Z-Domain Frequency-Dependent Network Equivalent for Electromagnetic Transient Studies

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***Abstract*—The size and complexity of modern power systems necessitate the use of Frequency Dependent Network Equivalents (FDNE) for part of the system. In order to give realistic simulations the frequency response of the equivalent must match that of the system it represents. In this paper a method for developing FDNE is presented and demonstrated. The FDNE is generated by Linearised Least Square fitting the frequency response of a z-domain formulation. The advantage of this approach is that a direct implementation occurs, which does not incur errors due to discretization inherent with implementing a fitted function in the s-domain. The developed FDNE is accurate and efficient, however, the disadvantage is that the z-domain formulation is time-step dependent and hence must be refitted every time the time step is altered.**

Keywords: Frequency Dependent Network Equivalents, Electromagnetic Transient Simulation

I. Introduction

Frequency Dependent Network Equivalents (FDNE) are required to represent part of the system that is external to the area of interest. In order to give accurate simulation the frequency response of the FDNE must match that of the system it represents.

Although the focus of this paper is on FDNE, the techniques for fitting the frequency response of a network are the same as fitting the frequency response of individual components to form accurate components models. For transmission lines the highly oscillatory nature of the terminal admittance response would require a high order function to accurately fit it. However the frequency response of the wave propagation matrix ($\exp(-G.\lambda)$) and characteristic admittance matrix are simpler and once a rational function is fitted a travelling wave model can be implemented.

Early techniques for developing equivalents for networks used the features (i.e. resonant points) of the network's frequency response to realise an RLC network that would mimic this frequency response [1]-[9]. The main reason for realising an RLC network is that these are easily implemented in existing transient analysis programs without requiring extensive modifications. However the assumed topology of the RLC network does influence the equations used for fitting and the accuracy that can be achieved. The parallel form (Foster circuit) proposed by Hingorani and Burbury [1] seems to work reasonable well for transmission network response but cannot model an arbitrary frequency response. Although the synthesis is direct, it is based on first ignoring the losses and determine the L and C values to give the right resonant frequencies then determining the R values to match the response at minima points. Most researchers have found that an iterative/optimisation procedure is necessary after this, to improve the fit.

An alternative approach is to fit a rational function to a response and implement the rational function directly in the transient program, without realizing an equivalent circuit. The fitting of a rational function may be performed in the s or z domain on time or frequency domain data. If in s-domain the bilinear transform is then used to "descretize" the function to allow it to be implemented in a digital computer. Other variations are to use direct ARMA Transfer Method or Prony Analysis. Both are similar in that they require an over-determined set on equations to be solved however differ in the way the equations are developed. In Prony analysis the discrete transfer function eigenvalues and residuals are determined from the ring down signal and using information about the input signal [10] [11]. Singular Value Decomposition (SVD) is the usual solution technique for these over-determined set of equations problems.

Singh and Abur [12] [13] proposed the use of two-sided recursion formulation (another name for ARMA model) for developing FDNE. Identification is directly in the z-domain of time domain data using a multi-sine excitation signal. This technique was then modified and applied to Transmission line modelling by Angelidis and Semlyen [14]. Frequency domain fitting was performed on the wave propagation matrix and characteristic admittance matrix. Noda, Nagaoka and Ametani continued this approach for transmission line modelling and made some significant improvements [15] [16]. The work of Hong *et al* [17] [18] is essential the same as the work of Singh and Abur except Prony Analysis formulation was used instead of a straight time domain formulation.

Todd *et al* [19] developed a FDNE using fitting frequency data by a rational function in the s-domain. Gustavsen and Semlyen have also developed an excellent method for fitting, called vector fitting. This was first applied to transmission line modelling and has subsequently been applied to FDNE [20] [21] [22]. Again s-domain fitting was used.

One of the motivations for investigating z-domain fitting is (as evident in the work of Singh *et al*, Noda *et al* and Angelidis *et al*), that it can be directly implemented in a digital simulation program without any loss of accuracy as it is already a discrete formulation. Fitting in the s-domain always requires "discretizing" a continuous system and the inherent approximations.

