Description of electrical machines with non-linear equivalent-circuits

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Abstract **- In this work equivalent-circuits are developed for description of electrical machines (here commutator motors and three-phase induction motors) and extended in such way that ferromagnetic saturation can be taken into account. Since all conventional equivalent-circuits of electrical motors are linear these electrical models are not valid for transient response (switching, short-circuit).**

In the modelling process the parameters of the non-linear equivalent-circuits are calculated by parameter estimation. The transient response of a electrical machine is measured. The parameter of the equivalent-circuits are changed in that way, the calculated response matched with the measured.

The parameters are estimated with evolutionary strategies. Evolutionary strategies are like genetic algorithms and evolution programming part of the class of optimization-algorithms based on the probability-theory. The evolutionary strategies imitate the process in biological evolution on subsequent levels. This method distinguishes itself being robust and resistent to noise on signal lines. The used method is applied to the example of an AC-motor and an induction-motor.

Keywords: Electric Machine, Saturation, Modelling, Transient effects.

I. INTRODUCTION

To calculate transient effects in electric machines two different ways are used. The numerical field calculation with time stepping [1] and the description of electric machines with equivalent circuits. The numerical field calculation is more exact, because parasitic effects such as saturation and skin effect can be considered. However the computing time is much larger, so that modern computers can only calculate a single machine in acceptable time. To consider several machines, i.e. to analyse electric power supply networks the description of electric machines with equivalent circuits is necessary. In this work equivalent circuits of electric machine are extented in such way that saturation can be considered.

Especially the propesed models could give better results for large synchronous machine, i.e. short circuit currents. With these results electric systems like switchers could better dimensioned.

II. NONLINEAR INDUCTIVE ELEMENTS

In complement to an ideal inductor the inducance of a real solenoid with saturation depends on the current. The inductance of a coil is given in equation (1):

$$L = \frac{\psi}{i} \tag{1}$$

The flux ψ is a function of the magnetic induction B and the current i is a function of the magnetic field strength H. The relation between B and H for a coil with iron core is nonlinear. This nonlinear function B(H) can be approximated with analytical functions [2]:

- broken lines

$$B = \begin{cases} \mu_s H & ; \text{ für } H < H_s \\ B_s + \frac{B_s - B_0}{H_s}(H - H_s) & ; \text{ für } H > H_s \end{cases} \tag{2}$$

- hyperbola

$$B = \frac{H}{a + bH} \tag{3}$$

- arctan-function

$$B = \alpha \arctan \alpha H \tag{4}$$

- polynom

$$H = a_1 B + a_2 B^3 + a_3 B^5 + \ldots.. \tag{5}$$

Often the polynom is limited on two terms:

$$H = \begin{cases} a_1 B + a_2 B^3 & ; \text{ high saturation} \\ a_1 B + a_2 B^9 & ; \text{ weak saturation} \end{cases} \tag{6}$$

III. ESTIMATION OF THE SATURATION-PARAMETERS

The main problem to describe saturation is not the analytical description, but the estimation of the saturation-parameters. These parameters are determined with suitable measurements by parameter-estimation. Therefore the currents of a transient effect, i.e. starting or short-circuit are measured. Then the parameters of the model are fitted in such way that best approximation can be obtained. So the parameter-estimation is an optimization problem.

IV. EVOLUTIONARY STRATEGIES

Optimization problems can be handled in many different ways. Some of these are tested on this problem.

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The best results are achieved with the evolutionary strategy. The principle of this method is shown.

The evolutionary-strategie (ES) was developed by Rechenberg [3] and Schwefel [4]. It belongs to the class of optimization-algorithms based on the probability-theory. The evolutionary strategies imitate the process in biological evolution on subsequent levels.

The fundamental idea is Darwin's principle „survival of the fittest". Based on a parents population children are generated, which are different from the parents (mutation). Than the quality of the children is determined. The best children of the population survive and form the next parents generation. It must be distinguished, whether the parents are considered for the next generation.

An important point of the evolutionary strategy is the step width adaptation. Therefore the step width δ is introduced. The step width specifies the maximum difference between the children and the parents after the mutation. If the child is near to the parents the step width is decreasing, otherwise it is increasing. Close to the optimum the step width gets smaller at all. On this way the optimum can be found and the algorithm converges.

