AN EFFICIENT TECHNIQUE FOR DETERMINING THE RESPONSES OF NONLINEAR CIRCUITS

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Abstract - **A new technique is proposed for determining the responses of nonlinear power circuits. The technique consists in connecting voltage sources in parallel with the nonlinear elements. Alternatively, current sources may be connected in series with the nonlinear elements. The mismatch currents through the voltage sources (or the mismatch voltages across the current sources) are calculated. The objective of the proposed technique is to determine the voltage or current sources for which the mismatch quantities are zero. The technique is general and it can be applied for simulating the transient and steady-state responses. The responses of typical nonlinear power circuits have been determined using the proposed technique.**

Keywords: Transient Response, Steady-state Response, Nonlinear Circuits, Power Electronic Circuits.

1. INTRODUCTION

The technique for simulating the transient response of nonlinear, power circuits usually consists in reducing the linear part of the circuit to its Thevenin equivalent and then applying the Newton-Raphson procedure for determining the currents in the nonlinear elements. Subsequently, the compensation theorem is applied to obtain the nodal voltages in the linear part. Efficient methods have been described to automate the procedure and diminish the solution time [1]. To reach the steady-state, either the transient response is computed over a large number of periods at the fundamental frequency or the convergence to the steady-state is accelerated [2].

The simulation of nonlinear, power circuits has generally progressed along two distinct lines. On the one hand, the circuit has been treated as one of fixed topology and the nonlinear devices have been represented by two-state resistors [3]. On the other hand, the power circuit has been decomposed into a sequence of circuit topologies and the state equations for each configuration are determined. Digital simulation then consists of integrating the appropriate state equations over a small time-step and of subsequently establishing the topology for the next time-step [4]. In another approach, the time delay in accounting for topology changes is avoided by introducing linear output prediction and correction procedures [5].

In this paper, a general methodology is proposed for determining the transient and steady-state responses of nonlinear, power circuits. The technique can handle any type of nonlinearity (including elements with hysteresis), provided that the element's behaviour is described by a suitable set of equations. It also allows the inclusion of detailed models for the semi-conducting devices and of control systems, which are treated as nonlinear blocks. The responses of typical nonlinear circuits are presented and the simulated responses are compared with the responses obtained with the MicroTran® version of the EMTP.

2. SOLUTION TECHNIQUE

Consider the nonlinear circuit shown in Fig.1a comprising the series connection of a voltage source, a linear resistor and a voltage-controlled nonlinear resistor. The voltage across the nonlinear resistor is the solution of the equations:

$$F = v_D + R_T\, i_D - e_0 \tag{1}$$

$$i_D = f(v_D), \tag{2}$$

where F is an objective function and $f(\cdot)$ is the characteristic of the voltage-controlled nonlinear resistor. The solution is approached iteratively by following the Newton-Raphson procedure:

$$\Delta v_D^k = -F^k \Big/ \left(\tfrac{\partial F}{\partial v_D}\right)^k \tag{3}$$

$$v_D^{k+1} = v_D^k + \Delta v_D^k, \tag{4}$$

where k is the iteration number and Δv_D^k is the correction to be added to the solution estimate v_D^k. Convergence is obtained when the magnitude of F is less than a specified tolerance.

The Newton-Raphson procedure may be shown to be equivalent to the sequential analysis of two circuits, if the correction Δv_D^k is written as:

$$\begin{aligned}\Delta v_D^k &= -\frac{v_D^k + R_T i_D^k - e_0}{1 + R_T\left(\frac{\partial i_D}{\partial v_D}\right)^k} \\ &= \frac{R_T R_D^k}{R_T + R_D^k}\left(\frac{e_0 - v_D^k}{R_T} - i_D^k\right) \\ &= \frac{R_T R_D^k}{R_T + R_D^k}\cdot i_M^k \end{aligned} \tag{5}$$

where i_M^k is the mismatch current and $R_D^k = \left(\frac{\partial v_D}{\partial i_D}\right)^k$ is the linearised (or small-signal) equivalent resistance of the nonlinear element.