This paper presents the formulation for developing FDNE using z-domain fitting of the frequency response and illustrates its use. This work is the z-domain equivalent of the work of Todd *et al* [19], however it does not suffer the implementation errors that existed in this work. It is also the frequency domain counterpart of the work of Singh and Abur [13] [12].

II. Fitting Issues

One of the advantage of the z-domain fitting over s-domain fitting is that it gives a more accurate fit for a given order of rational function. This is because all the poles and zeroes are constrained to be in the frequency of interest (i.e. below the Nyquist frequency). By contrast, fitting in the s-domain results in poles and zeroes being placed wherever needed over the complete frequency range of the fit, therefore some are likely to be positioned above the Nyquist frequency. This results in the main disadvantage of the z-domain fitting, which is that the fitting procedure must be repeated if the time step is changed where as the s-domain coefficients are independent of time-step. If the time step is halved the poles and zeros must be distributed up to the new Nyquist frequency in the z-domain whereas the s-domain is already fitted to the new frequency range.

Restricting the frequency range of the s-domain fit to below the Nyquist frequency results in a fit of similar accuracy to the z-domain, however the ability to halve the time-step without being required to refit is lost. If the time-step is halved the fit must be re-performed as the frequency range between the old and new Nyquist frequencies has not been matched. Ad hoc ways around this include fitting in the s-domain to a high frequency then "pruning" the terms down to the frequency of interest. If the simulation time-step is changed only the number of terms pruned needs to be changed.

Another advantage of z-domain fitting over s-domain is that the accuracy of the simulation is clearly seen. The s-domain rational function must be *discretized* so that it can be simulated on a digital computer. This is normally performed by using the Bilinear transform, which introduces additional errors above those of the fitting process. However the development of root matching techniques as a replacement for bilinear transform now allows highly accurate "discretizing" of the s-domain functions [23] [24]. Moreover FDNE developed by z-domain fitting does not suffer from numerical oscillations that are prevalent when a detailed representation is used [25].

The z-domain is a representation of a discrete system and hence there is no error in its implementation on a digital computer. The fitting errors alone are present. To facilitate implementation, a rational function in s-domain is normally split into partial fractions and implemented as the summation of many first order terms.

Stability of the fit is essential, without it the system can not be simulated. Testing the stability of the fit is easily achieved after performing the fit, however the illusive goal is to incorporate stability criteria as part of the fitting process. It can be achieved by fitting only real poles in left half plane (in s-domain) but this greatly restricts the accuracy that can be achieved. Other approaches have been to mirror poles in the right half plane into the left half plane to ensure stability or to remove them on the basis that corresponding residual is small. The left-hand half s-plane maps to the unit circle in the z-plane, therefore the stability criteria is that the pole magnitude is less than or equal to one. One way of determining this for both s and z domains is to find the poles by finding the roots of the characteristic equation (denominator) and checking that these criteria are met. Another method is to use the Jury table (Z-domain) or the s-domain equivalent of Routh-Hurwitz stability criteria [26]. As a general rule as the order of the rational function is increased the more accurate the fit but the less stable. So the task is to find the highest order stable fit. No simple approach, equivalent to s-plane approaches mentioned previously for turning an unstable fit into a stable fit, have been developed yet.

When simulating multi-terminal equivalents, such as three phase system with mutual coupling between phases, an admittance matrix rather scalar admittance must be fitted as functions of frequency. Although the fitting of each element in the matrix may be stable, inaccuracies in the fit can result in the complete system having instabilities at some frequencies. Again there is a need not to fit each element independently, but in such a way as to ensure the system of fitted terms are stable.

The ability to weight fundamental frequency has also been incorporated in the formulation given in appendix A. The least squares fitting process tends to smear the fitting error over the frequency range. Although this gives good transient response it results in a small, but noticeable steady-state error. By giving fundamental frequency a higher weighting (typically 100) the steady-state error is removed while the transient response is slightly deteriorated due to higher errors at other frequencies.

