ABCdq Model of a 3-Phase Induction Motor for Bus Transfer and Drives

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Abstract: In this paper, a three-phase induction motor has been modeled in a hybrid reference frame (ABCdq) to examine transients on the motor drives during bus transfer. The d,q model has been widely used to study single machine transients but it is difficult to use for multimachines during unbalanced faults if the effect of source impedance is considered. By using ABCdq model for the induction machine, the analysis of motor drive connected to the same bus with the induction motors is simpler. The bus impedance which is the coupling between the motors and motor drive is incorporated into the model.

1. Introduction

The detailed simulation of bus transfer and its effect on induction motors have been investigated in [1-6]. Multimachines connected to a common bus through a line exhibit a cross-coupling in the machine models. The solution of these equations in the conventional d,q reference frame during bus transfer is difficult since the actual abc variables have been transformed. The d,q reference frame has been used for multimachines connected to the infinite bus via source impedance [7]. In the solution of differential equations, the armature terminal voltage of the machine is computed with a delay of one time step.

The majority of drives installed in the power system have a line commutated rectifier, dc link and the motor driven by inverter. The rectifiers are either uncontrolled or controlled type. If the drive is capable of four quadrant operation, the drive can generate power to the other parallel connected loads during the bus transfer. In this paper, since a three-phase uncontrolled bridge rectifier is considered, the power does not flow to the loads from the drive. In the modeling of interactions of motor drive and motor during the bus transfer, the common tie line introduces a cross-coupling in the state equations. In addition, ac currents only stop flowing during a current zero (except if vacuum breakers are used) which are difficult to track if the conventional d,q model is used for the motor. Since an ABCdq model is used for the motor, it allows the tracking of its stator currents.

In this paper, the ABCdq model of a 3-phase induction machine [8] is used to investigate the interaction between the motor and drive connected in parallel to the same bus during the power system disturbances. The error of the calculation of terminal voltages one time step late is thus eliminated.

2. Mathematical Model of The Induction Motor

The induction motor is modeled in a hybrid reference frame (ABCdq) in order to preserve the stator states in their original form, while only transforming the rotor states to d,q axis variables. The phase transformation is firstly applied to the rotor states, then a commutator transformation is used such that the inductance matrix is independent of the rotor angle. In the model, it is assumed that the d-axis coincides with phase A of the stator while the q-axis leads the d by 90 degrees electrical.

The stator variables can be transformed from three-phase to two-phase to reduce the number of state variables, but the difficulty is to combine such a model with the model of the rectifier/inverter induction motor drive connected to the same bus.

The nonlinear differential equations of the ABCdq model can be obtained by applying two transformations in cascade. The orthogonal transformation matrix given below is used to transform the balanced three-phase rotor windings to a two-phase equivalent.

$$C_1 = \sqrt{\frac{2}{3}}\begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \quad (1)$$

The second transformation matrix transforms rotor variables into a reference frame stationary relative to the stator.

$$C_2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad (2)$$

where θ is the angle between the rotor phase a and the stator phase A. The impedance matrix, Z of the ABCdq model is obtained by performing the following operation.

$$Z = \begin{bmatrix} Z_{11} & Z_{12}C_1C_2 \\ C_{2T}C_{1T}Z_{21} & C_{2T}C_{1T}Z_{22}C_1C_2 \end{bmatrix} \quad (3)$$

where Z_{11}, Z_{12}, Z_{21} and Z_{22} are the corresponding

$$Z=\begin{bmatrix} R_s+L_{ss}p & 0 & 0 & Mp & 0 \\ 0 & R_s+L_{ss}p & 0 & \frac{-Mp}{2} & \frac{\sqrt{3}Mp}{2} \\ 0 & 0 & R_s+L_{ss}p & \frac{-Mp}{2} & \frac{-\sqrt{3}Mp}{2} \\ Mp & \frac{\sqrt{3}Mw_e-Mp}{2} & \frac{-\sqrt{3}Mw_e-Mp}{2} & R_r+L_rp & L_rw_e \\ -Mw_e & \frac{\sqrt{3}Mp+Mw_e}{2} & \frac{Mw_e-\sqrt{3}Mp}{2} & -L_rw_e & R_r+L_rp \end{bmatrix} \quad (4)$$

