Controller Modelling in Electromagnetic Transient Simulations

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***Abstract*—Recent advances in power electronics have led to rapid deployment of power electronic devices such as HVDC and FACTS controllers, in the power system. The controllers for these devices determine their operational behaviour, therefore accurate representation in system studies such as electromagnetic transient analysis. The details of the controls differ due to the differing characteristics and requirements of the system they are connected to. To cope with this electromagnetic transient programs allow flexibility in controller modelling by providing fundamental control blocks which can be interconnected in an arbitrary manner. Each block is simulated by a difference equation created by numerical integrator substitution into the appropriate equation. This however introduces a time-step delay in the data path as some variables are from the previous time step when evaluating a difference equation, as the difference equation evaluating it has not be solved yet. This introduces error and when sufficiently large, instabilities in the simulation. This is particularly true when the time-step is large relative to some of the time constants of the controller.**

This paper demonstrates the effect of this time-delay in data path and shows how the z-domain is a powerful tool in analysing the difference equations and data path delays involved with representing the controllers. From the z-domain considerations instabilities can be accurately predicted. Moreover these two ways of representing controllers is compared to the exponential form of difference equation derived using root-matching techniques.

Keywords: Control Representation, Electromagnetic Transient Simulation

I. Introduction

Controller representation is of great importance in electromagnetic transient simulations due to their great influence. Recent advances in power electronics have led to more power electronic devices, such as FACTS devices, being deployed in the power system and each requires some type of controls. In fact the performance of FACTS devices is more greatly influenced by the controller representation than the main circuit representation.

Ultimately the controller must be represented by difference equations for simulation purposes. This involves translating the control blocks, such as integrators, multipliers,..etc, into a discrete form for simulation on a digital computer. In other words the controller must be represented by difference equation(s) for simulation. Not only is the difference equation crucial for accurate simulation but also the mathematics of data flow.

There are a number of different approaches adopted by electromagnetic transient programs for enabling arbitrary controllers to be modelled. However they center on providing basic control function blocks that can be interconnected in an arbitrary manner. Although very powerful it invariably leads to one time-step delay in some data flow paths, which can lead to instabilities. The alternative, of deriving the transfer function for the complete controller removes the time-step delay in data flow but lacks flexibility. This paper demonstrates the effect of this time-delay in data path and shows how the z-domain is a powerful tool in analysing the difference equations and data path delays involved with representing the controllers. From the z-domain considerations instabilities can be accurately predicted. Moreover these two ways of representing controllers is compared to the exponential form of difference equation derived using root-matching techniques [1] [2] [3].

II. Illustrative Example

The first-order lag control system, depicted in Fig. 1, will be used to demonstrate the use of z-domain for prediction of instabilities.

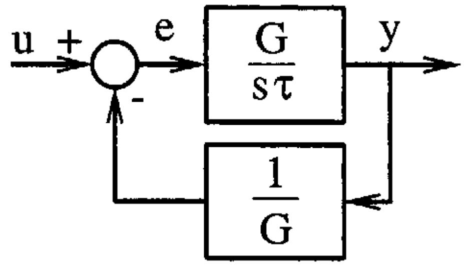

Fig. 1. First-Order Lag

$$\frac{y}{u} = \frac{1}{1+fg} = \frac{G}{1+s\tau} \tag{1}$$

where

f = feedback path = $1/G$
g = forward path = $\frac{G}{s\tau}$

The equations for the two blocks are:

$$e = u - \frac{1}{G}y \quad (2)$$

$$y = \frac{Ge}{s\tau} \quad (3)$$

Substitution of the Trapezoidal rule to form difference equations gives:

$$e_k = u_k - \frac{1}{G}y_k \quad (4)$$

$$y_k = \frac{\Delta t(e_k + e_{k-1})G}{2\tau} \quad (5)$$

From this the difference in data path becomes apparent. If solved as two separate difference equations then e_k must be calculated from y at previous time-step as y_k is not available, hence one time step delay in y data path. Swapping the order of equations will result in the same problem for e data path. Although an iterative approach could be used it is undesirable. Substituting one equation into the other and rearranging results in a difference equation with no delay in data path. This is equivalent to performing integrator substitution on the transfer function for the complete controller