In the first circuit, a voltage source v_D^k is connected across the nonlinear element as shown in Fig.1b and the current through this source is the mismatch current

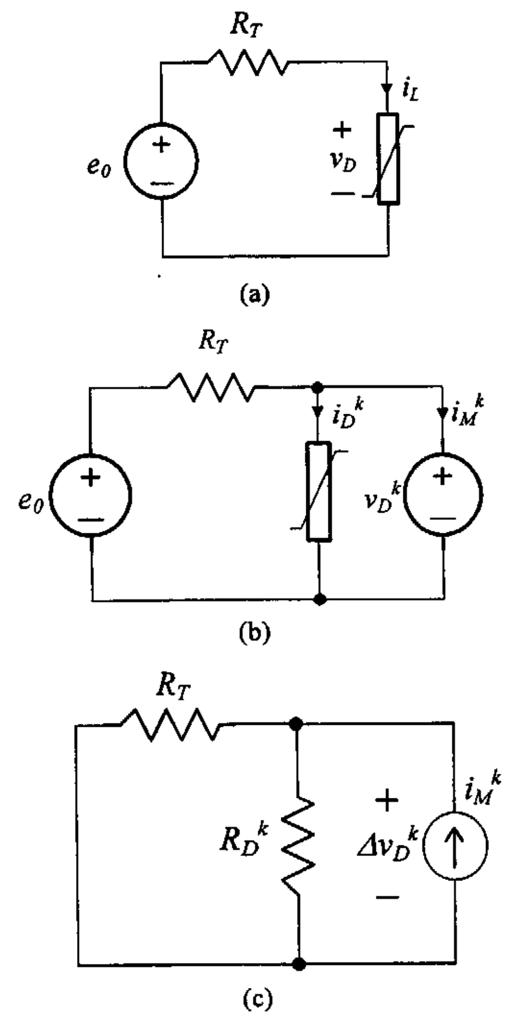

Fig. 1. Proposed solution technique. (a) a typical nonlinear circuit (b) connection of voltage source (c) small-signal equivalent circuit

i_M^k. The connection of the voltage source in parallel with the nonlinear element separates the circuit into a linear part and a nonlinear part. Therefore, the determination of the mismatch current is very much simplified.

The second circuit (which will be referred to as the small-signal equivalent) is required for calculating the correction Δv_D^k. This circuit is obtained by removing the external source e_0, replacing the nonlinear element by its linearised equivalent resistance and finally, substituting the voltage source v_D^k by the mismatch current i_M^k with polarity reversed as shown in Fig.1c. The voltage across the mismatch current source is the correction Δv_D^k to be added to the solution estimate before proceeding to the next iteration. The procedure has converged when $\left| i_M^k \right|$ is less than the tolerance.

When there are many nonlinear elements in a circuit, then voltage sources representing the estimated solution values are connected across each nonlinear element and the mismatch currents through these sources are determined. During this analysis, the linearised equivalents of the nonlinear elements are computed and stored. The small-signal equivalent circuit is set up, with the nonlinear elements replaced by their linearised equivalents and it is excited by the mismatch currents. The voltages developed across the mismatch currents are added to the previous estimates and the next iteration starts.

In some circuits, it may not be necessary to connect voltage sources across each nonlinear element. Appropriate connection of a lesser number of sources could separate the circuit into linear and nonlinear parts, as described in section 3.3.

The proposed technique may also be used to simulate the periodic steady-state response of nonlinear circuits. In this case, periodic voltage sources representing the estimated solution waveforms are connected across the nonlinear elements and the steady-state, periodic mismatch currents flowing through these sources are computed. These currents are the excitations for the small-signal equivalent in which the nonlinear elements are replaced by linear, periodically time-varying elements. The steady-state, periodic voltages across the mismatch current sources are the waveform corrections to be added to the previous estimates.