The whole principle is shown on a simple example the (1,λ)-ES. The notation was developed by Schwefel and it means that one parent gets λ children. The best child survives and the parent is not considered (comma-strategie).

The variations step:

$$\begin{array}{llll} \delta_{N1}^{g} = \delta_{E}^{g}\,\xi_{1} & \delta_{N2}^{g} = \delta_{E}^{g}\,\xi_{2} & \ldots & \delta_{N\lambda}^{g} = \delta_{E}^{g}\,\xi_{\lambda} \\ x_{N1}^{g} = x_{E}^{g} + \delta_{N1}^{g}\,z_{1} & x_{N2}^{g} = x_{E}^{g} + \delta_{N2}^{g}\,z_{2} & \ldots & x_{N\lambda}^{g} = x_{E}^{g} + \delta_{N\lambda}^{g}\,z_{\lambda} \end{array} \tag{7}$$

$x_{N\lambda}^{g}$ is the λ.-te child of the generation g

x_{E}^{g} parent of the generation g

z_{λ} random number

$$\left.\begin{array}{l} \xi_1 = \xi_2 = \ldots = \xi_{\lambda/2} = \alpha \\ \xi_{\lambda/2+1} = \xi_{\lambda/2+2} = \ldots = \xi_{\lambda} = 1/\alpha \end{array}\right\}\text{with}\,\alpha = 1{,}3$$

The selection step for the comma-strategie:

$$\begin{array}{l} x_{E}^{g+1} = Opt\left\{x_{N1}^{g},\ldots,x_{N\lambda}^{g}\right\} \\ \delta^{g+1} = \xi^{g}\delta^{g} \end{array} \tag{8}$$

This described algorithms is the simplest of the ES. More complex algorithm uses more biological elements. So sexuality is simulated in such way, that the parameters of two or more parents are joined (recombination). A detailed explanation about the ES is given by Rechenberg [3].

V. NON-LINEAR-EQUIVALENT CIRCUITS FOR ELECTRICAL MACHINES

A. Commutator machines

Commutator machines have a rotor and an exciter circuit. For each circuit an equation can be given:

$$\begin{array}{l} u_a = R_a i_a + L_a \dfrac{\mathrm{d}i_a}{\mathrm{d}t} + c_1\omega\psi \\ u_f = R_f i_f + \dfrac{\mathrm{d}\psi}{\mathrm{d}t} \end{array} \tag{9}$$

with

u_a : rotor voltage
u_f : exciter voltage
i_a : rotor current
i_f : exciter current
ω : angular velocity
ψ : flux linkage
R_a : rotor resistance
R_f : exciter resistance
L_a : rotor inducance
c_1 : motor constant

The differential equation of the machine changes in the way the machine is excitared (external, permanent, series, shunt excitation). In this work the ac-motor is analysed. AC-motors are commutator series wound machines. These machines can run with direct and alternating current. They are used in small drives up to 2000 W. Ac-motors were used as traction motors in locomotives. Today they are replaced by asynchronous machines.

To describe the ac-motor the equivalent circuit is used.

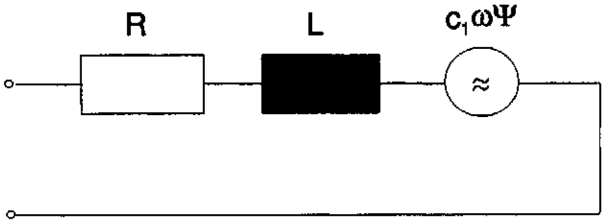

Fig. 1: Equivalent circuit of an ac-motor

This equivalent circuit is described by the following equation:

$$u_a = Ri_a + \frac{\mathrm{d}\psi}{\mathrm{d}t} + c_1\omega\psi \tag{10}$$

The nonlinear relation between electromagnetic flux and current, taking saturation into account, is given by the equation:

$$i_a = a_1\psi + a_2\psi^3 \tag{11}$$

The equation of motion is considered in order to take the variable speed of the shaft into account.

$$J\frac{\mathrm{d}\omega}{\mathrm{d}t} = c_1\psi i + k_{vent}\omega^2 + k_{Lager}\omega + M_{reib} \tag{12}$$

with a_1, a_2 : saturation parameter
k_{Ventl} : Const. for ventilator torque
k_{Lager} : Const.for bearing friction
M_{reib} : friction torque
J : moment of inertia

The model is based on the following assumptions:

1. The voltage drop at the brushes is neglected. The relation between the voltage drop and the armature current is nonlinear.
2. Armature- and field inductance are combinted in one total inductance. Therefore the degree of saturation is for both inductances the same.
3. Hysteresis and eddy-current-effects are also neglected.

