HVDC Ring Modelling and Simulation
-A New State Equation Development Algorithm for Modelling

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Abstract - **Dimensioning of electrical equipment requires calculation of maximum values of short-circuit currents. A method based on IEC 909 Standard allows to calculate these values in AC systems for sinusoidal transients. Short-circuit current calculation in power systems constructed with AC and DC networks is possible by using a simulation method of non linear electrical circuit analysis. The paper presents a method of the AC/DC power system modelling for short-circuit calculation which is an alternative for HVDC system modeling in place of EMTP like programs. This method is based on the new state equation development algorithm in case of AC/DC systems. A theory and results of computer calculation are presented.**

Keywords: Modelling, Simulation, HVDC link. Short-circuit calculations.

I. INTRODUCTION

DC power transmission devices are more and more frequently used to link different power systems or their parts [1], [2], [3], [4], [5].This can be made by the use of the DC transmission lines or back to back links connecting two systems, as well as by DC bus or DC ring connecting with converter stations more than two AC power systems. DC transmission is either used for connecting AC systems running on different speed or they can operate in parallel with already existing AC transmission lines to increase transmission capacity of the link. In any of mentioned cases, parameters of DC transmission system can not be neglected in short-circuit calculations.

Unfortunately IEC 909 does not define the method of short-circuit calculation for meshed AC/DC networks.

The problem is complicated because AC short-circuit current transient can be influenced by DC currents, what is thyristor valves operation and converter station control system behaviour during transients.

It seems that only efficient solution of this problem is an application to the short-circuit study a simulation method of adequate mathematical equations describing operation of both AC power system and DC links during transients.

For the modelling of transients in power system there are used following computer programs: EMTP [6] and PSCAD [7]. Their computation algorithms are based on the theory published in 1969 by Dommel [8]. The main advantage of this method is the simplicity of developed mathematical model of power system composed of many different devices.

The model is in a form of a set of differential and algebraic equations describing dynamics of all power system components (generators, transmission lines, transformers, etc.) completed with Kirchoff current equations for power system nodes.

To simplify simulation of above set of equations, all differential equations are replaced by algebraic equations, according to the trapezoid method. New set of pure algebraic equations, which can be solved with adequate numerical method is used in the simulation of transient phenomena of power system.

Above method of the development of mathematical model describing dynamic system is inconsistent with the theory of control. This theory models non-linear dynamic objects like power system, by state equations, i.e. a set of differential equations, where there are present only state variables describing explicit object trajectory in the state space. In this way we minimise number of variables in the calculations and for the solution of the set of state equations can be applied numerical procedures of differential equation set integration.

Therefore we can state that power system modelling according to the theory presented by Dommel, where in algebraic equations there are used all variables of all equations of different kind of power system devices, results in increased number of variables in computation procedures, compare to the number of state variables of such system. Trapezoidal method of integration applied to the solution of above model of power system, sometimes can produce numerical oscillations, which to be removed there is the need of the introduction to the mathematical model of the system additional impedance's non existent in physical object [9]. User needs to be enough experienced to distinguish the difference between numerical oscillations and oscillations which are characteristic for the modelled object. Due to the transient phenomena caused by operation of thyristor valves this could be very difficult in the case of the study of large power systems connected with DC links.

So there is a reason to develop computation method for power system transient study, which will be compatible with theory of control. This paper presents such methods, where using state equations of particular devices of AC power system and DC links, we set up the set of state equations of AC/DC power system.

Developed set of equations can be used for the analysis of power system transients, short-circuit study included. Together authors present computation example concerning application of presented method to the calculation of short-circuit currents in three AC power systems connected with both AC lines and DC rings.

II. STATE EQUATIONS BUILDING ALGORITHM

A. Structure of the power system model

The approach to power system modelling, presented in this work, combines models of AC networks and DC links. A structure of a modelled object consists of network nodes and connecting branches. Any substation busbar, as well as any short-circuit location in the network can be defined as a node. A transmission branch connects two nodes and can consist of single or multiple, connected in series, transmission lines, transformers and reactors. Load branches are connected only to one node and can consist either of different kind of typical load like for example synchronous or induction motors, fixed impedance, or of energy sources like for example generators or equivalent networks.

For either balanced or unbalanced faults, a short-circuit impedance is considered to be a load branch as well.

