Probability Density Function of the Lightning Crest Current at Ground Level. Estimation of the Lightning Strike Incidence on Transmission Line.

R. Lambert & A. Xémard
Direction des Etudes et Recherches
Electricité De France
1, avenue du Général de Gaulle
92141 Clamart Cedex
France

G. Fleury
Laboratoire de Mathématiques
Appliquées & CNRS
Université Blaise Pascal
63177 Aubière Cedex
France

E. Tarasiewicz & A.Morched
Analytical Methods and Specialized Studies
Ontario Hydro
700 University Avenue
Toronto, Ontario M5G 1X6
Canada

Abstract- **The Electrogeometric Theory is applied to analytically relate the probability density function of the lightning peak current at ground level to the available experimental distribution obtained from strokes to tall structures. Analysis of the influence of the height of the structure on the distribution is made. It suggests that the median value of the distribution at ground level is lower than the median value of the experimental distribution by 10kA. For practical calculations, a simplified expression is proposed to approximate the distribution at ground level. As an illustrative example the suggested function is applied to estimate the lightning strike incidence to transmission lines.**

I. INTRODUCTION

Actual methods, recommended by the CIGRE and the IEEE [1, 5, 6], to compute the shielding failure rate (SFR) of transmission lines (TL) are based on the 'Electrogeometric Theory'. They consist of defining a 'lightning exposure distance' for a shielding failure, as a function of the lightning crest current [1, 6]. The lightning crest current is a random variable with a range of possible values. Therefore estimating the SFR involves the integration of the lightning exposure distance weighted by the probability density function (pdf) of the crest current. This procedure can easily be extended to also estimate the total lightning strike incidence to TL as it will be explained in section 4.

Those methods for estimating the shielding failure rate or the total strike incidence require the knowledge of the crest current distribution, specifically the one at ground level, although not explicitly mentioned in [1, 5, 6].
Mostly used practices consider the IEEE [5] or CIGRE [1] approximate expressions of the latest available experimental distribution. However the latter is derived from measurements of lightning strokes to tall structures (less than 60 meters in height) [1].

This paper proposes an analytical expression to relate the probability density function at ground level to the experimental distribution, based on the Electrogeometric Theory. For practical studies an approximate and simple formulation of the distribution at ground level is suggested.

As an illustrative example the suggested distribution is applied to the estimation of the lightning strike incidence to transmission lines. Evaluation of the impact of correcting the distribution is provided.

II. THE ELECTROGEOMETRIC THEORY

Experiments have shown that the final stage of a lightning leader progressing downward involves the initiation of an upward rising connecting leader as the downward leader approaches within a certain critical distance the earth or a structure. This distance referred to as the striking distance is the key concept of the electrogeometric theory for which several ElectroGeometric Models (EGM) have been proposed. It permits the prediction of the point of impact of a stroke, that is the origin of the upward rising leader.
For example, if all striking distances, to the shield wire, to the phase conductors, and to earth are considered equal for a certain lightning current, then a vertical downward lightning leader would terminate on the closest object.

It is common to refer to the equation developed by Love (equation 1) to determine this distance, for which the above assumption of equal striking distances is made.

$$s(i) = 10.i^{0.65} \tag{1}$$

where $s(i)$ is the striking distance in meters,
i is the peak current amplitude in kA.

As one can see in equation 1, the striking distance is a function of the peak current amplitude of the return stroke.

Considering Love's EGM and one specific value of a stroke current, an illustration of the striking distance above a mast and the surrounding ground is given in Figure 1. The striking distance for the points at the ground describes a plane above the ground whereas the striking distance above the mast top (if represented as a point) describes half a sphere (half a circle for the sectional plan).

As we can see on Figure 1, a vertical downward lightning leader, during its progression to ground, will be intercepted by the mast if it will first meet the striking distance of the mast, i.e. if the downward leader is within a distance $r(i)$ of the mast.