To deal with a current-controlled nonlinearity, a current source is inserted in series with the nonlinear element and the mismatch voltage across the current source is calculated. The correction Δi_D^k is obtained from the small-signal equivalent circuit, in which the excitation is the mismatch voltage with reversed polarity.

3. ILLUSTRATIVE EXAMPLES.

The simulation of typical power electronic circuits is presented in this section. A 133MHz Pentium PC has been used in all the simulations. The number of time-steps per period at the switching frequency has been chosen to be 512, which besides being adequate for the transient responses, conveniently allows the harmonic analysis of the steady-state waveforms using the fast Fourier transform. The source codes have been written in the Fortran language and compiled with Microsoft® Fortran Power Station v1. Double-precision arithmetic has been used in all the computations. Semiconductor diodes and switches have been represented by two-state resistors with $r_{ON} = 0.18\Omega$ [5] and $r_{OFF} = 50M\Omega$. More detailed models may be easily included in the simulations.

Two of the examples have also been simulated with the MicroTran-EMTP. The MicroTran solution times have been minimised by allowing only one sample per period to be written to a file.

3.1 Buck regulator. The buck regulator with dynamic feedback shown in Fig.2a, is identical to that analyzed in refs. [4] and [5]. The feedback control system which dynamically controls the duty ratio, is also treated as a nonlinear element connected at the output terminals of the power stage. The duty ratio in this circuit has an upper limit of 0.9 and an over-current protection disconnects the power switch whenever the inductor current exceeds 8A.

The proposed technique for determining the transient response consists in connecting voltage sources e_T and e_C across the nonlinear elements, the former across the power switch and the latter across the output terminals of the power stage. Their magnitudes are the estimated solution values. The mismatch currents i_T, i_C flowing into the voltage sources are determined and the linearised equivalent resistances R_T, R_D of the power switch and the diode are also calculated.

The small-signal equivalent circuit is set up as shown in Fig.2b, in which the excitations are the mismatch currents and the nonlinear elements are substituted by their linearised equivalent resistances. The voltages Δe_T, Δe_C developed across the mismatch currents are added to the previous estimates e_T, e_C and the computation proceeds to the next iteration. Convergence is obtained when the mismatch currents are below specified tolerances.

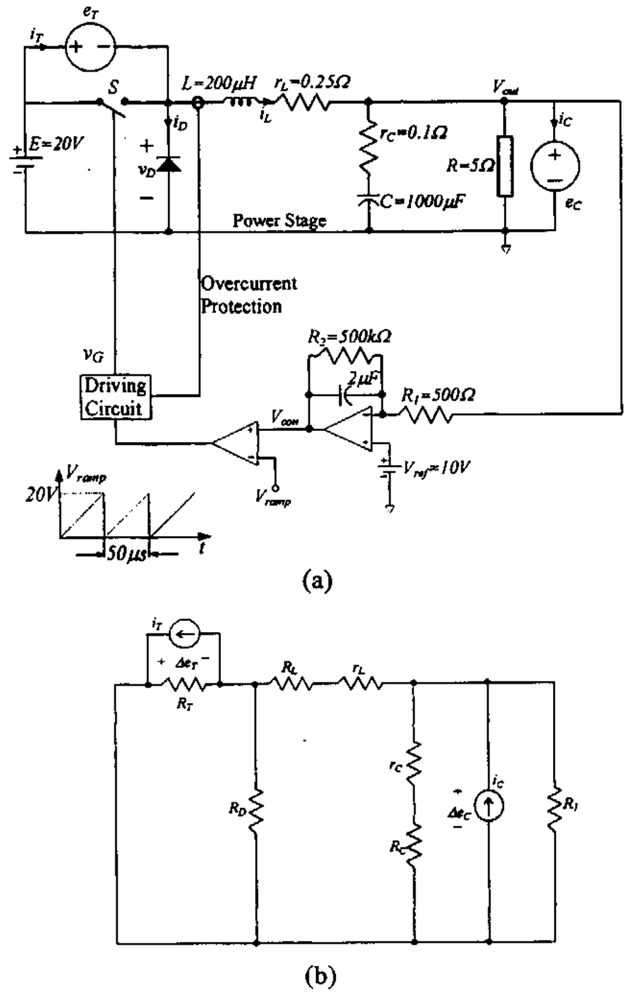

Fig. 2. Buck regulator with dynamic feedback. (a) buck regulator circuit (b) small-signal equivalent circuit