A. Time-step delay in data path

If there is a time-step delay in feedback path due to the way the difference equation for each block is simulated, then $e_k = (u_k - \frac{1}{G}y_{k-1})$

$$y = \frac{G}{s\tau} \quad (6)$$

Applying trapezoidal integration gives:

$$\begin{aligned} y_k &= y_{k-1} + \frac{\Delta tG}{2\tau}(e_k + e_{k-1}) \\ &= y_{k-1} + \frac{\Delta tG}{2\tau}(u_k - \frac{1}{G}y_{k-1} + u_{k-1} - \frac{1}{G}y_{k-2}) \\ &= (y_{k-1} - \frac{\Delta t}{2\tau}y_{k-1} - \frac{\Delta t}{2\tau}y_{k-2}) \\ &\quad + \frac{\Delta tG}{2\tau}(u_k + u_{k-1}) \end{aligned} \quad (7)$$

Tranforming the equation into the z-plane yields:

$$Y(1 - z^{-1}(1 - \frac{\Delta t}{2\tau}) + z^{-2}\frac{\Delta t}{2\tau}) = \frac{\Delta tG}{2\tau}(1 + z^{-1})U \quad (8)$$

Rearranging gives:

$$\begin{aligned} \frac{Y}{U} &= \frac{\frac{\Delta tG}{2\tau}(1 + z^{-1})}{(1 - z^{-1}(1 - \frac{\Delta t}{2\tau}) + z^{-2}\frac{\Delta t}{2\tau})} \\ &= \frac{\frac{\Delta tG}{2\tau}z(z + 1)}{(z^2 - z^1(1 - \frac{\Delta t}{2\tau}) + \frac{\Delta t}{2\tau})} \end{aligned} \quad (9)$$

The roots are given by:

$$z_1, z_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (10)$$

$$\begin{aligned} z_1, z_2 &= \frac{1}{2}(1 - \frac{\Delta t}{2\tau}) \pm \sqrt{(1 - \frac{\Delta t}{2\tau})^2 - 4\frac{\Delta t}{2\tau}} \\ &= \frac{1}{2}(1 - \frac{\Delta t}{2\tau}) \pm \sqrt{(1 - 3\frac{\Delta t}{\tau}) + \frac{\Delta t^2}{4\tau^2}} \end{aligned} \quad (11)$$

Stability is assured so long as the roots are withing the unit circle $|z| \leq 1$.

B. No Time-step delay in data path

If no delay in the feedback path implementation then $e_k = (u_k - \frac{1}{G}y_k)$

Applying trapezoidal integration gives:

$$\begin{aligned} y_k &= y_{k-1} + \frac{\Delta tG}{2\tau}(e_k + e_{k-1}) \\ &= y_{k-1} + \frac{\Delta tG}{2\tau}(u_k - \frac{1}{G}y_k + u_{k-1} - \frac{1}{G}y_{k-1}) \\ &= (y_{k-1} - \frac{\Delta t}{2\tau}y_k - \frac{\Delta t}{2\tau}y_{k-1}) \\ &\quad + \frac{\Delta tG}{2\tau}(u_k + u_{k-1}) \end{aligned} \quad (12)$$

Tranforming the equation into the z-plane yields:

$$Y(1 - z^{-1}(1 + \frac{\Delta t}{2\tau}) + z^{-1}(\frac{\Delta t}{2\tau} - 1) = \frac{\Delta tG}{2\tau}(1 + z^{-1})U \quad (13)$$

Rearranging gives:

$$\begin{aligned} \frac{Y}{U} &= \frac{\frac{\Delta tG}{2\tau}(1 + z^{-1})}{(1 - z^{-1}(1 + \frac{\Delta t}{2\tau}) + z^{-1}(\frac{\Delta t}{2\tau} - 1))} \\ &= \frac{\frac{\Delta tG}{2\tau}(z + 1)}{z(1 + \frac{\Delta t}{2\tau}) + (\frac{\Delta t}{2\tau} - 1))} \end{aligned} \quad (14)$$

Pole is:

$$z = \frac{(1 - \frac{\Delta t}{2\tau})}{(1 + \frac{\Delta t}{2\tau})} \quad (15)$$

Note that $|z_{pole}| \leq 1$ for all $\frac{\Delta t}{2.\tau} > 0$, therefore is always stable. However this does not mean that numerical oscillations will not occur due to errors in trapezoidal integration.