component impedance matrix of a 3-phase ABCabc induction motor in a 'natural' reference frame as given in [9]. I is the identity matrix. C_{1T} and C_{2T} are the transpose of C_1 and C_2 respectively. The complete motor equations obtained by performing the matrix multiplication in (3) is

$$\begin{bmatrix} e_{sa} & e_{sb} & e_{sc} & v_d & v_q \end{bmatrix}^T = Z\begin{bmatrix} i_a & i_b & i_c & i_d & i_q \end{bmatrix}^T$$

where

$$v_d=\sqrt{\frac{3}{2}}V_m\mathrm{Cos}(sw_st+\theta)$$

$$v_d=\sqrt{\frac{3}{2}}V_m\mathrm{Sin}(sw_st+\theta)$$

$$p=\frac{d}{dt}, \qquad p\theta=w_e,$$

where

$$s=\frac{w_s-w_e}{w_s}$$

V_m is the peak value of the rotor phase voltage. In this work, it is assumed that the rotor terminal voltages (v_d and v_q) are zero because a squirrel cage induction machine is considered.

The electromagnetic torque developed by the machine is given below;

$$T_e=\frac{P}{4}M\left(\sqrt{3}i_di_b-2i_qi_a+i_qi_b-\sqrt{3}i_di_c+i_qi_c\right)$$

The mechanical motion is incorporated in the torque balance equation given below and the friction and windage loss is neglected.

$$T_e=T_L+(\frac{2}{P})J_m\frac{dw_e}{dt} \quad (5)$$

3.Model of Three-Phase Bridge Rectifier

The three-phase bridge rectifier was analyzed in [10]. In the same study, the model of PWM inverter for simulation purpose was given. During the continuous current mode of operation there are 12 possible status of 6 diodes employed in three-phase bridge rectifier. Each diode is conducting or blocking due to the level of anode-cathode voltage and anode current. These twelve states (J=1,2,3,4,5,6,7,8,9,10,11,12) are configuring the connection of source to the output of rectifier. In this section, the state space form of model for rectifier is given. The source is represented by its Thevenin equivalent. Each phase current flowing through source inductance and filter capacitor voltage are defined as state variables. Infinite bus voltages and the rectifier output current, i_i, are described as the inputs. The selection of the rectifier output current as input in the model enables to combine the rectifier model with the model of induction motor driven by the inverter.

Twelve conduction sequences can be classified in two categories. Either two or three diodes are conducting simultaneously.

3.1. Conduction of two diodes

During the conduction of two diodes, one phase current and the capacitor voltage are defined as the state variables. Two phase voltages are feeding power to the rectifier and third phase is open. The model of the rectifier given in (6) has the variables f_1 (source voltage) and f_2 (phase current) defined in table 1 for all conduction sequences, **J.**

$$p\begin{bmatrix} f_2 \\ kv_i \end{bmatrix}=L_1^{-1}\begin{bmatrix} f_1-(r_f+2r_s)f_2-kv_i \\ \frac{ki_i-f_2}{C} \end{bmatrix} \quad (6)$$

where;

$$L_1^{-1}=\left(\frac{-1}{2L_s+L_f}\right)\begin{bmatrix} -1 & 0 \\ 0 & 2L_s+L_f \end{bmatrix}$$