Fig.3a shows the start-up transient for the buck regulator and the cpu time for generating these waveforms is 0.99s. Fig.3b shows the transient waveforms when there is a step change in the regulator's input voltage from 20V to 40V at 18ms. The cpu time for this simulation is 3.4s.

The inductor current and the capacitor voltage waveforms show the existence of a 20kHz oscillation due to the periodic switching of the power switch.

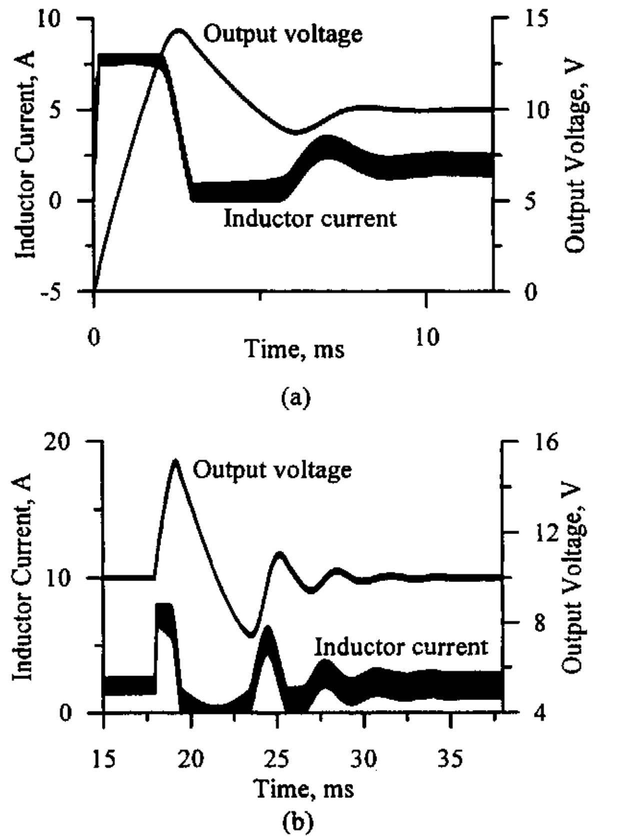

Fig. 3 Response waveforms for the buck regulator. (a) start-up transients (b) transients due to step change in input voltage

3.2 Zero-voltage switched, quasi-resonant boost converter.

The ZVS-QR boost converter takes a long time to settle down to the steady-state. Therefore, it is worthwhile determining the steady-state response with the proposed technique without having to march through a transient.

Fig.4 shows a ZVS-QR boost converter [6]. The switching frequency of the converter is 420kHz. The small-signal equivalent for this converter is shown in Fig.4b. The start-up transient computed with the proposed method requires 638 periods of computation to reach the steady-state and this corresponds to 2.8s of cpu time. The MicroTran-EMTP [7] takes 1min13s to compute the 638 periods of start-up transient. Direct determination of the steady-state response using the proposed technique is accomplished in 12 iterations, which require 0.33s of cpu time.