B. Fundamental equations

The mathematical models of the all mentioned above devices of AC/DC power system are known in the bibliography [10]. These models can be written in different reference systems, e.g. $\alpha\beta0$, qd0, xy0 co-ordinates. For three phase balanced devices we can apply any of the above systems. For unbalanced devices reference system has to be fixed (stationary) on a non-symmetrical circuit. For that reason equations of synchronous machine need to be written in qd0 system, but DC lines equations have to be written in $\alpha\beta0$ reference system. Modelling the unbalanced branches of AC networks like unbalanced transmission lines and power transformers or modelling the phenomena of non-simultaneously pole switching in circuit breakers sequence, we need to apply the $\alpha\beta0$ reference system. The same approach will be also used for modelling of unbalanced short-circuit. It should be underlined that each of these mathematical models will get now individual reference system for model variables, specially terminal voltages and load currents.

When we add to the set of state equations of all power system components the set of Kirchoff's current equations, we will receive a set of n equations with n unknown quantities. Generally, it is possible that the same quantity (e.g. generator current) engaged in two different equations is written in two different reference systems "a" and "b". Because this gives us the possibility that the same quantity will be represented by two different variables, a set of n equations has to be completed with additional equation describing transformation of this quantity from reference system "a" to reference system "b".

It is necessary to distinguish following kinds of transformation:

1. transformation between two reference systems running on different speeds (e.g. transformation between qd0 system and $\alpha\beta0$ system applied for synchronous machines - transformation is a function of rotor angle),
2. transformation between two reference systems which are relatively stationary (e.g. transformation between $\alpha\beta0$ reference system on primary side of transformer and $\alpha\beta0$ reference system on the secondary side of transformer - transformation is a function of transformer ratio and connection group),
3. "controlled" transformation (e.g. transformation between $\alpha\beta0$ reference system on the AC side of converter and $\alpha\beta0$ reference system used on the DC side of converter - this transformation is a function of actually operating pair of thyristor valves).

If we complete the set of n equations containing n unknown quantities with equations describing transformation of all vector values written in different reference systems, we will receive a set of m equations with m unknown variables which will be the mathematical model of the considered object.

C. Variable reduction

According to the last paragraph mathematical model of the object consist of:

1. the state equations of all components of the object,
2. the nodal equations of the object,
3. the transformation equations for quantity represented in different reference systems.

In the above structure the number of variable derivatives exceeds the number of object state variables. Hence the developed model can not be considered as the object's set of state equations and can not be used for the study of transients. The numerical methods used for a solution of differential equation set can not be applied, either.

(This problem can be explained by example of the system consisting of reactive (inductive) load fed from equivalent network through k different underground lines. A mathematical model of this object contains k+2 differential equations, i.e. state equations of all components of this object. We can easily reduce the number of variables and eventually we will get a single differential equation representing state equation of the whole system).

A method of reduction of model variables to model state variables for the AC networks, has been presented in papers [11]. According to this method in the meshed networks the structure of radial network branches needs to be distinguished from branches closing the network loops. Now all these state equations and variables concerning radial network branches can be eliminated, according to the equations (1) and (2):

$$\mathbf{i}_z = \mathbf{P} \times \mathbf{i} \tag{1}$$

$$\frac{d\mathbf{i}_z}{dt} = \mathbf{P} \times \frac{d\mathbf{i}}{dt} + \frac{d\mathbf{P}}{dt} \times \mathbf{i} \tag{2}$$

where

- $\mathbf{i}_z$ - vector of currents of the radial network,
- i - vector of load currents and currents of branches closing network loops, x - left components of state vector concerning load branches,
- P - the matrix of transformation of vectors of load currents and currents of branches closing network loops, to reference systems of branches of radial network.

and the state equation of the radial network branches can be replaced by the set of algebraic equations.
In the case of vector transformation between both stationary and rotating reference systems, elements of the matrix P can be defined as the rotate operator for the change of reference system and elements of the matrix $\frac{d\mathbf{P}}{dt}$ describe the difference between rotating speeds of two reference systems.
In the case of "controlled" transformation, relative displacement of reference systems for AC and DC systems change almost stepwise according to the sequence of valve commutations. Between commutations value of $\frac{d\mathbf{P}}{dt}$ is equal to 0, but during commutation $\frac{d\mathbf{P}}{dt}$ is a function of the commutation process.
There are two possible solutions:

1. Commutation process can be neglected (as in [12]), assuming that it doesn't influence energy balance due to the negligible commutation time.
2. Commutation process can be modelled by introducing an additional short-circuit on the AC side of the two commuting valves. This additional circuit is present during the commutation period only. During the commutation, when both valves conduct the current, this additional circuit connects two points of practically the same potential. Therefore, introduction of this additional short-circuit doesn't change current flow in the adjacent AC and DC branches of the system.
   After commutation finishes, when the current flows in the newly connected valve only, a new pair of conducting valves and at this same time is a new value of the matrix P are defined. The additional short-circuit is also removed at the end of commutation. This way, despite the changes in the model structure, the continuity of the current flow in the AC and DC branches of the model is preserved.
   The above described method was utilised in this work. It allows commutation process can be taken into account as well as step changes of the matrix P during the calculations. The additional circuit introduced into the system will further be termed as commutation short-circuit.

After elimination of the currents of radial network branches, algebraic equations of radial network branches and state equations of left network elements (i.e. all load branches and branches closing network loops) set up together state equations of modelled power system.

D. State equations

Following the above presented algorithm, the state equations of all load branches and transmission branches closing the network loops can be described in a general form by following equations (1), (2) and (3):

$$\frac{d\mathbf{i}}{dt} = \mathbf{v} + \mathbf{Y} \times \mathbf{P}^T \times \mathbf{u} \tag{3}$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{i}, \mathbf{x}, \mathbf{e}) \tag{4}$$

where:

$$\mathbf{v} = -\mathbf{B} \quad \times \mathbf{i} + \mathbf{g}(\mathbf{i}, \mathbf{x}, \mathbf{e}) \tag{5}$$

and

- **i, x** - the components of state vector of modelled object:
  - **i** - vector of load currents, currents of branches closing network loops and currents of commutation short-circuits,
  - **x** - left components of state vector concerning load branches,
- **e** - the input vector,
- **u** - the vector of nodal voltages,
- $\mathbf{P}^T$ - the matrix of transformation of nodal voltages to reference systems of respective load branches or branches closing network loops.
- **Y, B** - the parameter matrixes of load and network loops closing branches respectively.

According to the paragraph 2.3 we can eliminate all of the variables of the radial network so that their differential equations will be replaced by a set of equations which can be then simplified to the following, single formula:

$$\mathbf{u} = (\mathbf{A} + \mathbf{T} \times \mathbf{P} \times \mathbf{Y} \times \mathbf{P}^T)^{-1} \times (\mathbf{B}_{RN} \times \mathbf{T} \times \mathbf{P} \times \mathbf{i} + \mathbf{T} \times \mathbf{P} \times \mathbf{v}) \tag{6}$$

where:

- **A, $\mathbf{B}_{RN}$, T** - the parameters matrixes of the radial network branches,
- **P** - the matrix of transformation of the current vectors in the load branches, network loops closing branches and currents of commutation

short-circuits, to the reference system of the radial network.

The equations (1), (2), (3) and (4) form the set of state equations of the modelled object.

III. COMPUTATION EXAMPLE

Above presented method for the case of the transformation between two, both rotating or stationary systems has been used for the written in Pascal power system simulator developed in Electrical Power Engineering Institute, Technical University of Lodz. Tool description, its applications and results of simulation concerning first of all short-circuit current calculations have been published in the past [11], [13], [14],[15].

Alternation of the method concerning controlled transformation allowed to develop written in programming language of MATLAB, simulation engine for AC/DC power networks. Software has been tested on simple HVDC ring model with three AC networks connected with parallel operating AC and DC transmission lines, presented method can be applied to any configuration of HVDC transmission network (bus, ring, back-to-back), for unlimited number of connected AC networks.

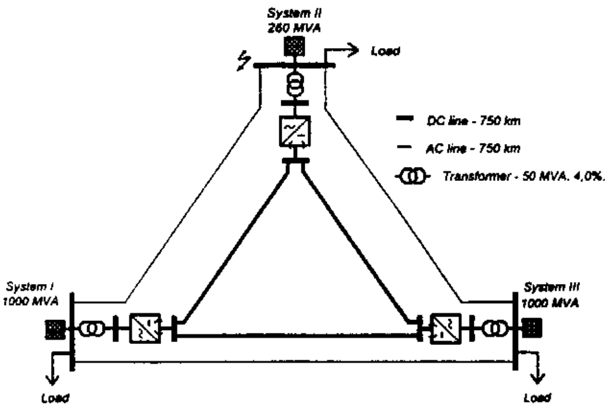

Fig. 1. HVDC ring - test network

In computed system two 6-pulse converter stations operates as a rectifier and the third one operates as an inverter. Harmonic filters are switched off. DC transmission system is controlled by the mean of thyristor valve firing angle control system described in [16] and by the tap changers actions regarding converter stations transformers.