and

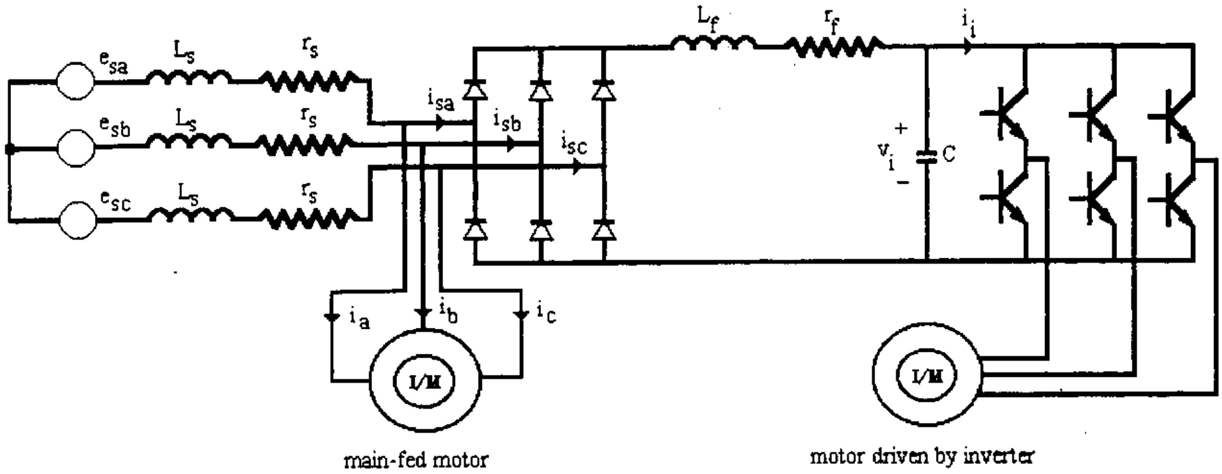

Figure 1. Induction Motor and Drive

Table 1. State variables during the conduction of two diodes.

	J =1 or 7	J =3 or 9	J =5 or 11
$f_1 =$	e_{ac}	e_{bc}	e_{ba}
$f_2 =$	i_{sa}	i_{sb}	i_{sc}

3.2. Overlap period

The overlap angle is assumed to be less than 60 degrees, therefore three diodes conduct during this period. In this mode, three phases are feeding power to the rectifier. f_3 and f_4 are phase voltages while f_5 and f_6 are phase currents in (7). These are defined in table 2 for all states. Two phase currents and filter capacitor voltage are chosen as the state variables.

$$p\begin{bmatrix} f_5 \\ f_6 \\ kv_i \end{bmatrix} = L_2^{-1}\begin{bmatrix} f_3 - r_s f_5 + r_s f_6 \\ f_4 - (r_s + r_f) f_5 - (2r_s + r_f) f_6 - kv_i \\ \dfrac{ki_i - f_5 - f_6}{C} \end{bmatrix} \tag{7}$$

where

$$L_2^{-1} = \frac{-1}{F}\begin{bmatrix} -2L_s - L_f & -L_s & 0 \\ L_s + L_f & -L_s & 0 \\ 0 & 0 & L_s(3L_s + 2L_f) \end{bmatrix}$$

and

$$F = L_s(3L_s + 2L_f)$$

Table 2. State variables during overlap period.

	J =2 or 8	J =4 or 10	J =6 or 12
$f_3 =$	e_{ab}	e_{ac}	$-e_{cb}$
$f_4 =$	$-e_{cb}$	e_{cb}	$-e_{ac}$
$f_5 =$	i_{sa}	i_{sa}	i_{sb}
$f_6 =$	i_{sb}	i_{sc}	i_{sc}

where;

$e_{ab} = e_{sa} - e_{sb}$, $e_{ac} = e_{sa} - e_{sc}$, $e_{cb} = e_{sc} - e_{sb}$

and

$$k = \begin{cases} 1, & \text{for } J = 1, 2, 3, 5, 6, 10 \\ -1, & \text{for } J = 4, 7, 8, 9, 11, 12 \end{cases}$$

4. Modeling of the motor in parallel to drive

The drive system given in figure 1 has been modeled using two possible configurations of the rectifier. The first occurs when two diodes conduct and the second occurs during the overlap period. The coupling parameter, that is the source impedance, between the rectifier and main-fed motor is included into the model.