C. Root-Matching Technique

Root-Matching

$$\begin{aligned} \frac{Y}{U} &= \frac{G(1 - \epsilon^{-\frac{\Delta t}{\tau}})}{(1 - z^{-1}\epsilon^{-\frac{\Delta t}{\tau}})} \\ &= \frac{zG(1 - \epsilon^{-\frac{\Delta t}{\tau}})}{(z - \epsilon^{-\frac{\Delta t}{\tau}})} \end{aligned} \quad (16)$$

$$Y = \epsilon^{-\frac{\Delta t}{\tau}}z^{-1}y + G(1 - \epsilon^{-\frac{\Delta t}{\tau}})u \quad (17)$$

Transforming to the time domain yields the difference equation:

$$y_k = \epsilon^{-\frac{\Delta t}{\tau}}y_{k-1} + G(1 - \epsilon^{-\frac{\Delta t}{\tau}})u_k \quad (18)$$

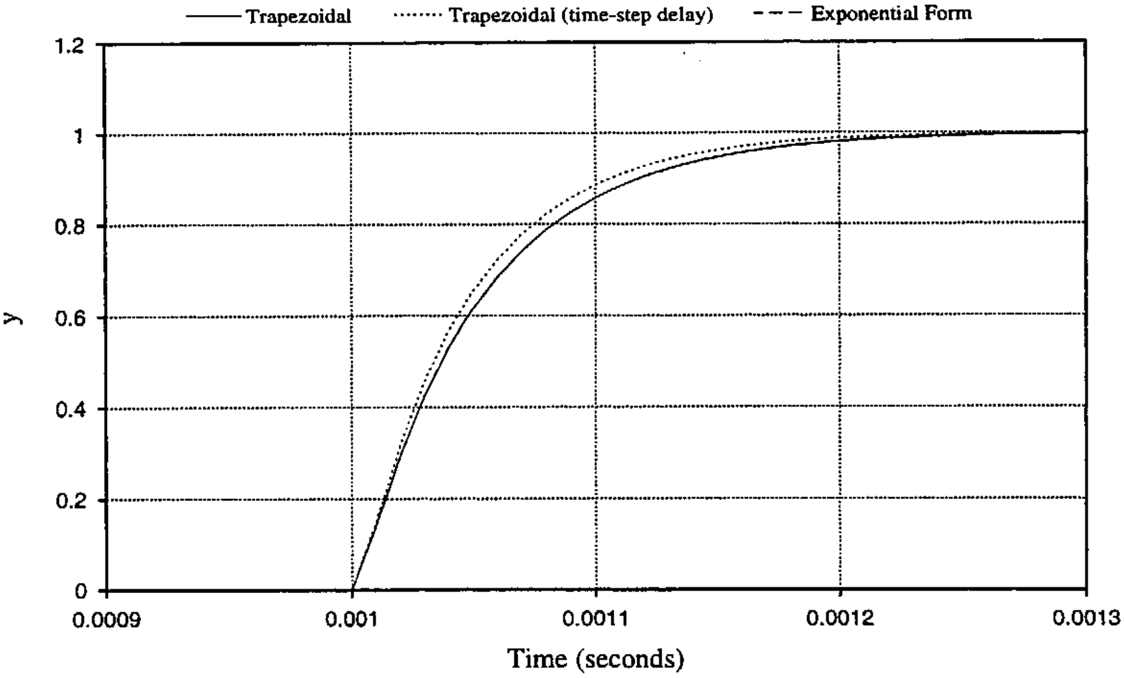

Fig. 2: Simulation results for time-step=5 us

Pole in the z-plane is:

$$z_{pole} = \epsilon^{-\frac{\Delta t}{\tau}} \quad (19)$$

Note that $|z_{pole}| \leq 1$ for all $\epsilon^{-\frac{\Delta t}{\tau}} \leq 1$ hence for all $\frac{\Delta t}{\tau} \geq 0$