B Asynchronous machine

In electromechanical drive systems the asynchronous machine is that type of electric machine which is used mostly. The reason therefore is the simple and robust construction of the machine. The equivalent circuit of the asynchronous machine [5]:

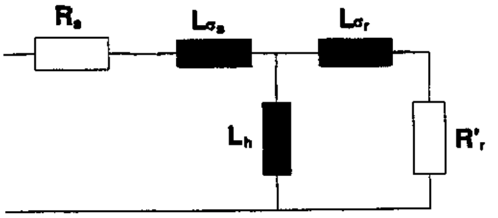

Fig. 2: Equivalent circuit of an asynchronous machine

The inductances of the asynchronous machines depend all on the degree of their saturation. Here one has to distinguish between the saturation which is caused by the main and leakage field. An investigation of the parameter sensitivity shows that only the total leakage inductance of the stator and rotor $L_s = L_{\sigma s} + L_{\sigma r}$ has an influence on the stator current. It is impossible to distribute the total leakage inductance on the rotor and stator. Now it would be possible to use the equivalent circuit of fig. 2 where both leakage inductances have the same values $L_{\sigma s} = L_{\sigma r}$. Instead of the use of this assumption only one total leakage inductance is considered within the rotor circuit of the diagram, fig. 3. This implies that the saturation effect is no longer considered to be separate for the stator and rotor but as a combination of both.

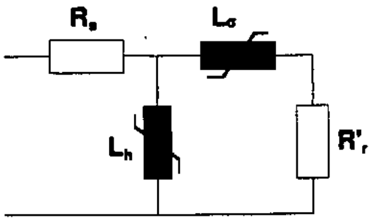

Fig. 3: Equivalent circuit of an asynchronous machine with main field and leakage saturation

This equivalent circuit of the asynchronous machine is a graphical interpretation of the following differential equation system:

$$\begin{aligned} u_{s\alpha} &= R_s i_s + \frac{\mathrm{d}\psi_h}{\mathrm{d}t} \\ 0 &= R_r^{'} i_r^{'} + \frac{\mathrm{d}\psi_r}{\mathrm{d}t} + j\omega\psi_r \end{aligned} \tag{13}$$

The currents are given in equation (14):

$$\begin{aligned} i_\mu &= a_{1h}\psi_h + a_{2h}\psi_h^3 \\ i_r^{'} &= a_{1r}(\psi_r - \psi_h) + a_{2r}(\psi_r - \psi_h)^3 \\ i_s &= i_\mu + i_r^{'} \end{aligned} \tag{14}$$

with a_{1h}, a_{2h}, a_{1r}, a_{2r} : saturation parameter
i_μ : magnetizing current

The equation of motion completes the system for the machines' description:

$$J\frac{\mathrm{d}\omega}{\mathrm{d}t} = j\rho L_h\left(i_s^{*} i_r^{'} - i_s i_r^{'*}\right) + M_a \tag{15}$$

with ρ : pair of poles
M_a : motive moment
* : conjugate complex

The equivalent circuit of the asynchronous machines does not include the skin effect in the bars of the squirrel-cage. In order to take the skin effect into account the equivalent circuit of fig. 3 must be extended, see fig. 4.

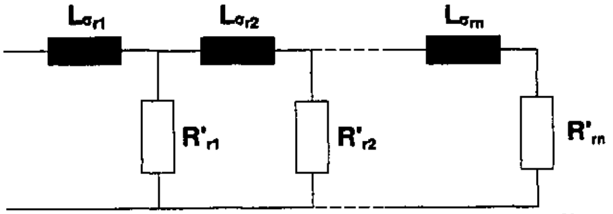

Fig. 4: Multisection network to describe the skin effect for a rotor circuit

C Synchronous machine

The synchronous machine is described by modified Park's equivalent circuit [6]:

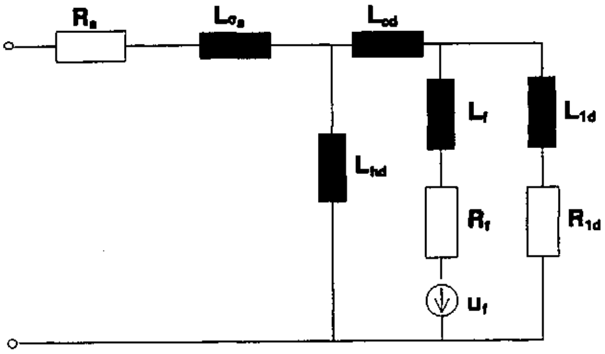

Fig. 5: Equivalent circuit of the d-axis of a synchronous machine

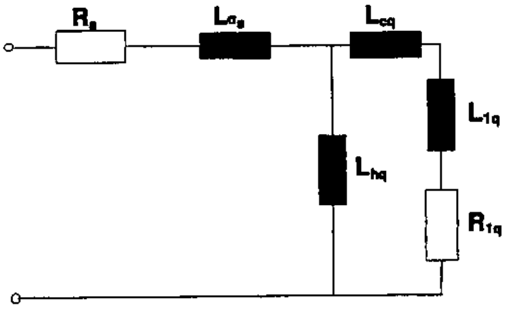

Fig. 6: Equivalent circuit of the q-axis of a synchronous machine

Leakage inductances of the armature, the field winding and amortisseur within synchronous machines are saturated during transients like the three phase short circuit. Now these inductances are no longer constant but they depend on the current in the stator and rotor.
This is shown in figure 7:

The q-axis:

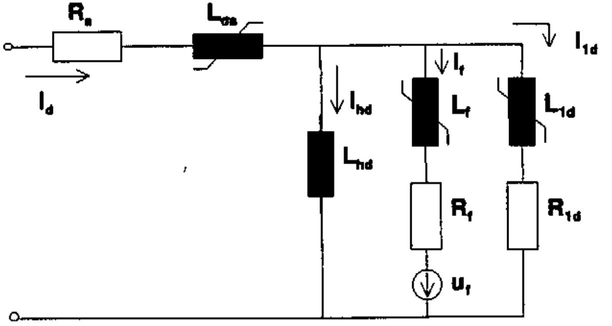

Fig. 7: Equivalent circuit of the d-axis of a synchronous machine with non linear inductances

The q-axis:

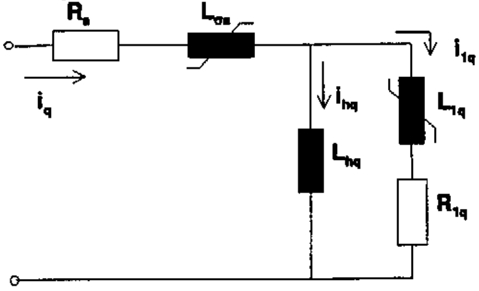

Fig. 8: Equivalent circuit of the q-axis of a synchronous machine with non linear inductances

The non linear differential equation of the synchronous machine is given in equation (16).

$$
\begin{aligned}
u_d &= R_a i_d + \frac{\mathrm{d}\psi_{d\sigma}}{\mathrm{d}t} + \frac{\mathrm{d}\psi_{hd}}{\mathrm{d}t} - \omega\psi_q \\
u_f &= R_f i_f + \frac{\mathrm{d}\psi_f}{\mathrm{d}t} + \frac{\mathrm{d}\psi_{hd}}{\mathrm{d}t} \\
0 &= R_{1d} i_{1d} + \frac{\mathrm{d}\psi_{1d}}{\mathrm{d}t} + \frac{\mathrm{d}\psi_{hd}}{\mathrm{d}t} \\
u_q &= R_a i_q + \frac{\mathrm{d}\psi_{q\sigma}}{\mathrm{d}t} + \frac{\mathrm{d}\psi_{hq}}{\mathrm{d}t} + \omega\psi_d \\
0 &= R_{1q} i_{1q} + \frac{\mathrm{d}\psi_{1q}}{\mathrm{d}t} + \frac{\mathrm{d}\psi_{hq}}{\mathrm{d}t} \\
\frac{\mathrm{d}\omega}{\mathrm{d}t} &= (\psi_{\sigma d} + \psi_{hd}) i_q + (\psi_{\sigma q} + \psi_{hd}) i_d
\end{aligned}
\tag{16}
$$