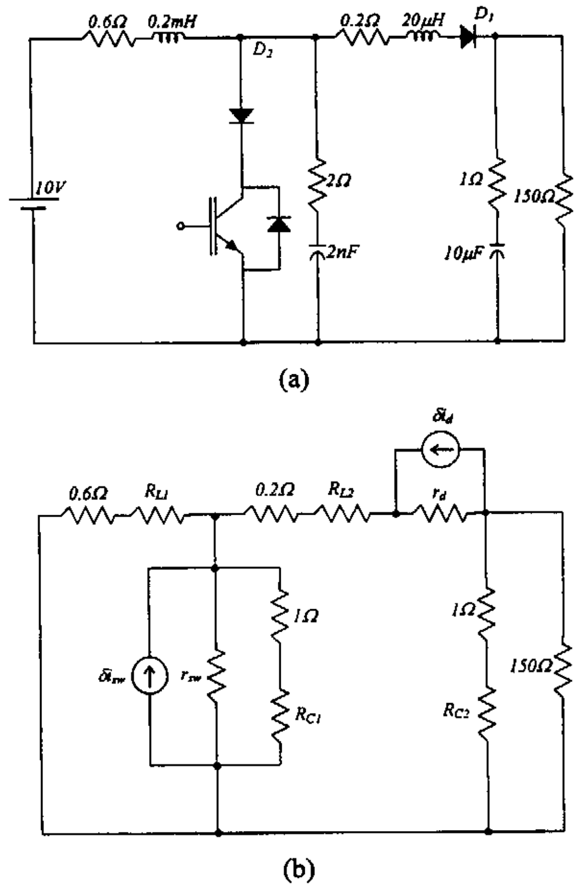

Fig.4. (a) Zero-voltage switched, quasi-resonant boost converter.
(b) Small-signal equivalent for the ZVS-QR boost converter.

Fig.5 shows the steady-state waveforms. The maximum differences between the waveforms obtained with the proposed technique and the MicroTran-EMTP waveforms are 0.7% in the current waveforms and 0.3% in the voltage waveform.

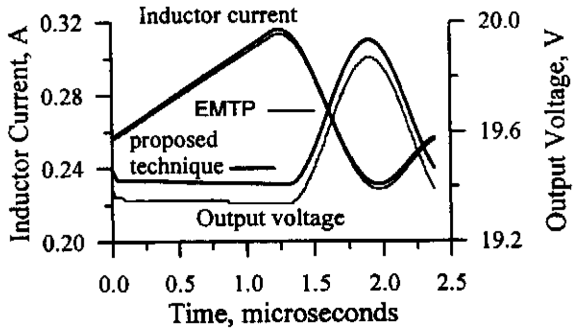

Fig.5. Steady-state response waveforms for ZVS-QR boost converter

3.3. AC/DC Converter. Fig.6a shows a typical AC/DC converter. The AC system has been represented by a delta-connected 3-phase unbalanced and distorted voltage source and the DC system consists of a smoothing reactor with its series resistance and an opposing emf. The gating pulses for the individual valves of the 6-pulse bridge are supplied by a control system (Fig.6b). The parameter values for the converter (Table 1) correspond closely to the CIGRE HVDC benchmark [8]. Since the procedure for simulating the steady-state response closely follows that for simulating the transient response, only the computation of the transient response will be described below.

At a given time-step, initial estimates of the voltages e_{AB}, e_{BC} and the current i_{LD} are applied as external sources (Fig.6a). These three sources are sufficient to separate the nonlinear part comprising the six valves from the linear parts comprising the AC and DC power systems. The currents i_{LA}, i_{LB} and the voltage e_{LD} are obtained from an analysis of the linear parts. The current i_{LD} is the input signal to the control system, whose output determines the firing instants and the conducting states of the individual valves. The currents i_{NA} and i_{NB}, the voltage e_{ND} and the linearised equivalents of the valves are obtained from an analysis of the nonlinear part of the circuit. The description of this analysis will be presented later.