4.1 Conduction of two diodes

The induction machine model given in (4) is combined with the rectifier model in (6) to analyze the system described by (8). The coupling parameter between the rectifier and main-fed motor is found during each state and it is given in table 3.

$$\begin{bmatrix} e_{sa} & e_{sb} & e_{sc} & 0 & 0 & f_1 & \frac{ki_i}{C} \end{bmatrix}^T = Z'\begin{bmatrix} i_a & i_b & i_c & i_d & i_q & f_2 & kv_i \end{bmatrix}^T \tag{8}$$

where

$$Z' = \begin{bmatrix} & & & & & a & 0 \\ & & & & & b & 0 \\ & & Z & & & c & 0 \\ & & & & & 0 & 0 \\ & & & & & 0 & 0 \\ a & b & c & 0 & 0 & 2r_s + r_f + (2L_s + L_f)p & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{C} & -p \end{bmatrix}$$

where

Table 3: The coupling parameters during the conduction of two diodes.

	J =1 or 7	J =3 or 9	J =5 or 11
a=	$r_s + L_s p$	0	$-r_s - L_s p$
b=	0	$r_s + L_s p$	$r_s + L_s p$
c=	$-r_s - L_s p$	$-r_s - L_s p$	0

4.2. Overlap period

The rectifier model given in (7) is combined with the induction machine model in (4). The resulting equation is given in (9). The coupling parameter is obtained from the explicit form of formulation during each conduction sequence of overlap period and it is given in table 4.

$$\begin{bmatrix} e_{sa} & e_{sb} & e_{sc} & 0 & 0 & f_3 & f_4 & \frac{ki_1}{C} \end{bmatrix}^T = Z''\begin{bmatrix} i_a & i_b & i_c & i_d & i_q & f_5 & f_6 & kv_1 \end{bmatrix}^T \quad (9)$$

where

$$Z'' = \begin{bmatrix} & & & & & u_1 & u_2 & 0 \\ & & & & & v_1 & v_2 & 0 \\ & & Z & & & w_1 & w_2 & 0 \\ & & & & & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 \\ k_1 & l_1 & m_1 & 0 & 0 & r_s + L_s p & -r_s - L_s p & 0 \\ u_2 & v_2 & w_2 & 0 & 0 & z' & y & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{C} & \frac{1}{C} & -p \end{bmatrix}$$

where

$$z' = r_s + r_f + (L_s + L_f)p$$

$$y = 2r_s + r_f + (2L_s + L_f)p$$

Table 4: The coupling parameters during the overlap period.

	J =2 or 8	J =4 or 10	J =6 or 12
$k_1 =$	$r_s + L_s p$	$r_s + L_s p$	0
$l_1 =$	$-r_s - L_s p$	0	$r_s + L_s p$
$m_1 =$	0	$-r_s - L_s p$	$-r_s - L_s p$
$u_1 =$	$r_s + L_s p$	$r_s + L_s p$	$-r_s - L_s p$
$v_1 =$	0	$-r_s - L_s p$	$r_s + L_s p$
$w_1 =$	$-r_s - L_s p$	0	0
$u_2 =$	0	0	$-r_s - L_s p$
$v_2 =$	$r_s + L_s p$	$-r_s - L_s p$	0
$w_2 =$	$-r_s - L_s p$	$r_s + L_s p$	$r_s + L_s p$

5. Results

The computer simulation program that uses the standard 4 th order Runga Kutta integration algorithm is prepared in order to analyze a three-phase induction motor and an induction motor drive (three phase bridge rectifier, filter circuit and PWM inverter) connected to the same bus with nonzero source impedance.

A three-phase, 4-pole, 208 V, 1.4 A induction motor and its drive have the following parameters per phase referred to stator side.

$$L_{ss1} = 0.29 \text{ H}, \quad L_{rr1} = 0.29 \text{ H},$$
$$L_{m1} = 0.272 \text{ H}, \quad r_{ss1} = 5.6\ \Omega, r_{r1} = 4.9\ \Omega,$$
$$J_{m1} = 0.0004 \text{ Kg.m}^2, T_{L1} = 0.00457 w_{e1}$$

The filter and source parameters are as follows;

$$r_f = 0.33\,\Omega, \quad L_f = 0.02\,\text{H},$$
$$C = 0.1\,\text{mF}, \quad L_s = 0.5\ \text{mH}, \ r_s = 10\,\text{m}\Omega$$

The amplitude modulation ratio and frequency modulation ratio of the PWM inverter are 1.4 and 18 respectively. The snubber losses are measured at the operating voltage while the inverter output terminals are open-circuited and represented in the model by a resistance in parallel to the filter capacitor. The magnitude of this resistance is found to be 140 Ohms.