The nonlineare relation between the fluxes and currents is given in equation (16). The d-axis:

$$
\begin{aligned}
i_d &= a_{d\sigma}\psi_{d\sigma}^3 + b_{d\sigma}\psi_{d\sigma} \\
i_f &= a_f\psi_f^3 + b_f\psi_f \\
i_{1d} &= a_{1d}\psi_{1d}^3 + b_{1d}\psi_{1d} \\
i_{hd} &= i_d - i_f - i_{1d}
\end{aligned}
\tag{17}
$$

The q-axis:

$$
\begin{aligned}
i_q &= a_{q\sigma}\psi_{q\sigma}^3 + b_{q\sigma}\psi_{q\sigma} \\
i_{1q} &= a_{1q}\psi_{1q}^3 + b_{1q}\psi_{1q} \\
i_{hq} &= i_d - i_q
\end{aligned}
\tag{18}
$$

with the saturation parameters $a_{\sigma d}$, $b_{\sigma d}$, a_f, b_f, a_{1d}, b_{1d}, a_{1q}, b_{1q}

VI. RESULTS

A A.C.-motor

Measurements have been done for a 50 W A.C.-motor. The measurement system is shown in fig. 9:

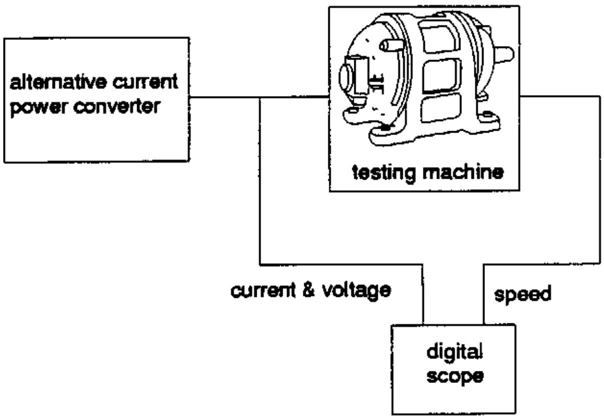

Fig. 9: Test assembly

The motor is supplied by an a.c. power converter. In a first step the machine parameters are determined from a running up at rated voltages.

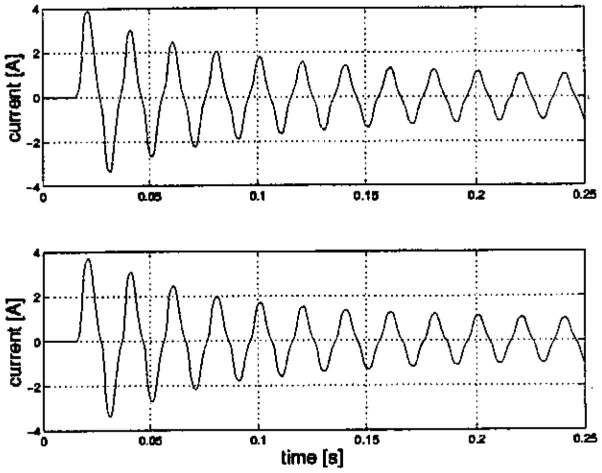

Fig. 10: Course of measured (—) and calculated (---) stator current for a 50 W a.c.-motor

With these parameters another running up has been performed with firing angle α = 45°. This second measurement is compared with the calculation which uses the parameters from the first one.
The main subject of this paper is to determine the non linear electromagnetic parameters of electrical machines.

Indeed the mechanical behaviour of electrical machines with small ratings is strongly determined by their friction. For the electromagnetic parameters this mechanical influence need not be taken into account, if the machine speed is measured directly. The speed is one of the variables which is used in the parameter estimation then.

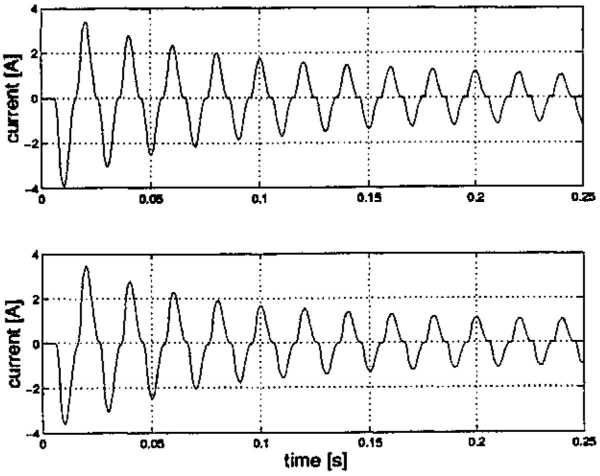

Fig. 11: Verification of the estimated parameters with firing angle $\alpha = 45°$

The calculation results match fairly well with the measurement. So good results cannot be achieved with the linear model.

