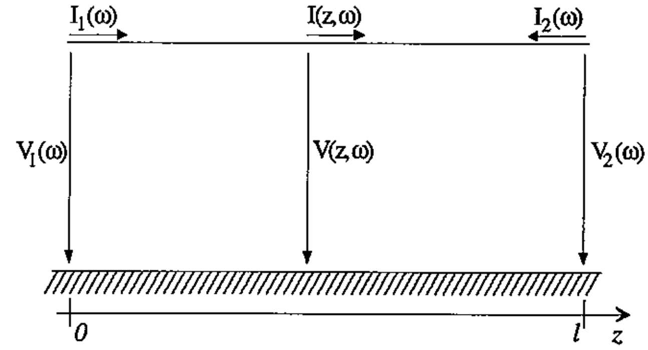

Figure 1. Single-phase transmission line representation

From the general solution of the transmission line equations (1), the relationship between voltages and currents at both ends of the line can be established:

$$\begin{aligned} V_2(\omega) &= \cosh[\gamma(\omega)\,l]\,V_1(\omega) - Z_c(\omega)\sinh[\gamma(\omega)\,l]\,I_1(\omega) \\ I_2(\omega) &= \frac{1}{Z_c(\omega)}\sinh[\gamma(\omega)\,l]\,V_1(\omega) - \cosh[\gamma(\omega)\,l]\,I_1(\omega) \end{aligned} \tag{2}$$

where l is the line length; Z_c is the characteristic impedance of the line and γ the propagation factor, which are given by:

$$\begin{aligned} Z_c(\omega) &= \sqrt{Z(\omega)/Y(\omega)} \\ \gamma(\omega) &= \sqrt{Z(\omega)\cdot Y(\omega)} \end{aligned} \tag{3}$$

From equations (2) the relationship between forward travelling waves ($V+Z_cI$) and backward travelling waves ($V-Z_cI$) can be derived:

$$\begin{aligned} V_1(\omega) - Z_c(\omega)I_1(\omega) &= H(\omega)\,\left[V_2(\omega) + Z_c(\omega)I_2(\omega)\right] \\ V_2(\omega) - Z_c(\omega)I_2(\omega) &= H(\omega)\,\left[V_1(\omega) + Z_c(\omega)I_1(\omega)\right] \end{aligned} \tag{4}$$

where $H(\omega)$ is the propagation defined as:

$$H(\omega) = \frac{1}{\cosh[\gamma(\omega)\,l] + \sinh[\gamma(\omega)\,l]} = e^{-\gamma(\omega)\,l} \tag{5}$$

The propagation function given by (5) can be expressed as the product of a delay function and a distortion function:

$$H(\omega) = H_a(\omega)\,H_d(\omega) = e^{-j\omega\tau}\,H_d(\omega) \tag{6}$$

where:

$H_a(\omega)$: delay function
$H_d(\omega)$: distortion function
$\tau = \frac{l}{v}$: minimum travelling time
v : maximum wave velocity

Finally, the frequency-domain line model equations are achieved from (4):

$$\begin{aligned} I_1(\omega) &= Y_c(\omega)\,V_1(\omega) - J_1(\omega) \\ I_2(\omega) &= Y_c(\omega)\,V_2(\omega) - J_2(\omega) \end{aligned} \tag{7}$$

where $Y_c(\omega) = \frac{1}{Z_c(\omega)}$ is the characteristic admittance of the line and $J_1(\omega)$, $J_2(\omega)$ are equivalent source currents:

$$\begin{aligned} J_1(\omega) &= e^{-j\omega\tau}\,H_d(\omega)\left[Y_c(\omega)\,V_2(\omega) + I_2(\omega)\right] \\ J_2(\omega) &= e^{-j\omega\tau}\,H_d(\omega)\left[Y_c(\omega)\,V_1(\omega) + I_1(\omega)\right] \end{aligned} \tag{8}$$

The time-domain model is achieved by applying the inverse Fourier transform to (7) and (8):

$$\begin{aligned} i_1(t) &= y_c(t) * v_1(t) - j_1(t) \\ i_2(t) &= y_c(t) * v_2(t) - j_2(t) \end{aligned} \tag{9}$$

where the symbol * stands for convolution integral and small letters represent the time-domain counterparts of the respective frequency-domain functions, achieved by inverse Fourier transform.

The time-domain equivalent source currents are given by:

$$\begin{aligned} j_1(t) &= h_d(t) * \left[y_c(t) * v_2(t-\tau) + i_2(t-\tau)\right] \\ j_2(t) &= h_d(t) * \left[y_c(t) * v_1(t-\tau) + i_1(t-\tau)\right] \end{aligned} \tag{10}$$

To notice that the two ends of the line are decoupled at a given instant provided that the minimum travelling time τ is greater than the simulation timestep.