Modelled network has been tested under steady state and transient conditions, short-circuit faults on AC network side included. Some of the results of this study will be presented below.

In the figures 4 - 9 the sequence of the events was as described below:

0 s - all converter stations switched on,
0.1 s - transformer tap changer operation in the inverter station,
0.25 s - short-circuit inception,
0.35 s - short-circuit removed,
0.45 s - transformer tap changer operation in the inverter station.

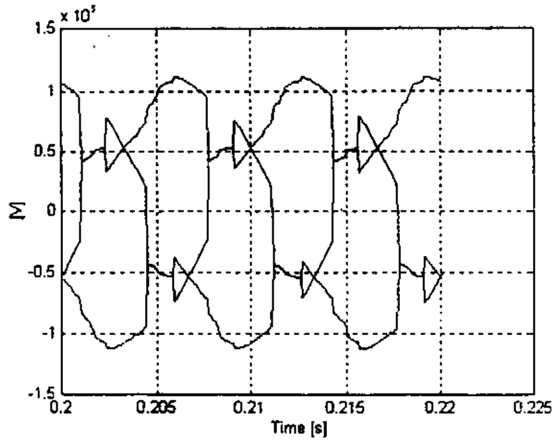

Fig. 2. AC Voltage waveforms at the inverter II busbar

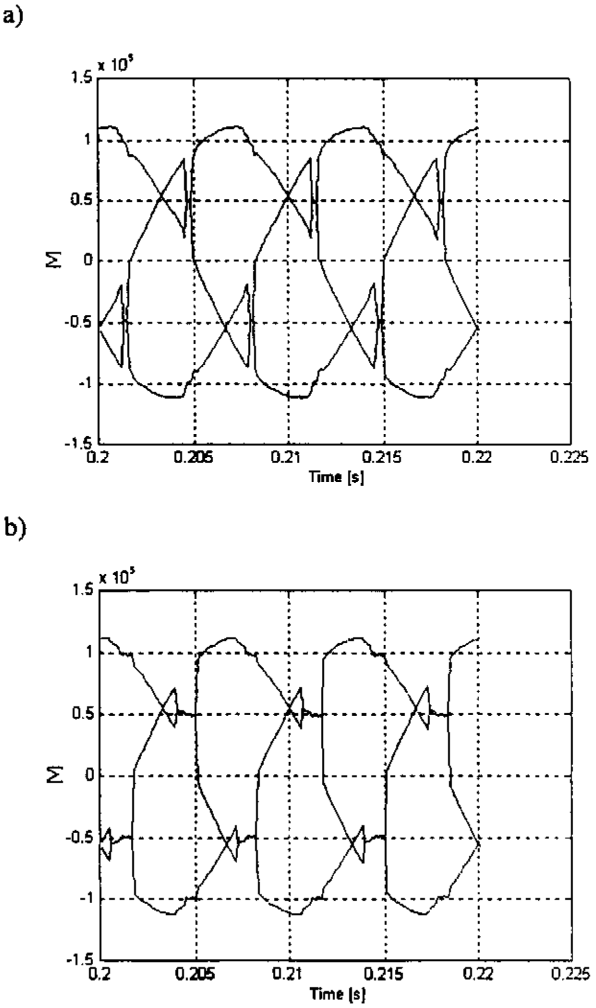

Fig. 3. AC Voltage waveforms at the busbar of a) rectifier I, b) rectifier III

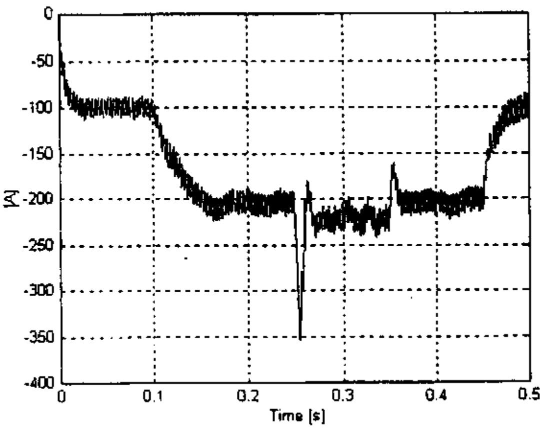

Fig. 4. DC current of the inverter II

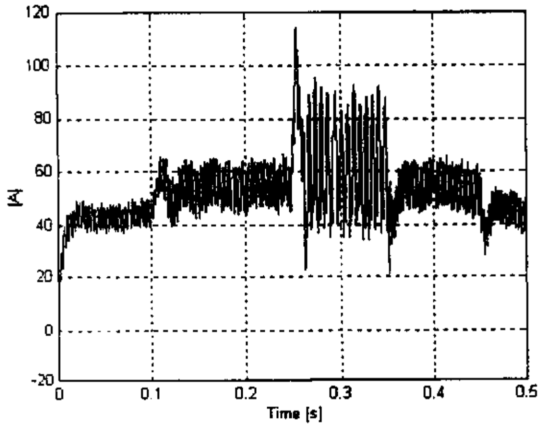

Fig. 5. DC current of the rectifier I

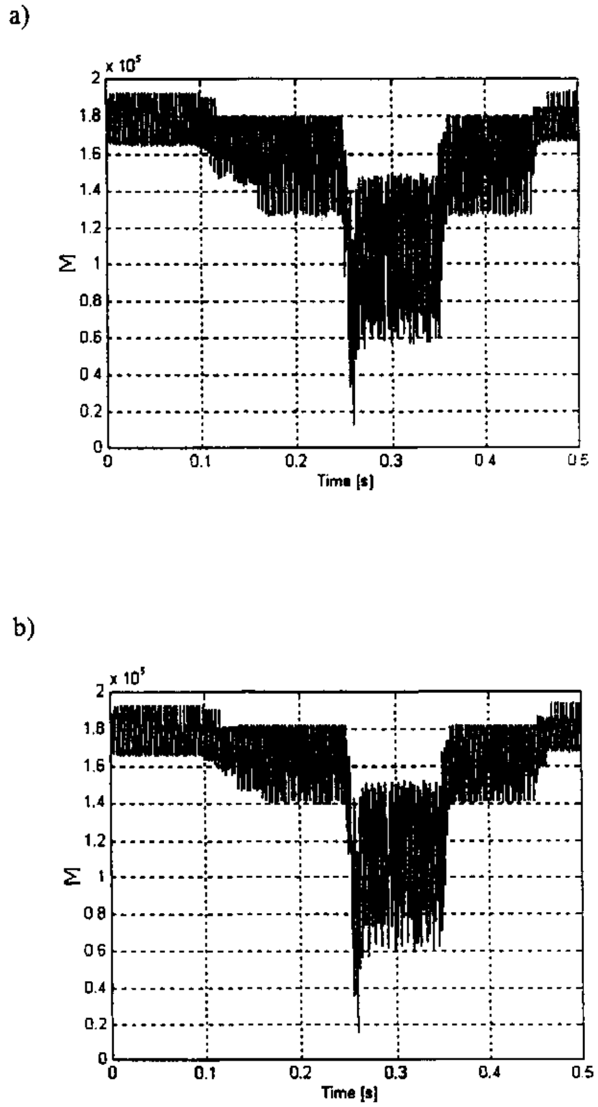

Fig. 7. DC Voltage at the busbar of a) rectifier I, b) rectifier III

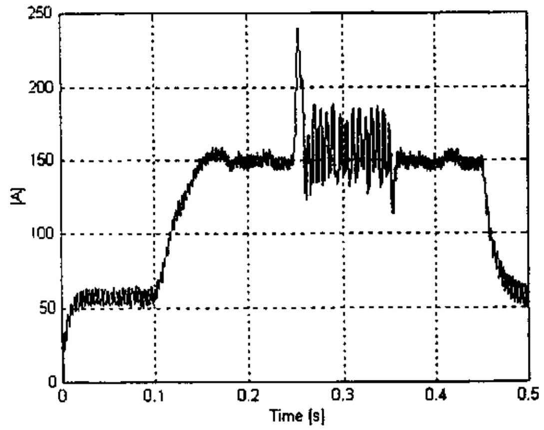

Fig. 6. DC current of the rectifier III

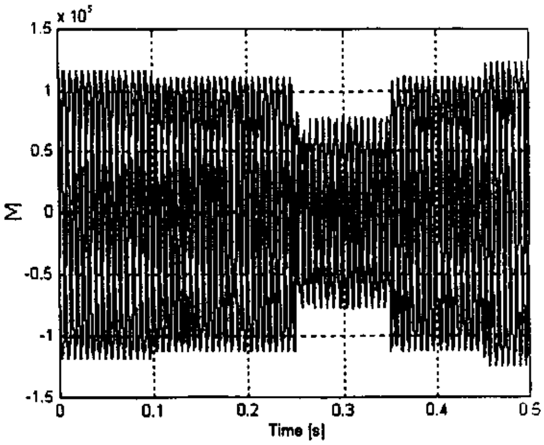

Fig. 8. AC Voltage at the busbar of inverter II

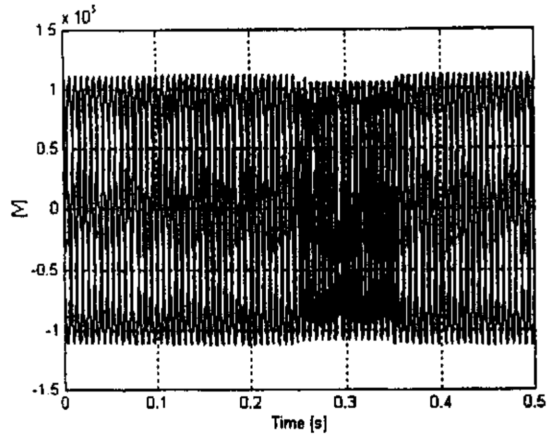

Fig. 9. AC Voltage at the busbar of rectifier I

IV. CONCLUSIONS