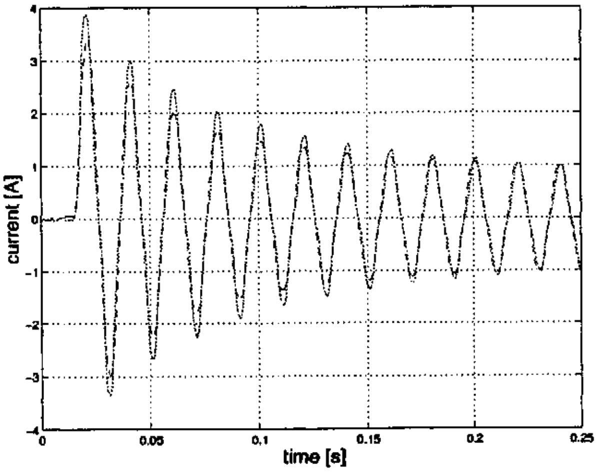

Fig. 12: Course of measured (—) and calculated (---) stator current with the linear machine model

Neglecting the saturation the machine model gives currents which are too small in comparison with the measurement for the first seven periods. After this time the current has reached an amplitude where no saturation occurs any more.

Taking the saturation into account the extended machine model gives the current which is about 5 % for the first four maximums and lower than 3 % for all other periods. It is supposed that the relative strong deviation between calculation and measurement is caused by the change in the brush contact resistance. For this see figure 13.

The nonlinear model is also capable to simulate the non sinusoidal course of the current.

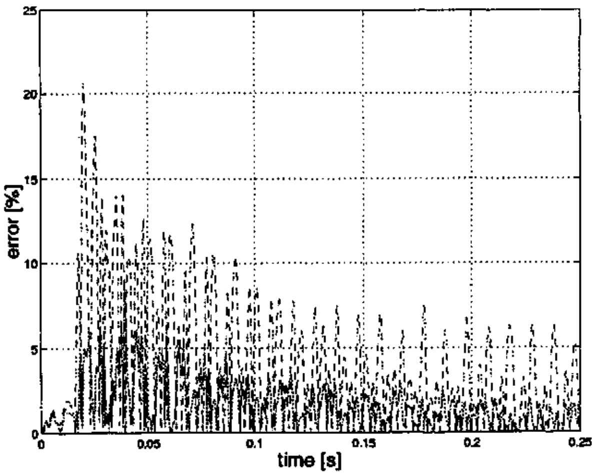

Fig. 13: Difference between calculated and measured stator current with nonlinear (—) and linear (---) machine model

B *Asynchronous machine*

A 7.5 kW slip ring asynchronous motor is used in order to verify the method. The resistance of the stator winding is measured whereas all other parameters are determined during an estimation process. This process uses the fact that the main field saturation depends especially upon the stator voltage. Therefore the machine run up has been done for two different voltages $U_s = 400$ V and $U_s = 231$ V. The results of the evaluated run ups are given in figure 14.

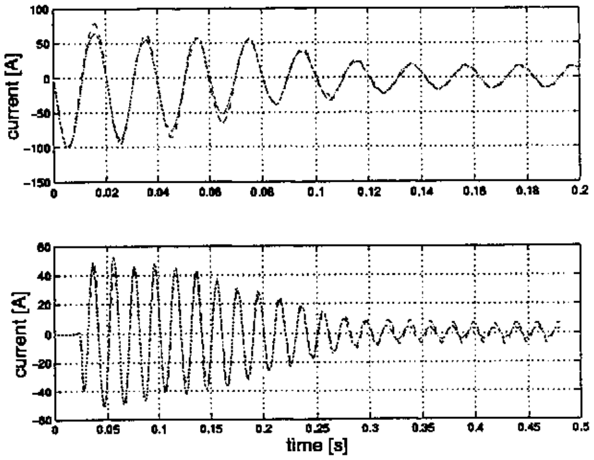

Fig. 14: Course of measured (—) and calculated (---) stator current of a slip ring asynchronous machine for two different degrees of main field saturation

The error between the measured and estimated stator current for the linear and nonlinear machine is given in figure 15. The results of the nonlinear model are better than the results of the linear model. Admittedly the total error is quite large. This is caused by the change in the slip ring contact resistance.