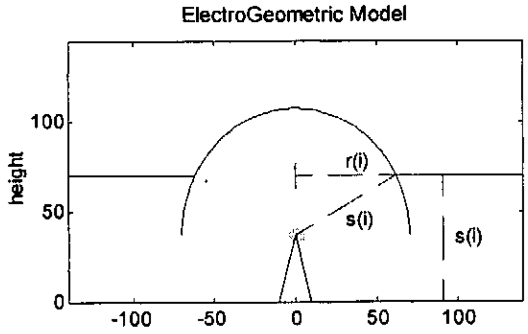

Figure 1: representation (a) of the striking distance s(i) to a mast and its surrounding ground and (b) of the corresponding attractive radius r(i) of the mast.

The geometric zone around the mast within which the process of lightning interception takes place may be expressed through the attractive radius $r(i)$ as follows using Pythagore's theorem :

$$r(i) = \sqrt{s(i)^2 - \left(s(i) - h\right)^2} \quad \text{for}\{i \,/\, s(i) > h\}$$
$$r(i) = s(i) \quad \text{for}\{i \,/\, s(i) \le h\} \qquad (2)$$

where $r(i)$ is the attractive radius
s(i) is the striking distance
h is the mast height
i is the peak current amplitude

In this model the mast creates an electrical shadow on the ground which forces the lightning flashes that would otherwise terminate inside this zone to strike the mast head instead.

I. DISTRIBUTION OF STROKE CURRENT

A. Experimental distribution

A statistical sample of 408 observations of negative downward flashes to structures less than 60m in height has permitted the derivation of the frequency distribution of lightning peak current amplitude [1]. Several formulations have been proposed to approximate this experimental distribution [1]. The one suggested by Anderson [2] (adopted by the IEEE Working Group on Estimating the Lightning Performance of Transmission Lines [1, 5]) and used in this paper is as follows:

$$P(I > i) \quad = 1 - F_I(i) \qquad (3)$$

$$= \frac{1}{1 + (i/31)^{2.6}}$$

where $F_I(.)$ is the cumulative distribution function.

Methods based on the EGM that are used to predict the lightning strike incidence on transmission lines or their shielding failure rate require the knowledge of the lightning peak-current distribution at ground level. Therefore, it is of interest to relate the distribution at ground level to the measured distribution.

A. Distribution at ground level

Mathematical Formulation

According to the electrogeometric theory, we have seen that a mast in an open field will create in its vicinity a circular shadow of radius $r(.)$ function of the peak current amplitude and of the mast's height (see section 2). Let the electrical shadow be denoted by $\Delta(i)$, disc of center the mast stand and of radius $r(i)$.

Let now the probability of observing a lightning flash at $(x, y) \in \Re^2$ and of peak current $i \in \left]0,+\infty\right[$ be of density $f_{I,X,Y}(i,x,y)$ for $(i,x,y) \in \Re_+^* \times \Re \times \Re$.

Then the probability that the head mast is struck by a lightning flash is :

$$m = \int_0^\infty \left(\iint_{\Delta(i)} f_{I,X,Y}(i,x,y)dxdy \right).di \qquad (4)$$

Therefore the density of the peak current at the head mast $f_{I_{mast}}(.)$, given that the head mast is struck by lightning, is:

$$f_{I_{mast}}(i) = \frac{\iint_{\Delta(i)} f_{I,X,Y}(i,x,y)dxdy}{\int_0^\infty \left(\iint_{\Delta(j)} f_{I,X,Y}(j,x,y)dxdy \right).dj} \qquad (5)$$

Given that any point of the ground in an open field is equally likely to be hit by a lightning flash irrespective of the crest current value, that is to say that the peak current and the point of impact are independent random variables, we may therefore write :

$$f_{I,X,Y}(i,x,y) = f_{I_{ground}}(i).g_{X,Y}(x,y) \quad (6)$$

where the probability density $g_{X,Y}$ is uniform in the vicinity of the mast.