Fig. 2 shows the equivalent circuit that represents the time-domain frequency-dependent line model.

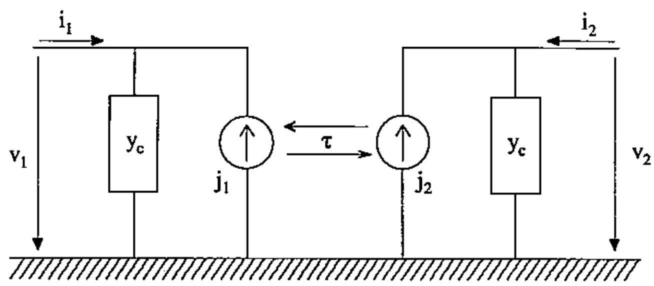

Figure 2. Equivalent circuit for time-domain simulation

B. Multiphase Line Model

The above described model is generalised to multiphase transmission lines by using the modal theory. In the general case, the corresponding transformation matrices are complex and frequency-dependent. However, for single-circuit overhead transmission lines, these modal transformation matrices can be considered as real and evaluated at a given frequency.

In the present line model, the transformation matrices are evaluated directly from the line primary parameters at a frequency chosen by the user according to the typical transients expected in the simulation. The imaginary part of each transformation matrix is first minimised by rotation and it is then neglected.

III. MODEL IMPLEMENTATION

A. Fitting the propagation function and the characteristic admittance

One of the most important steps to achieve an efficient and accurate line model is to find the approximate functions for the propagation function and characteristic admittance of the line.

It is known that the evaluation of the convolution integrals by efficient recursive schemes [6] requires the impulse response h_d and the characteristic admittance y_c to be represented, in the time-domain, as a sum of exponential terms:

$$h_d(t)=\sum_{k=1}^{n} H_k e^{-\alpha_k t} \tag{11}$$

$$y_c(t)=Y_\infty+\sum_{k=1}^{m} Y_k e^{-\lambda_k t} \tag{12}$$

This representation corresponds, in the frequency domain, to rational expressions for $H_d(\omega)$ and $Y_c(\omega)$:

$$H_d(\omega)=G_h\frac{(j\omega+z_1)(j\omega+z_2)...(j\omega+z_j)}{(j\omega+p_1)(j\omega+p_2)...(j\omega+p_n)} \tag{13}$$

$$Y_c(\omega)=G_y\frac{(j\omega+z_1)(j\omega+z_2)...(j\omega+z_m)}{(j\omega+p_1)(j\omega+p_2)...(j\omega+p_m)} \tag{14}$$

Therefore, it is necessary to fit the original functions derived from the line parameters with the rational expressions given by (13) and (14).

The fitting in the frequency-domain has the advantage of avoiding a numerical inverse Fourier transform of the original propagation function and characteristic admittance as explained in [7].

In regard to real-time simulation, it is known that the order of the model should be restricted and pre-established, in order to achieve a constant and lower timestep. Therefore, meeting the requirements of calculation time and accuracy requires an optimal fitting algorithm to be implemented.

Optimal fitting means to minimise the difference between the original and approximate characteristic admittance and propagation functions using a given number of poles. This number, also known as the model order for each mode, is determined by the calculation time available which, in turn, depends on the simulation timestep and the network size.

The implementation of the fitting process is done in two main steps. First, the Bode's asymptotic approximation is applied to trace the original functions, providing an initial model with the desired number of poles.

Then, using the achieved representation as initial values for the gain, poles and zeros, an optimal fitting algorithm is used. Targeting a minimum error, the optimal fitting changes the values of the model parameters (gain, poles and zeros) keeping the model order unchanged and equal to the previously specified value. The implemented optimal fitting algorithm is based on the Simplex search method and the objective function to be minimized was based on the least squares. Furthermore, only real and negative poles and zeros are allowed for the line representation.

Consequently, an improved result is achieved with a restricted model order for the representation of the wave propagation function and the line characteristic admittance.

B. Algorithm Description

In order to provide compatibility with the real-time simulator where the line model is implemented, the line equivalent circuit was developed as a constant admittance in paralell with a source current that depends only on past known values.

According to this representation, the frequency-dependent line model is considered as a linear element being the model admittance given once, in the beginning of the simulation, and the current source updated at each timestep.