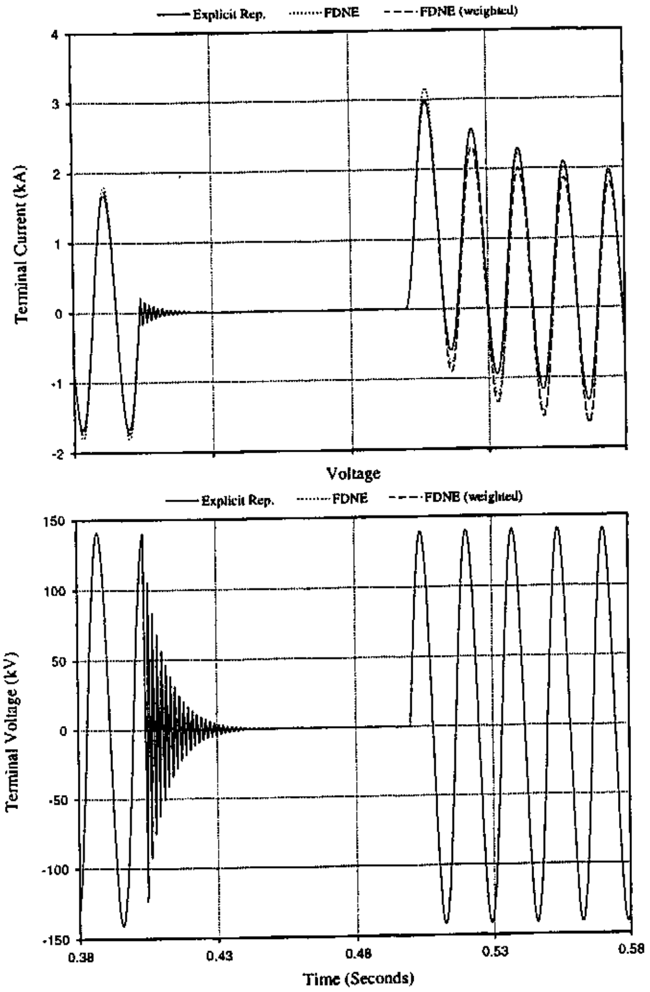

Fig. 1. Simple 1-Port Network

III. Illustrative Example

To illustrate the use of FDNE developed using z-domain fitting of the frequency response a few simple single-phase examples will be given. The simulation package PSCAD/EMTDC was used to demonstrate the techniques and a custom component written to implement the z-domain rational function. Most of the complexity is in the derivation of the rational function coefficients which is performed by a utility program. The techniques can equally be applied to any electromagnetic transient program. Only the ability to represent a z-domain rational function is required. Due to lack of space only the salient points will be given. The test system is a simple ac system supplying a load via a transmission line with frequency-dependent parameters. The 1-port FDNE represents the transmission line and load. The 2-port takes the receiving end as the second port.