III. Illustrative Example

With reference to Fig. 1, three time-steps will be considered ($\Delta t = \frac{1}{\tau}, \tau, 10\tau$). For each time-step the results from using three different difference equations are given; Trapezoidal with no feedback (data path) delay; Trapezoidal with data path delay and exponential form using Root-matching techniue. These results are shown in Fig. 2 to Fig. 4 for $\Delta t = \frac{1}{\tau}, \tau, 10\tau$ respectively.

When $\frac{\Delta t}{\tau} = \frac{1}{10}$ solving equation (11) results in two real roots exist, they are $z_1 = 0.0559$ and $z_2 = 0.894$. As both are less than one the resulting difference equation is stable. This can clearly be seen in Fig. 2. The exponential form and Trapezoidal with no data path delay are indistinguishable while the error introduced by data path delay is noticeable.

When $\frac{\Delta t}{\tau} = 1$ a pair of complex conjugate roots result ($z_1, z_2 = 0.5 \pm j0.6614$). They lie inside the unit circle ($|z_1| = |z_2| = 0.82916 < 1$) hence stable. Fig. 3 shows that although considerable error has been introduced by the time-step delay in data path, the difference equations are stable. The result from the difference equation with data-path delay shows a large error (overshoot) which dies down in approximately 20 time-steps. A slight difference can be seen between no data delay using Trapezoidal integrator and exponential form.

When $\frac{\Delta t}{\tau} = 10$ two real roots exist. They occur at $z_1, z_2 = 1.382$ *and* 3.618. Since these lie outside the unit circle in the z-plane the system of difference equations are unstable. This is shown in the simulation results in Fig. 4.

As predicted by equation (15), the difference equation with no data path delay is always stable but close examination of an expanded view (displayed in Fig. 5) shows numerical oscillation resulting from using trapezoidal integrator. This numerical oscillation will increase as the step-length increases. Fig. 5 also shows the theoretical curve and exponential form of difference equation. The Exponential form of difference equation gives the exact answer at every point it is evaluated. The exponential form has been derived for the overall transfer function (i.e. no time delays in data paths). If a modular building block approach is adopted, the exponential form of difference equation can be applied to the various blocks and the system of difference equations solved in the same way as for Trapezoidal integrator. However, the errors due to data path delays will be incurred.