and

$$m = \int_0^{\infty} f_{I_{ground}}(i).\left(\iint_{\Delta(i)} g_{X,Y}(x,y)dxdy\right).di$$

$$= \pi. \int_0^{\infty} r(i)^2 . f_{I_{ground}}(i).di$$

(7)

$$f_{I_{mast}}(i) = \frac{f_{I_{ground}}(i). \iint_{\Delta(i)} g_{X,Y}(x,y)dxdy}{\int_0^{\infty} f_{I_{ground}}(j).\left(\iint_{\Delta(j)} g_{X,Y}(x,y)dxdy\right).dj}$$

$$= \frac{f_{I_{ground}}(i). r(i)^2}{\int_0^{\infty} r(j)^2. f_{I_{ground}}(j). dj}$$

(8)

Conversely the inverse relation can be established to derive the probability density at ground level from the probability density for high structure:

$$f_{I_{ground}}(i) = k. \frac{f_{I_{mast}}(i)}{r(i)^2} \qquad (9)$$

where the normalizing constant is :

$$k = \left[\int_0^{\infty} \frac{f_{I_{mast}}(j)}{r(j)^2} dj\right]^{-1} \qquad (10)$$

Application to Anderson's approximation

Knowing that the probability density function is the derivative of the cumulative distribution function :

$$f_I(i) = \frac{d}{di}\left(F_I(i)\right) \qquad (11)$$

and substituting Anderson's distribution approximation (equation 3) into equation 9 we obtain :

$$f_{I_{ground}}(i) = k. \frac{1}{r(i)^2} . \frac{d}{di}\left(1 - \frac{1}{1 + (i / 31)^{2.6}}\right)$$

(12)

The cumulative distribution function (cdf) may be obtained by integrating the probability density function as follows:

$$F_{I_{ground}}(i) = \int_0^{i} f_{I_{ground}}(i)di \qquad (13)$$

Numerical evaluation of equation 12 and 13 is shown graphically in Figure 2and Figure 3. A thirty meter average mast height has been considered in computing the attractive radius $r(i)$.

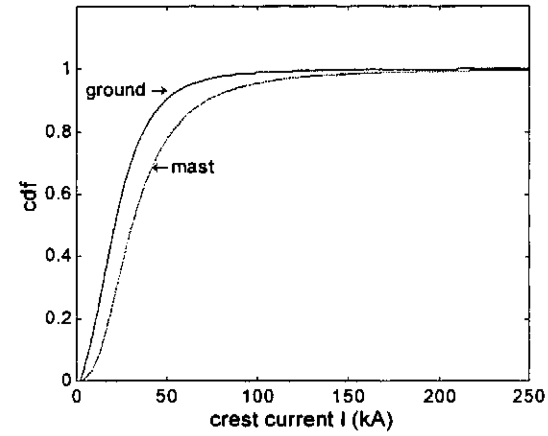

Figure 2 :Anderson's cdf approximation (mast) and associated cdf at ground level.

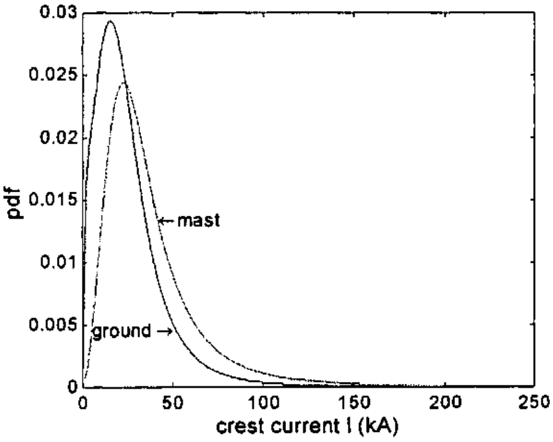

Figure 3 : pdf corresponding to Anderson's cdf approximation and associated pdf at ground level..

To compare the two distributions and illustrate the differences,

Table 1 presents the value i_p such that $F_I(i_p) = p$ for some set of arbitrarily chosen values of p, where p is any given number such that $0 < p < 1$.