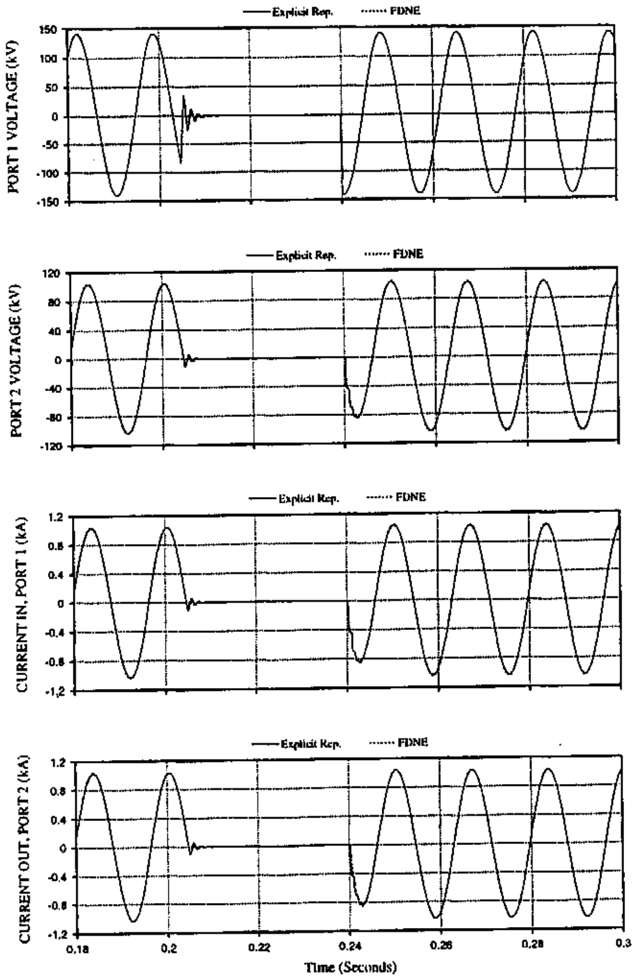

Fig. 2. Simple 2-Port Network

Figure 1 displays the comparison of terminal current and voltage between two different 1 port FDNE's and explicit representation of a simple system consisting of a transmission line (frequency-dependent) and load, with the circuit breaker opening and closing. The first FDNE has equal weighting on all frequencies while the second weights the fundamental higher. Careful inspection of the current given by the FDNE shows that it is better than that given by the explicit representation. The explicit representation exhibits numerical noise when the circuit breaker is opened. This numerical noise is due to the well known truncation errors involved with the Trapezoidal rule that is used to *discretize* the continuous system. FDNE developed using z-domain matching do not suffer from this type of error. The two FDNE's are both 10^{th} order models and perform reasonably well. As one might expected, the first FDNE gives more steady-state error but better transient representation while the second FDNE smaller steady-state error and less accurate transient representation.

Making the load-side a port and explicitly representing the load gives a simple 2-port model. Figure 3 shows the comparison of terminal current and voltage between a 1 port FDNE and explicit representation. The match is very good. Careful inspection of the switch ON (Figure 3) shows that the current surges arrive at slightly different times. This is because the explicit model uses an interpolated transmission line model that accurately models travelling times that are non-integer multiples of the step-length, while the FDNE can only represent multiple time step-length delays at present.