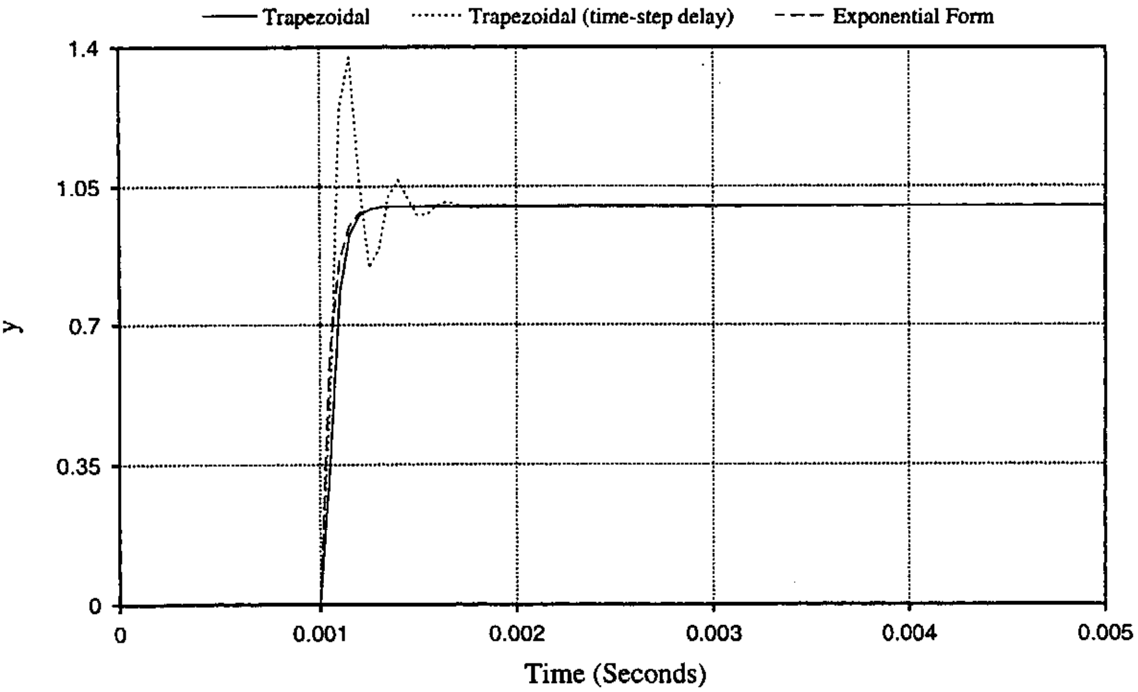

Fig. 3: Simulation results for time-step=50 us

IV. Conclusion

This paper has highlighted the effect of the time-delay in data path that results from having a modular approach to controller representation. The use of z-domain in analysing the difference equations and data path delays has been demonstrated. From the z-domain analysis instabilities can be accurately predicted.

Modelling the complete controller transfer function is preferable to a modular building block approach as it avoids the data path delays and inherent error associated with it, which can lead to instabilities. However the error due to the use of trapezoidal integrator still exists for which the solution is to use the exponential form of difference equation derived using root-matching techniques.

V. Acknowledgement

The author would like to express their appreciation to Jos Arrillaga, Ani Gole, and Dennis Woodford for their help and encouragement in this work. They also wish to thank the Manitoba HVDC Research Centre, Winnipeg, Canada and the University of Canterbury, Christchurch, New Zealand for enabling this work to be performed.

VI. References

[1] Watson N.R. and Irwin G.D, *Electromagnetic Transient Simulation of Power Systems using Root-Matching Techniques* Proceedings IEE, Part C, Vol. 145, No. 5, Sept. 1998, pp 481-486

[2] Watson N.R. and Irwin G.D, *Accurate and Stable Electromagnetic Transient Simulation using Root-Matching Techniques* International Journal of Electrical Power & Energy Systems, Volume 21, Number 3, March 1999, pp 225-234

[3] Watson N.R. and Irwin G.D, *Comparison of Root-Matching Techniques for Electromagnetic Transient Simulation* Submitted to IEEE