The line model can be described as a sequence of operations having the voltages in the phase domain as input variables and the contribution to the current source as the output. This algorithm is summarised in the following steps:

- Receive from the network the line voltages in the phase domain
- Transform the phase voltages to modal domain

- Evaluate modal currents
- Evaluate recursive convolution buffers
- Receive contribution from the other line terminal
- Evaluate contribution to the other line terminal
- Evaluate modal contribution to the past history current
- Transform contribution to the phase domain
- Send contribution to the network current source in the phase domain

IV. MODEL VALIDATION

A. Test Conditions

The validation of the frequency-dependent transmission line model implemented in the ARENE is made by comparison with EMTP simulations using the J. Marti model.

Among all the simulations performed, the energization of the three-phase untransposed line is of special interest to test the accuracy of the model.

The test line configuration is represented in Fig. 3 and the corresponding data given in Table I.

The 380 kV system represented in Fig. 4 was modelled both in the EMTP and in the real-time simulator in order to perform the open-ended line energization test.

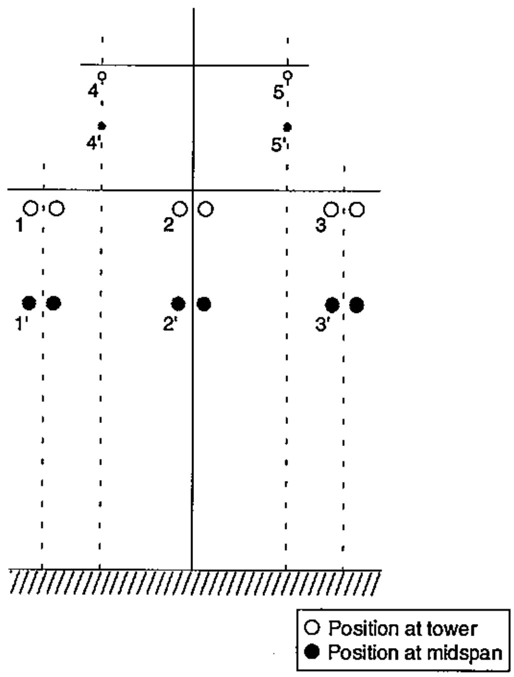

Figure 3. Transmission line geometry

TABLE I. Transmission line data

Conductor	*1*	*2*	*3*	*4*	*5*
Horizontal coordinate	-12 m	0 m	12 m	-8 m	8 m
Height at tower	26 m	26 m	26 m	36 m	36 m
Height at midspan	18 m	18 m	18 m	30 m	30 m

Line length : 100 km
Earth resistivity : 100 Ω.m

Phases (conductors 1, 2, 3 of Fig. 3):
Bundle radius : 20.0 cm
Sub-conductor radius : 1.56 cm
Resistivity : 3.2×10^{-8} Ω.m
Relative permeability : 1.0

Ground Wires (conductors 4, 5 of Fig. 3):
External diameter : 1.42 cm
Resistivity : 2.0×10^{-7} Ω.m
Relative permeability : 1.0×10^3

System Data (Fig. 4):
r = 1 mΩ
l = 50 mH
t_1 = 20 ms

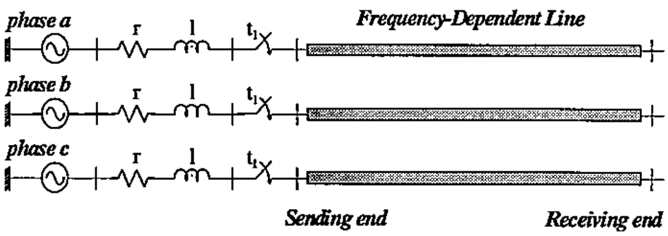

Figure 4. System used for validation test

In order to perform a real-time simulation with a specified timestep of 50μs, the number of poles given in the Table II was used for the propagation function and characteristic admittance representation. The fitting was done in the frequency range of 0.1 Hz to 100 kHz and the transformation matrices were evaluated at 1 kHz.