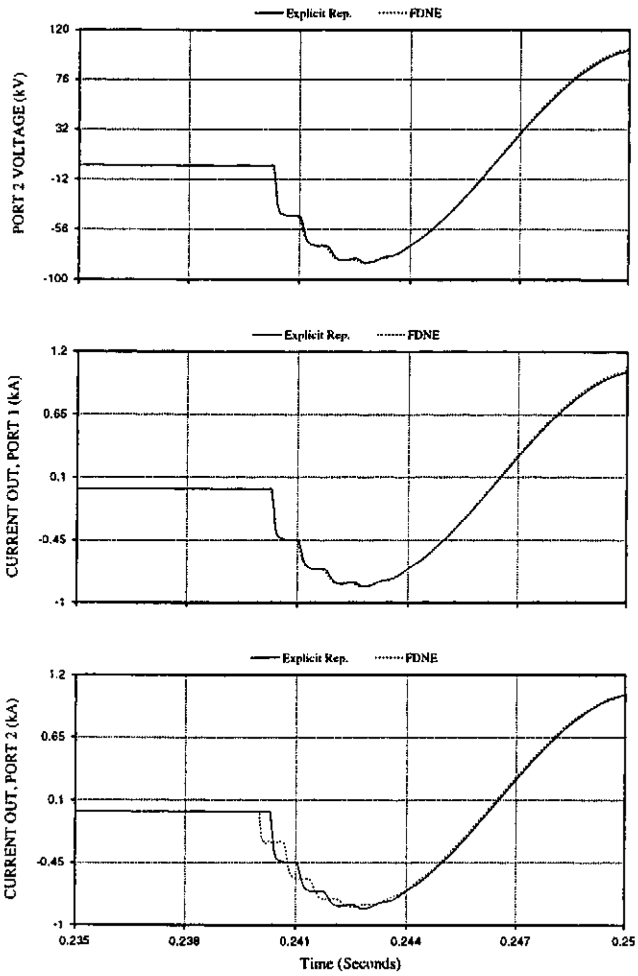

Fig. 3. Expanded view of simple 2-Port Network

IV. Conclusion

This paper has presented the use of z-domain fitting of a frequency response for developing frequency dependent network equivalents and the various issues have been discussed. The advantages are:

- accuracy due to pole/zero placement in the simulation frequency spectrum
- ease of implementation and that no error involved in its implementation.

The main disadvantage of z-domain fitting is the need to re-perform the fitting processes whenever the time step is varied. As varying the time-step is likely to be required for a multitude of reasons this is a major consideration. Moreover the development of root-matching techniques allow highly accurate "discretizing" of the s-domain, thereby diminishing the accuracy advantage of z-domain fitting.

Ensuring stability is also more difficult in the z-domain with no *ad hoc* methods available other than reducing the order and re-fitting. Incorporating stability criteria as part of the fitting process still needs to be achieved.

V. Acknowledgement

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VII. Appendix A - Formulation

$$H(z)=\frac{a_0+a_1z^{-1}+a_2z^{-2}+...+a_mz^{-m}}{1+b_1z^{-1}+b_2z^{-2}+...+b_mz^{-m}} \quad (1)$$

Evaluating the frequency response of the rational function and equating it to the required value gives:

$$H(j\omega)=\frac{\sum_{k=0}^{m}(a_k z^{-kj\omega\Delta t})}{1+\sum_{k=1}^{m}(b_k z^{-kj\omega\Delta t})} \quad (2)$$

Multiplying both sides by the denominator and rearranging gives:

$$H(j\omega) = -\sum_{k=1}^{m}((b_k.H(j\omega)-a_k)\epsilon^{-kj\omega\Delta t})-a_0$$

Using $H(j\omega)=c(j\omega)+jd(j\omega)$ gives:

$$\begin{aligned}-H(j\omega) &= \sum_{k=1}^{m}((b_k.(c(j\omega)+jd(j\omega)) \\ &\quad -a_k)\epsilon^{-kj\omega\Delta t})-a_0 \\ -c(j\omega)-jd(j\omega) &= \sum_{k=1}^{m}((b_k.c(j\omega) \\ &\quad -a_k)+jb_k.d(j\omega))\epsilon^{-kj\omega\Delta t})-a_0\end{aligned}$$

Splitting into Real and Imaginary components (using $\epsilon^{-kj\omega\Delta t}=\cos(k\omega\Delta t)+j\sin(k\omega\Delta t)$)gives:

$$\begin{aligned}-c(j\omega) &= \sum_{k=1}^{m}((b_k.c(j\omega)\cos(k\omega\Delta t)+ \\ &\quad b_kd(j\omega)\sin(k\omega\Delta t))-a_0 \quad (3) \\ -d(j\omega) &= \sum_{k=1}^{m}(-(b_k.c(j\omega)-a_k)\sin(k\omega\Delta t)+ \\ &\quad b_kd(j\omega)\cos(k\omega\Delta t))-a_0 \quad (4)\end{aligned}$$

rearranging yields:

$$\begin{aligned}-c(j\omega) &= \sum_{k=1}^{m}(b_k.(c(j\omega)\cos(k\omega\Delta t)+d(j\omega) \\ &\quad \sin(k\omega\Delta t))-a_k\sin(k\omega\Delta t))-a_0 \\ -d(j\omega) &= \sum_{k=1}^{m}(b_k.(d(j\omega)\cos(k\omega\Delta t)-c(j\omega)) \\ &\quad -a_k\sin(k\omega\Delta t)) \quad (5)\end{aligned}$$