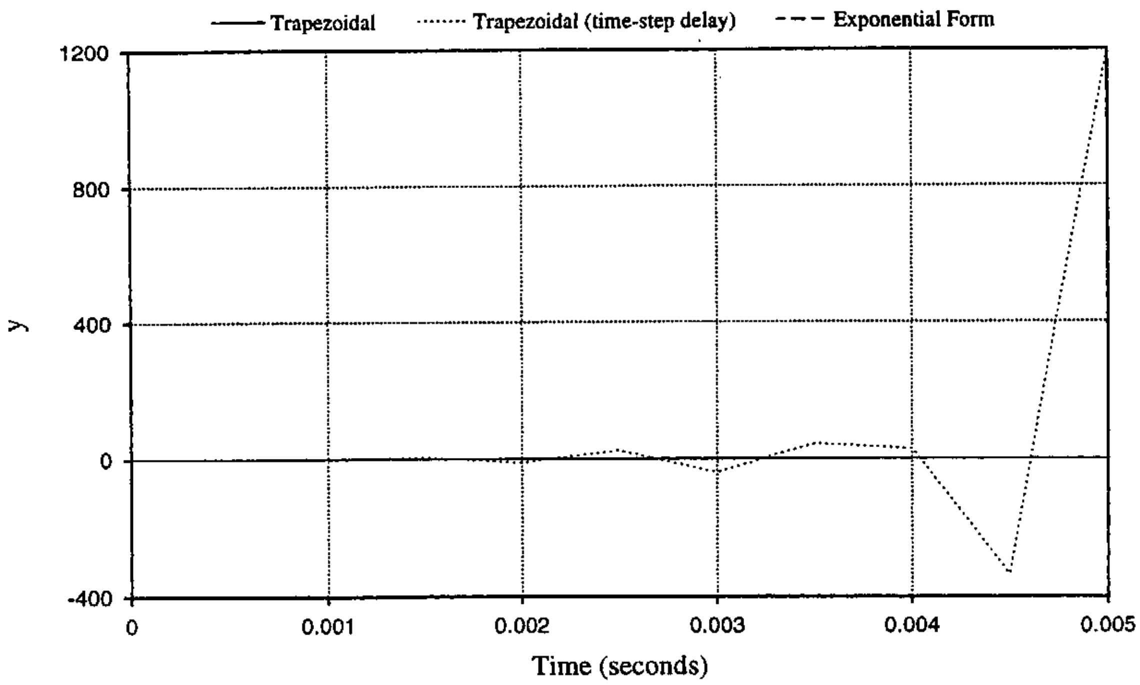

Fig. 4: Simulation results for time-step=500 us

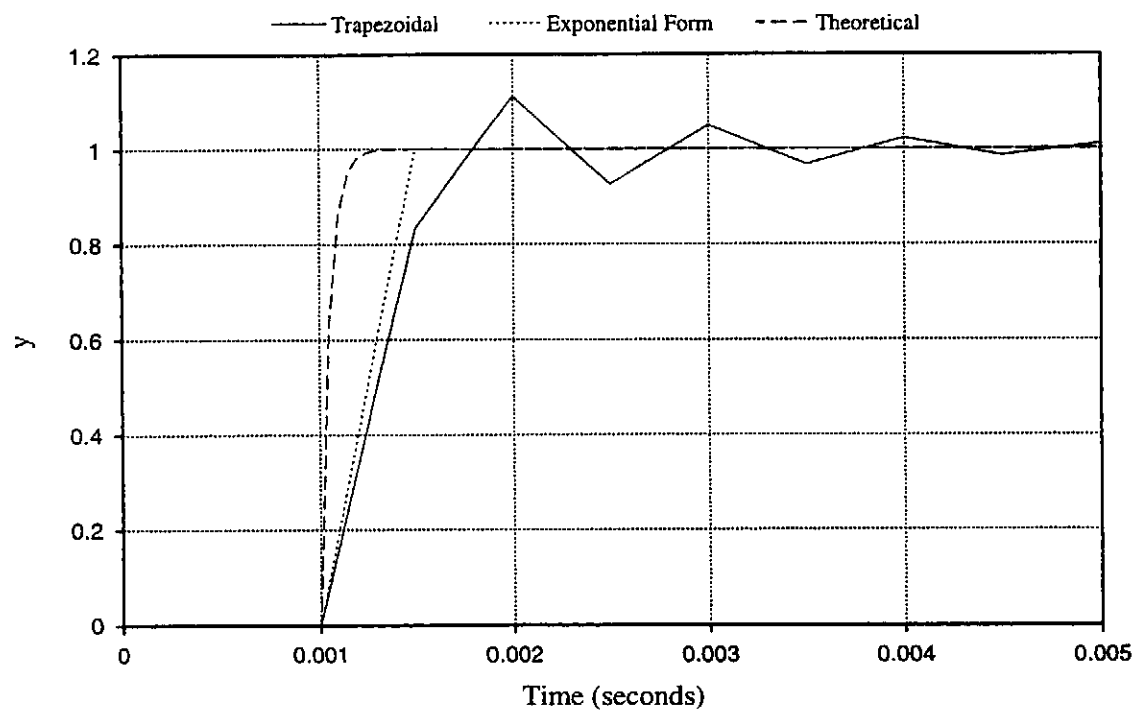

Fig. 5: Expanded view of Simulation results for time-step=500 us