Table 1: data extracted from the inverse function of Anderson's approximation (Eq.3.) and from approximation (Eq.13).

$P(I > i)$ (%)	i_{ground}	i_{mast}
99	2.6	5.3
95	4.7	10
90	6.9	13.3
80	10.9	18.2
70	14.4	22.4
60	17.8	26.5
50	**21.4**	**31**
40	25.4	36.2
30	30.3	42.9
20	37.1	52.8
10	49.3	72.2
5	63	96
1	105	181
0.1	187	250

Table 1 shows that the median value at the head mast is 31 kA and at the ground level is 21.4 kA.

Ground distribution approximation

An approximate solution of the same type as Anderson's distribution formula (see equation 3) may be found for the ground distribution of lightning peak current amplitude in the following form :

$$\hat{F}_{I_{ground}}(\alpha, i) = 1 - \frac{1}{1 + (i / I_{50})^{\alpha}} \qquad (14)$$

where I_{50} is the median value.

A non-linear fitting in the least-squares sense of the lightning peak current's cdf at ground level leads to minimize the following expression :

$$\min_{\alpha \in \Re}\left(\sum_k \left|F_{I_{ground}}(i_k) - \hat{F}_{I_{ground}}(\alpha, i_k)\right|^2\right) \qquad (15)$$

Matlab's Levenberg-Marquardt routine gives the following solution :

$$\hat{F}_{I_{ground}}(i) = 1 - \frac{1}{1 + (i / 21.4)^{2.5}}, \qquad (16)$$

for which the relative error is shown in Figure 4.

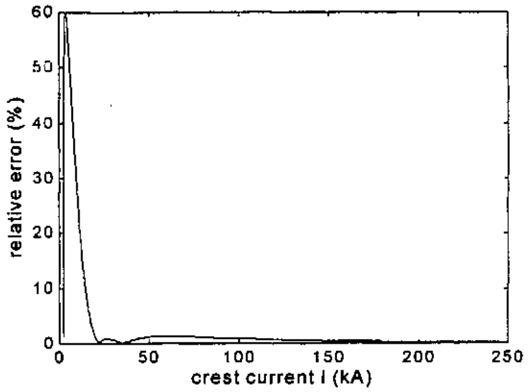

Figure 4 : relative error due to the fitting of the cdf of the lightning peak current at ground level.

One can see that the approximation of the ground distribution (equation 16) does not introduce a significant error for peak current values above 20 kA.

Sensitivity to the mast height

The approximate solution given in equation 16 is based on the analytical description of the interception of a lightning flash by the mast given by the EGM for which an assumption of an average mast height of thirty meters has been made. The impact of this assumption on the approximation of the distribution is studied hereafter. The probability density at ground level for different mast heights using the relation given by equation 12 was computed. The plots of the results are portrayed in Figure 5.

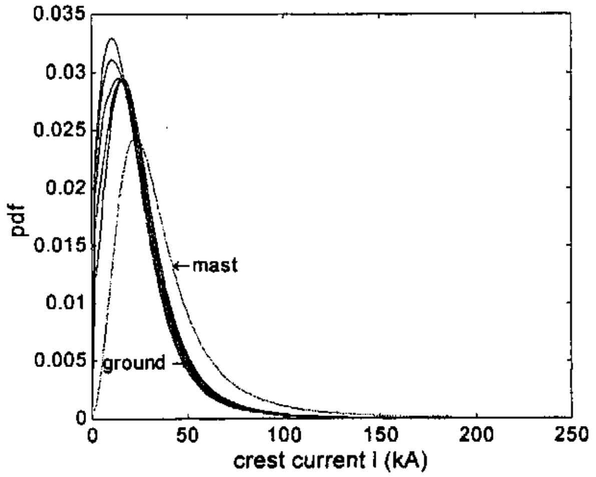

Figure 5 : probability density functions of the peak current at ground level, considering different mast heights in equation 2 .

The corresponding median values and parameters α of the approximate function (equation 14) are given in Table 2.