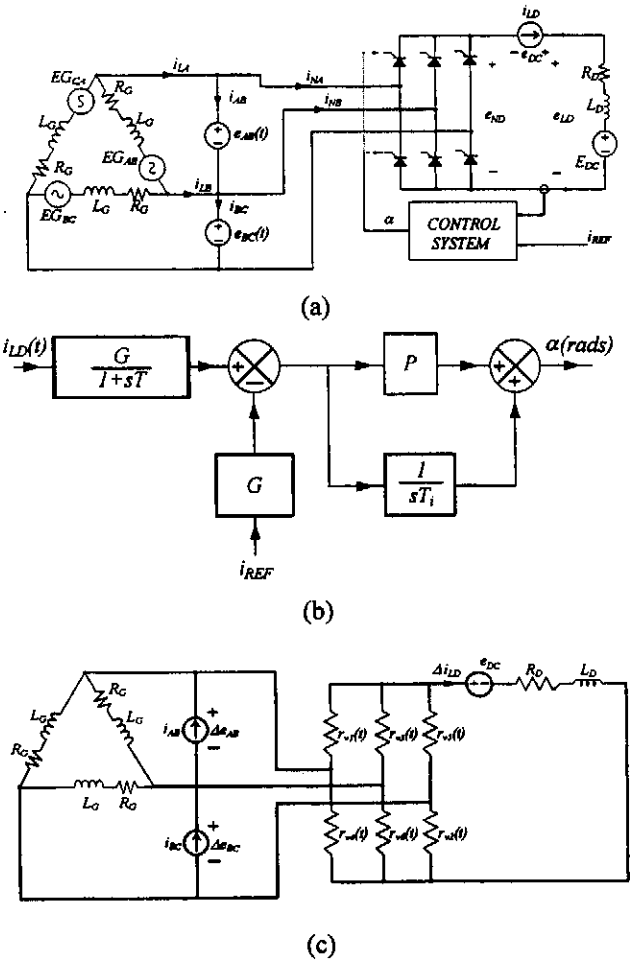

Fig. 6. AC/DC converter with current control. (a) 6-pulse converter (b) current controller (c) small-signal equivalent circuit

The mismatch currents flowing into the voltage sources e_{AB}, e_{BC} and the mismatch voltage across the current source i_{LD} are given by:

$$i_{AB} = i_{LA} - i_{NA} \tag{6}$$

$$i_{BC} = i_{LB} - i_{NB} + i_{AB} \tag{7}$$

$$e_{DC} = e_{LD} - e_{ND}, \tag{8}$$

and these are the excitations for the small-signal equivalent for the AC/DC converter shown in Fig.6c. The voltages across the mismatch current sources and the current through the mismatch voltage source are added to the initial estimates e_{AB}, e_{BC} and i_{LD}. The iterations continue until convergence and the computations proceed to the next time-step.

Analysis of the nonlinear part of the circuit will be described now. The currents i_{NA}, i_{NB} and the voltage e_{ND} are determined by applying the proposed technique recursively. Voltage sources v_{ND} and v_{LD} are connected as shown in Fig.7a and the mismatch currents i_{MD} and i_{KD} flowing into these sources are determined. The small-signal equivalent for the nonlinear module is then established as shown in Fig.7b. The voltages Δv_{ND}, Δv_{LD} across the mismatch current sources are computed and added to the initial estimates v_{ND} and v_{LD}. Convergence is obtained in a few iterations.

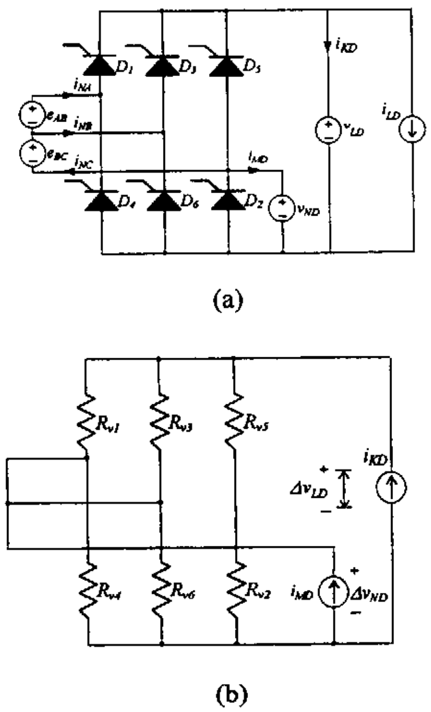

Fig. 7 Analysis of the nonlinear part of the AC/DC converter. (a) model for nonlinear part (b) small-signal equivalent circuit

Fig. 8a shows the steady-state response waveforms for the AC/DC converter and the cpu time for this simulation is 0.96s. The start-up transient has been subsequently computed using the valve firing angles obtained from the previous simulation as input data and de-activating the control system. With fixed firing angles, the start-up transient requires 34 periods to reach the steady-state and the cpu time for this task is 2.47s. The MicroTran-EMTP requires 12s to compute the 34 periods of start-up transient. Fig. 8b shows the steady-state waveforms obtained with the MicroTran-EMTP.