In matrix form the set of equations to be solved is:

$$\begin{pmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{pmatrix}\begin{pmatrix}\underline{a}\\ \underline{b}\end{pmatrix}=\begin{pmatrix}\underline{C}\\ \underline{D}\end{pmatrix} \quad (6)$$

where:

$$\begin{aligned}\underline{a}^T &= [a_0,a_1,a_2,....a_m] \\ \underline{b}^T &= [b_1,b_2,....b_m] \\ \underline{C}^T &= [-c(j\omega_1),-c(j\omega_2),....-c(j\omega_n)] \\ \underline{D}^T &= [-d(j\omega_1),-d(j\omega_2),....-d(j\omega_n)]\end{aligned}$$

$$A_{11} = \begin{pmatrix}-1 & -\cos(\omega_1\Delta t) & \dots & -\cos(m\omega_1\Delta t)\\ -1 & -\cos(\omega_2\Delta t) & \dots & -\cos(m\omega_2\Delta t)\\ \vdots & \vdots & \ddots & \vdots\\ -1 & -\cos(\omega_n\Delta t) & \dots & -\cos(m\omega_n\Delta t)\end{pmatrix}$$

$$A_{21} = \begin{pmatrix}0 & -\cos(\omega_1\Delta t) & \dots & -\cos(m\omega_1\Delta t)\\ 0 & -\cos(\omega_2\Delta t) & \dots & -\cos(m\omega_2\Delta t)\\ \vdots & \vdots & \ddots & \\ 0 & -\cos(\omega_n\Delta t) & \dots & -\cos(m\omega_n\Delta t)\end{pmatrix}$$

$$A_{12} = \begin{pmatrix}R_{11} & R_{12} & \dots & R_{1m}\\ R_{21} & R_{22} & \dots & R_{2m}\\ \vdots & \ddots & \vdots & \\ R_{n1} & R_{n2} & \dots & R_{nm}\end{pmatrix}$$

$$A_{22} = \begin{pmatrix}S_{11} & S_{12} & \dots & S_{1m}\\ S_{21} & S_{22} & \dots & S_{2m}\\ \ddots & \vdots & \vdots & \\ S_{n1} & S_{n2} & \dots & S_{nm}\end{pmatrix}$$

$$\begin{aligned}R_{ik} &= c(k\omega_i)\cos(k\omega_i\Delta t)+d(k\omega_i)\sin(k\omega_i\Delta t) \\ S_{ik} &= d(k\omega_i)\cos(k\omega_i\Delta t)-c(k\omega_i)\sin(k\omega_i\Delta t) \\ m &= Order\ of\ fit \\ n &= Number\ of\ frequency\ sample\ points\end{aligned}$$

As the number of sample points exceeds the number of unknown coefficients singular value decomposition is used to solve this equation.

Least squares fitting smears the error across the frequency spectrum. This is undesirable as it is important to obtain accurately the steady-state condition. Adding weighting factors allows this to be achieved. Adding weighting factors gives:

$$\begin{aligned}w(j\omega).(H(j\omega)) &= -\sum_{k=1}^{m}(w(j\omega)(b_k.H(j\omega) \\ &\quad -a_k)\epsilon^{-kj\omega\Delta t})-w(j\omega)a_0\end{aligned}$$

$$\begin{aligned}-w(j\omega).c(j\omega) &= \sum_{k=1}^{m}(b_k.w(j\omega).(c(j\omega)\cos(k\omega\Delta t) \\ &\quad +d(j\omega)\sin(k\omega\Delta t)) \\ &\quad -a_k.w(j\omega)\sin(k\omega\Delta t))-w(j\omega).a_0 \\ -w(j\omega).d(j\omega) &= \sum_{k=1}^{m}(b_k.w(j\omega).(d(j\omega)\cos(k\omega\Delta t) \\ &\quad -c(j\omega))-a_k.w(j\omega)\sin(k\omega\Delta t))\end{aligned}$$