Table 2 : parameters of the approximate function and distribution's median values as a function of the mast height.

Mast height (meters)	α_0	Median value for the peak current (kA) I_{50}
20	2.6	22.3
30	**2.5**	**21.4**
40	2.4	20.3
50	2.3	19.3
60	2.2	18.4

It can be concluded that the differences between the experimental distribution and the corrected distribution may be important for lightning strike incidence calculation. The sensitivity to the mast height within a reasonable and realistic range of values may seem to be insignificant with respect to the uncertainties of the lightning input data. Therefore, the use of the appropriate formula for the ground distribution seems to be advisable, in particular for calculation of the lightning strike incidence on transmission lines.

I. LIGHTNING STRIKE INCIDENCE

The number of flashes to a transmission line (TL) is related to the lightning activity of the region in which the transmission line is. The regional lightning activity is often characterised by the so-called lightning ground flash density N_g which expresses the number of flashes to ground per unit area (square kilometer) per year.

If all lightning leaders are considered vertical, the 'electrical shadow' created on the ground by the TL may be assumed to be a band along the TL. The width of the band represents the transversal exposure distance of the TL, given by the sum of the distance between the shield wires with the radius of attraction on each side as follows :

$$2.\, r(i) + d_{sw} \qquad (17)$$

where $r(i)$ is the attractive radius of the line (see section II)
d_{sw} is the distance between the two shield wires, if one ground wire $d_{sw} = 0$.

We have seen in section II that the attractive radius is a function of the crest current and of the structure's height. For the latter we can consider a weighted average height as follows :

$$h_a = h_m + \frac{1}{3}(h_t - h_m) \qquad (18)$$

where h_a is the weighted average height of the shield wire(s),
h_m is the shield wires' height at midspan,
h_t is the shield wires' height at the towers.

Now considering the ground flash density and integrating with respect to i the geometric zone of exposure weighted by its probability density we obtain an estimation of the number of strokes to the line:

$$N_s = N_g \cdot \int_0^{\infty} L_l \cdot (2.\, r(i) + d_{sw}) \cdot f_{I_{ground}}(i) di \qquad (19)$$

where N_s is the number of lightning strokes terminating on the transmission line,
L_l is the line length,
$f_{I_{ground}}(.)$ is the probability density function of the peak current at ground level.

Strike incidence per year and 100km line length as a function of the average line height is shown in Figure 6. A unit ground flash density ($N_g = 1$) has been assumed and several distances between the two ground wires have been considered.

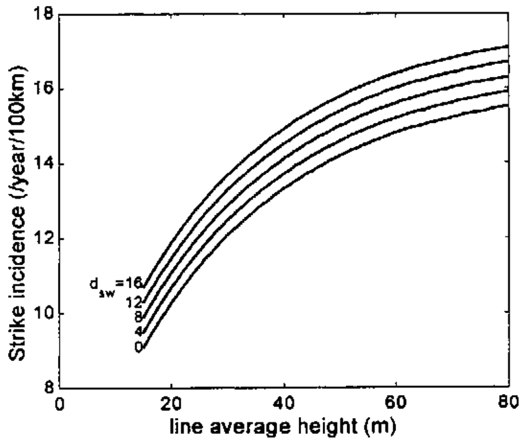

Figure 6 : Number of flashes to a line (per 100km/year) versus line height for different spacing between the two ground wires (Ng=1).

The distance between the two ground wires introduces a small offset on the estimation of the strike incidence (see Figure 6).

It may be noted that in the referenced literature [1, 2], the estimation of the lightning strike incidence on transmission line was calculated without differentiating between the experimental distribution and the one at ground level. To illustrate the importance of using the correct distribution the number of flashes to the line was computed for a 230kV line and for a wide range of transmission line average height.

Numerical Example

To illustrate the difference, we will estimate the expected lightning strike incidence for a shielded 230kV transmission line considering both distributions, the IEEE approximated form of the experimental distribution, and the corrected distribution computed at ground level. The strike incidence estimation is computed using Eq.19.