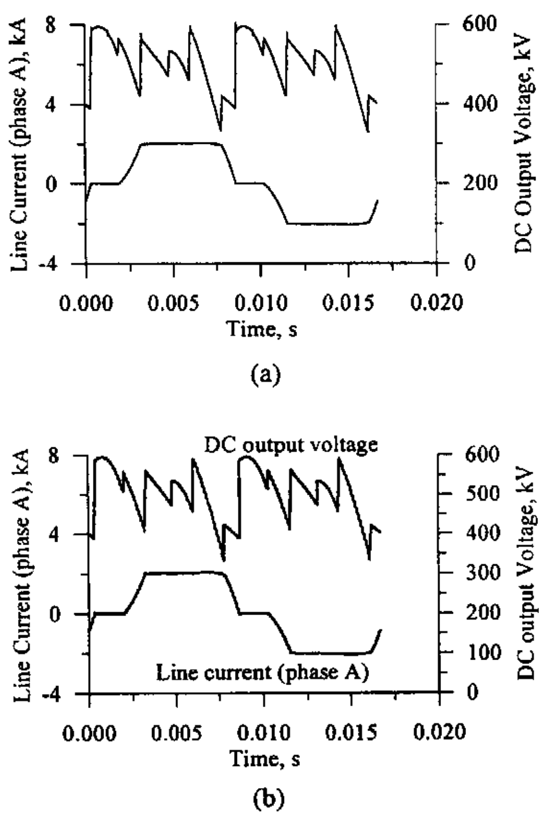

Fig. 8. AC/DC converter response waveforms. (a) proposed technique (b) MicroTran EMTP waveforms

4. CONCLUSIONS

A general approach for determining the transient and steady-state responses of nonlinear power circuits has been described. The proposed technique is rapidly-convergent and detailed semi-conducting device models may be easily included in the computations.

The responses of typical power electronic circuits have been computed and the response waveforms have been presented. These responses have been checked against the waveforms obtained with the MicroTran-EMTP.

5. REFERENCES

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Table 1. Parameter values for AC/DC converter

Parameter	Value
R_G	$0.1\ \Omega$
L_G	$0.216\ H$
R_D	$5\ \Omega$
L_D	$0.82888\ H$
EG_{PK}	$348{,}571.7\ V$
EG_{AN}	$EG_{PK}\{\sin\omega t + 0.071\sin(\omega t + 1.034) + 0.011\sin(3\omega t - 0.439) + 0.102\sin(3\omega t + 1.357)\}$
EG_{BN}	$EG_{PK}\{\sin(\omega t - 2.0944) + 0.071\sin(\omega t + 3.1284) + 0.011\sin(3\omega t - 2.5334) + 0.102\sin(3\omega t + 3.4514)\}$
EG_{CN}	$EG_{PK}\{\sin(\omega t + 2.0944) + 0.071\sin(\omega t - 1.0604) + 0.011\sin(3\omega t + 1.6554) + 0.102\sin(3\omega t - 0.7374)\}$
EG_{AB}	$EG_{AN} - EG_{BN}$
EG_{BC}	$EG_{BN} - EG_{CN}$
EG_{CA}	$EG_{CN} - EG_{AN}$
E_{DC}	$495{,}000\ V$
i_{REF}	$2{,}000\ A$
G	$1\ rad/A$
T	$1\ s$
P	1.0989
T_i	$0.0091\ s$