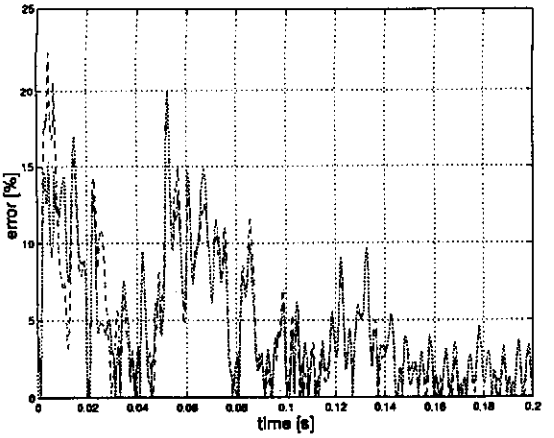

Fig 15 : Error between measured and calculated stator current of a 7.5 kW slip ring motor ($U_s = 400$ V)

The measurement will reach any relevance in practice only if it is posible to estimate the parameters of a squirrel cage induction motor, too. On account of the skin effect the multisection network calling to section V is used. The rotor is simulated with five circuits. The measurement procedure is the same as for the slip ring motor. Again two run ups with different stator voltages are analysed. The result is given in figure 16.

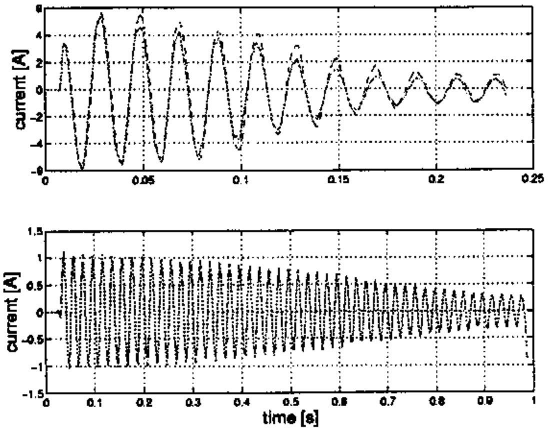

Fig. 16: Course of measured (—) and calculated (---) stator current of a 0.25 kW squirrel cage motor for two different degress of main field saturation

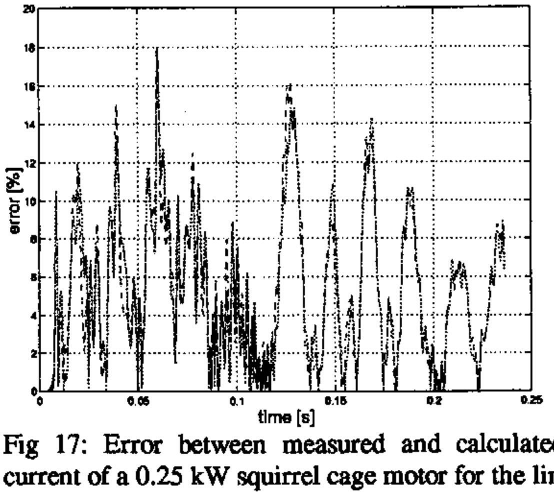

Fig 17: Error between measured and calculated stator current of a 0.25 kW squirrel cage motor for the linear (--) and nonlinear (—) motor model.

Figure 17 shows no significant difference between both models. In case of a small asynchronous machine the stator current does not depend on saturation effects.

VII. CONCLUSION

Within this paper equivalent circuits are presented which take the saturation effects of the main flux and of the leakage flux into account. The parameters of these circuits are determined with the help of measurements. Here dynamic behaviour during running ups of three phase short circuits is used. The parameter identification algorithms which uses the evolutionary strategy gives the best results. In comparison with equivalent circuits which don't take saturation into account the calculation results are much more accurate. The well known kind of the equivalent circuits stays the same and its parameters can still be interpreted in the typical manner.

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