Line Data :

line : L24A, single shield wire
towers : ST6-69
height of the shield wire at tower : 37.2 m
height of the shield wire at midspan : 30.8 m
regional ground flash density : $N_g = 1.44$ flash/km²/year

Results :

weighted average height : 32.9 m
$N_{s\_b} = 18$ flashes/100km/year
$N_{s\_c} = 20.7$ flashes/100km/year
where

N_{s_b} : is the number of flashes to the line based on the biased distribution
N_{s_c} : is the number of flashes to the line based on the corrected distribution

$$\text{relative error (\%)} = \frac{|N_{s\_b} - N_{s\_c}|}{N_{s\_c}} * 100$$

The estimation based on the experimental distribution overestimates by 15% the lightning strike incidence.

Standard average height

Let us now estimate for both distributions the expected lightning strike incidence of a transmission line as a function of its weighted average height. The distance between ground wires is assumed equal to zero (one overhead ground wire), and a ground flash density is assumed equal to 1 ($N_g = 1$ flash/km²/year). Results are represented graphically in Figure 7. The values from the previous example are indicated. (14.4*1.44=18 and 12.5*1.44=20.7 as calculated in numerical example).

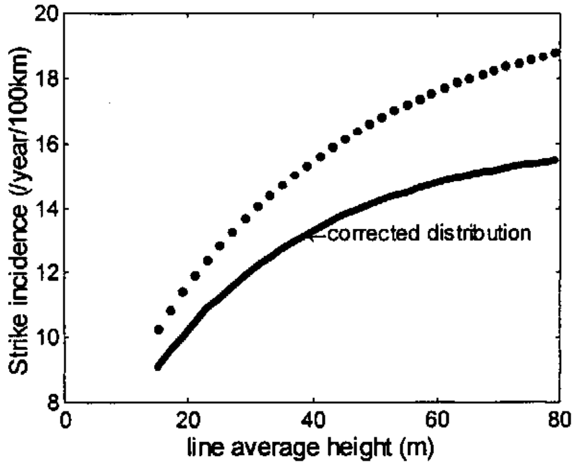

Figure 7: Number of flashes to the line versus the transmission line average height for both Anderson's distribution approximation (referred to as the biaised pdf) and the approximated ground distribution as in equation 17 (referred to as the corrected pdf).

According to the electrogeometric concepts, the estimation based on the experimental distribution would overestimate by 15 to 20% the lightning strike incidence. This is not a significant difference considering the fact that both estimations are mainly affected by the validity of the EGM and the accuracy of the lightning data.

I. CONCLUSION

In this paper we have presented an analytical expression to relate the probability distribution of the lightning peak current at ground level to the available experimental distribution, which is derived from measurements of strokes to tall structures. The approach is based on the ElectroGeometric Theory, for which all striking distances have been assumed equal.

Considering Love's Electrogeometric Model and the IEEE approximation of the stroke current experimental distribution, a simple approximate formulation is proposed for the distribution at ground level:

$$P(I > i) = \frac{1}{1 + (i / 21.4)^{2.5}}$$

The suggested distribution has been applied to the estimation of the lightning strike incidence to transmission lines. Comparison of the predicted number of flashes terminating on transmission lines using the IEEE standard distribution and the corrected distribution suggests that use of the standard distribution will overestimate the lightning strike incidence by 15-20%.
Finally a chart (Figure 6) is proposed to quickly estimate the number of flashes to a power line, given its average height.

REFERENCES

[1] CIGRE Working Group 01 (Lightning) of Study Committee 33 (Overvoltages and Insulation Coordination), 'Guide to procedures for estimating the lightning performance of transmission lines', CIGRE Brochure #63, October 1991, Paris.

[2] J.G. Anderson, Transmission Line Reference Book, 345kV and Above, Second Edition, Chapter 12, Electric Power Research Institute, Palo Alto, California, 1982.