A three-phase, 230 V, 13 A, 4-pole induction motor connected in parallel to drive has the following parameters in ABCdq reference frame:

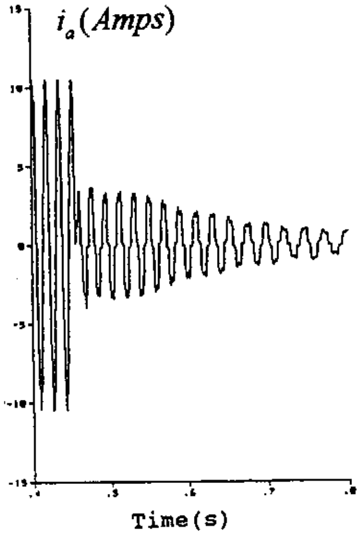

Figure 2a. Predicted stator current

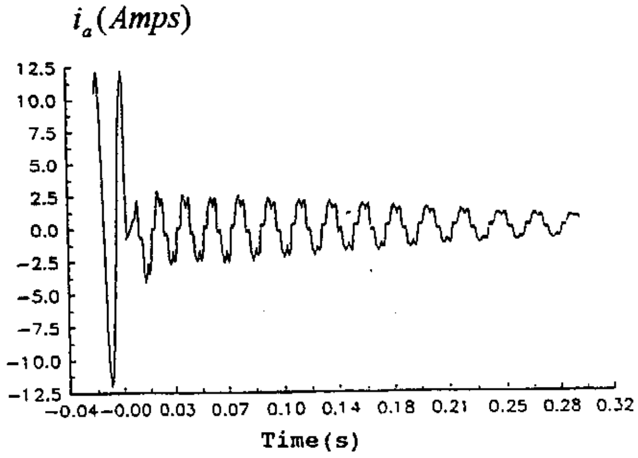

Figure 2b. Measured stator current

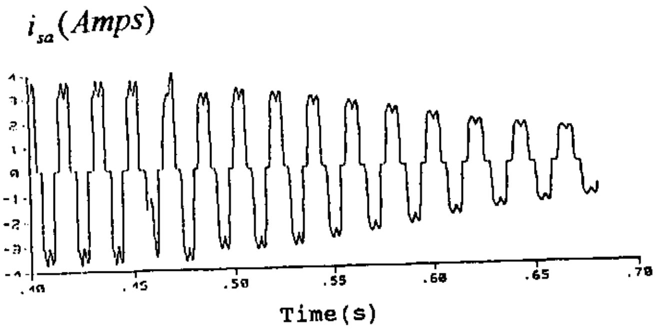

Figure 3a. Predicted rectifier input current

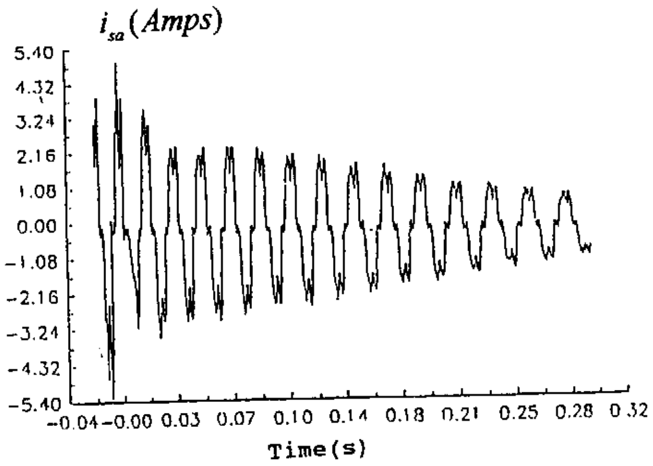

Figure 3b. Measured rectifier input current

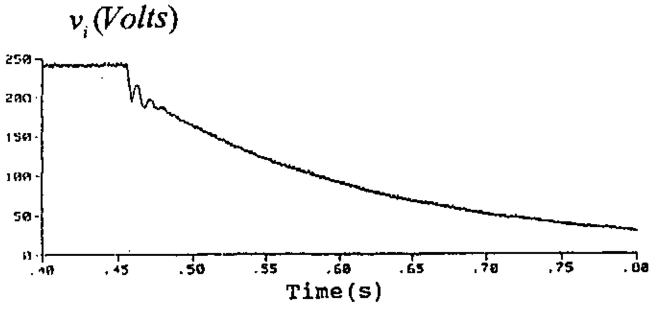

Figure 4. Predicted filter capacitor voltage

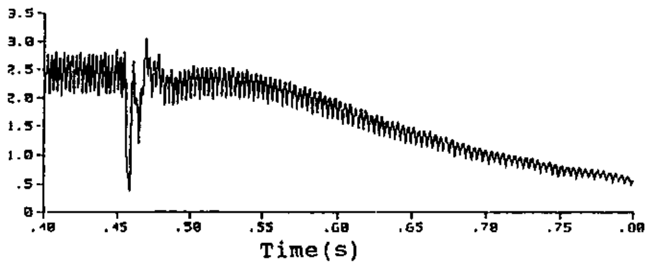

Figure 5. Predicted rectifier output current

$$L_{ss} = L_r = 0.074\text{H},\ R_{ss} = 0.4\,\Omega,\ R_r = 0.296\Omega$$

$$M = 0.058\text{H},\qquad J_{m2} = 0.0497\text{Kg.m}^2$$

The load torque equation was found to be [11]

$$T_{L2} = 27.4(0.018 + 0.38(1-s) - 0.06(1-s)^3)\ \text{N.m}$$

A three-phase source with 180 V at 60 Hz has been used during the experimental work.

Three phase source is disconnected from the motor and drive by using a three-phase circuit breaker during the experimental work. In the simulation, a large resistance (1000 Ohms) is inserted into each source line while the phase current is at zero level.

Figures 2a and 3a show the predicted stator current of the main-fed motor and rectifier input current respectively while figures 2b and 3b show the corresponding experimental results. The experimental and simulation results are not synchronized on time basis. In the computer simulation, as the system is running at the steady state around 0.45 seconds of simulation time three phase source is disconnected. Three cycles of stator current of main-fed motor before disconnection is printed out by the computer simulation program in order to compare with the experimental