TABLE II. Number of poles of the line model

	Mode 1	Mode 2	Mode 3
Hd	9	9	7
Yc	9	8	7

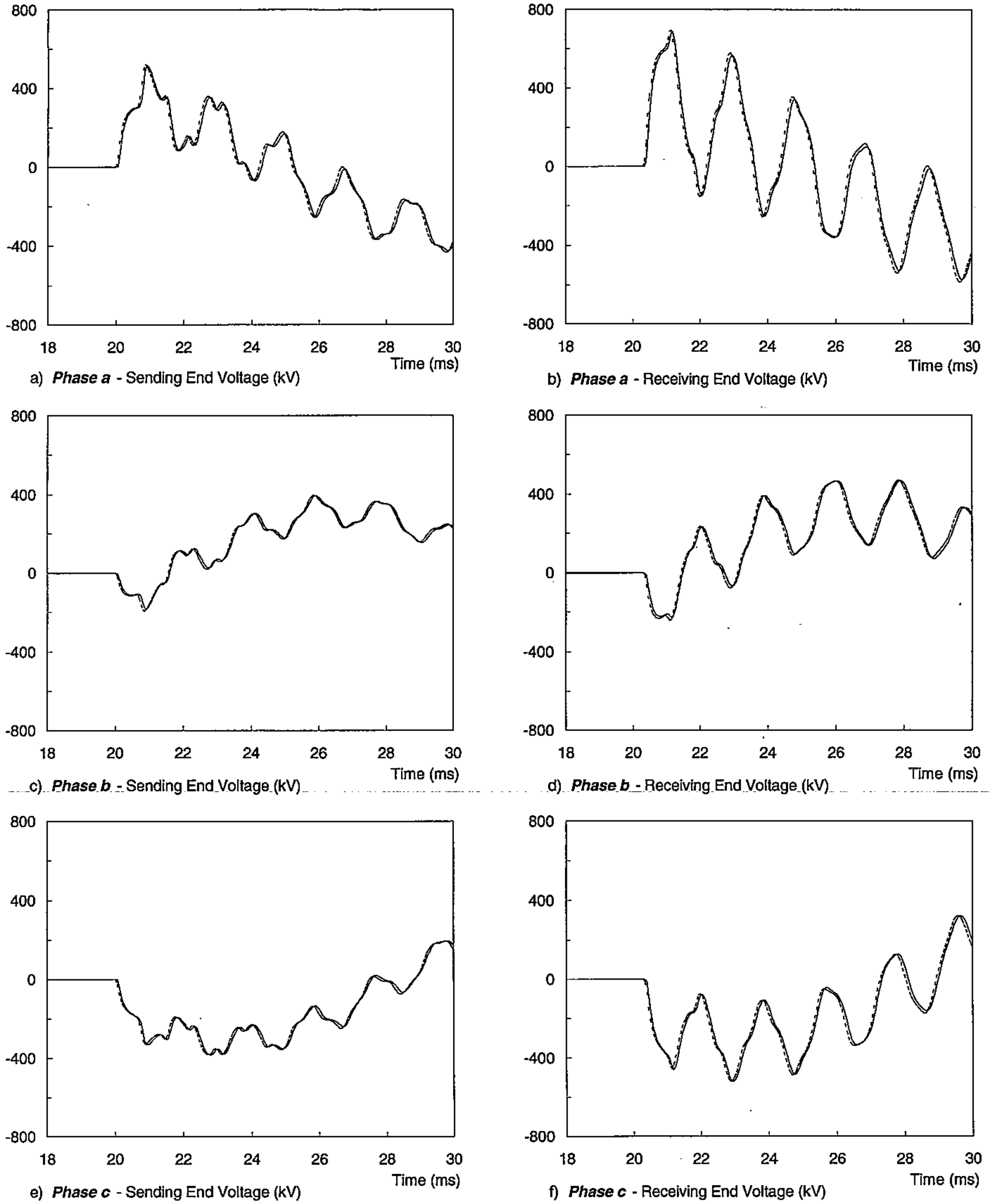

Figure 5. Open-ended line energization at 20 ms. Sending and receiving end voltages.
- - EMTP simulation using J. Marti model ——ARENE simulation using the developed line model

B. Test Results

The energization test is done by closing simultaneously the switches at the sending end of the line at the instant t_1 (20 ms) being the receiving end opened.

The results presented in Fig. 5 show the comparison between the sending and receiving end voltages in phases *a*, *b* and *c* using the EMTP J. Marti line model (dashed line in the graphics) and the developed line model for the real-time simulator (solid line in the graphics).

Concerning the speed of the model, a calculation time of 37μs was measured in a computer based on a HP RISC PA7000/100MHz processor.

V. CONCLUSIONS

A frequency-dependent transmission line model was successfully implemented and tested in the digital transient network analyser ARENE.

The line model is derived by recursive convolution in modal domain, and real frequency-constant transformation matrices are considered. Using an optimal fitting algorithm for the propagation function and characteristic admittance representation, an accurate low order model was achieved, allowing real-time applications.

The efficiency of the transmission line model is evaluated both by speed and accuracy criteria. The developed model computation time (proportional to the number of poles) has decreased to values suited to real-time simulation (37μs in the test case presented). Concerning the accuracy of the model, comparisons were made with EMTP simulation using the frequency-dependent J. Marti model and a very good agreement was found.

VI. REFERENCES

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