[3] J.R. Currie, A.C. Liew, and M. Darveniza, 'Monte-Carlo Determination of the Frequency of Lightning Strokes and Shielding Failures on Transmission Lines', IEEE Trans. on PA&S, Vol. 90, Sept/October 1971, pp. 2305-2312.

[4] A.J. Eriksson, 'The incidence of Lightning Strikes to Transmission Lines', IEEE Trans. PWRD, Vol. 2, July 1987, pp. 859-870.

[5] IEEE Working Group on Estimating the Lightning Performance of Transmission Lines, 'Design Guide for Improving the Lightning Performance of Transmission Lines', IEEE Standards PAR 1243, Draft 12, July 1996.

[6] IEEE Working Group Report, 'Estimating Lightning Performance of Transmission Lines II - Updates to Analytical Models', IEEE Trans. on PWRD, Vol. 8, No. 3, July 1993.

[7] I.S. Grant and J.G. Anderson, 'A Comparison of Electro-Geometric Theory and the Shadow Method for Determination of the Incidence of Lightning to a Transmission Line Conductor', CIGRE 33-85 (WG01) IWD.

[8] A.M. Mousa, and K.D. Srivastava, 'The Implications of the Electrogeometric Model Regarding Effect of Height of Structure on the Median Amplitude of Collected Strokes', IEEE Trans. on PWRD, Vol. 4, No. 2, April 1989, pp.1450-1460.

[9] A.M. Mousa and K.D. Srivastava, 'The Deviation from the Log-normal Format of the Frequency Distribution of the Amplitudes of the Lightning Currents Collected by Ground Objects', Proc. of Canadian Conf. on Electrical and Computer Engineering', Montreal, Quebec, Canada, ISSN 0840-7789, 1989, pp. 617-621.

[10] F.A.M. Rizk, 'Modeling of Transmission Line Exposure to Direct Lightning Strokes', IEEE Trans. on PWRD, Vol. 5,No. 4, October 1990, pp. 1983-1997.

BIOGRAPHIES

Gérard Fleury received his Ph.D. degree in Computer Science from Blaise Pascal University (Clermont-Ferrand, France) in 1993. He is Maître de conférences in the Laboratory of Applied Mathematics of Blaise Pascal University and CNRS. His research interests concern probabilistic numerical methods.

Rémi Lambert was born in Nancy, France, on May 10, 1973. He graduated from the National Institute of Applied Sciences in Lyon, France, with an engineering degree and a diploma (DEA) in electrical engineering in 1996. He is a Ph. D. student, and has been working on the definition of probabilistic methods for insulation coordination studies at the Research Division of EDF.

Atef S. Morched (senior member, IEEE) received a B. Sc. in Electrical Engineering from Cairo University in 1964, a Ph. D. and a D. Sc. from the Norwegian Institute of Technology in Trondheim in 1970 and 1972. He has been with Ontario Hydro since 1975 where he currently is the Manager of the Analytical Methods and Specialized Studies of the Grid System Strategies and Plans Division.

Eva Tarasiewicz received the B.Sc., M.Sc., and Ph.D. degrees in electrical engineering from Poznan University of Technology, Poland, in 1977, 1978 and 1982, respectively. From 1982 to 1983 she has worked as an Assistant Professor at Poznan University of Technology. From 1983 to 1987 she continued her work on numerical methods in computation of electromagnetic fields transients at the University of Manitoba, Canada and at Mc Master University, Hamilton, Canada, as an Assistant Professor in the Department of Electrical and Computer Engineering. In 1987 she joined Ontario Hydro, Toronto, Canada, where she is now a senior engineer. She is a member of the IEEE Surge Protective Devices Committee and chair of its Working Group Arrester Protection and Coordination with Transformer Insulation.

Alain Xémard was born in France on December 20, 1961. He graduated from the National Institute of Applied Sciences in Lyon, France, with an engineering degree in electrical engineering in 1985. His research interests include insulation coordination, and development of tools for electromagnetic transient calculation. He has been working at the Research Division of EDF since 1992.