work. The peak value of the current is found around eleven ampers from the ABCdq model and experimental work. The motor input current is sinusoidal before the bus is disconnected. The reference for the time axis of measured results is set to be zero when the disconnection of the three phase supply takes place and one cycle before disconnection is recorded.

After the disconnection of source, the stator current of the main-fed motor has the same waveform with the rectifier input current. During this transition the motor feeds power to the rectifier. Thus the motor acts as a generator during the period while there is no external supply connected. The peak of stator current decays from the initial steady state value as energy is transferred from the motor to the inverter drive and the motor load. As the back emf of the motor decreases, the magnitude of its armature current also decreases, as well as the filter capacitor voltage. Also, the frequency of the rectifier input current that is proportional to rotor speed decreases in time. The predicted filter capacitor voltage and rectifier output current are shown in Figure 4 and Figure 5 respectively.

6. Conclusion

In this paper, a hybrid model (ABC/dq) of an induction machine is used to analyze the motor drive transients during the bus transfer. The source inductance is included into the model in order to investigate the interaction of the machine and drive. It is observed that when the power source is disconnected, the main-fed machine feeds the power stored in its inertia to the motor drive. The standard d,q model requires the use of fictitious d,q voltages. This hybrid model provides a superior advantage to include couplings and unbalance effects from the supply side.

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