Presented methods is suitable for the digital simulation of transients caused in short-circuit in AC power systems connected with DC ring. In spite of the presence of non-linear elements like thyristor valves in converter stations, authors did not observe during computations any effects of numerical instability of the model developed according to the theory presented in this paper.

Results of the simulation permit to state that DC transmission lines influence values of peak short-circuit current.

Short-circuit current transients depend on the operation on the control systems of converter stations, which can decrease value of steady state current by limiting the current in the DC link.

V. REFERENCES

[1] J. Arrillaga, *High Voltage Direct Current Transmission*, Peter Peregrinus Ltd., London, UK, 1983.

[2] G.K. Carter, C.E. Grund, H.H. Happ, R.V. Pohl, „The dynamics of AC/DC Systems with Controlled Multiterminal HVDC Transmission", *IEEE Transaction on Power Apparatus and Systems*, vol. PAS-96, March/April 1977, pp. 402-413.

[3] C.E Grund et al., „Functional Model of Two-Terminal HVDC Systems for Transient and Steady-State Stability", IEEE Committee Report, *IEEE Transaction on Power Apparatus and Systems*, vol. PAS-103, June 1984, pp. 1249-1255.

[4] N.G. Kangorani, J.L. Hay, R.E. Crosbie, „Dynamic simulation of HVDC transmission systems on digital computer", *IEE Proceedings*, vol. 112, May 1966, pp. 793-802.

[5] T. Haregawa et al., „System design of Kii HVDC link", *CIGRE 1996*, paper 14 -103.

[6] *EMTP Theory Book*, Bonneville Power Administration, 1986.

[7] *PSCAD/EMTDC - Power System Simulator Software*, Winnipeg, Manitoba HVDC Research Centre, 1994.

[8] H.W. Dommel, „Digital Computer Solution of Electromagnetic Transients in Single - and Multiphase Network", *IEEE Transaction on Power Apparatus and Systems*, vol. PAS-88, April 1969, pp.388-399,.

[9] P. Sowa, *Modelowanie dynamiczne układów elektroenergetycznych*, Skrypty Uczelniane, Politechnika Śląska, Gliwice, Poland, 1992, (p.16, p.153).

[10] P.M. Anderson, A.A. Fouad, *Power System Control and Stability*, The Iowa State University Press, Ames, Iowa, 1977.

[11] R. Mienski, „A simulator of the industrial power network for short-circuit calculation", *Proceedings of IV International Symposium „Short-Circuit Currents in Power System"*, pp. 261-266, September 1990, Liege.

[12] R. Mienski, T. Siewierski, „Short-Circuit Current Computation in AC and DC Network", *Proceedings of VIII International Symposium „Short-Circuit Currents in Power System"*, pp. 55-60, October 1998, Brussels.

[13] R. Mienski, „Electrical network simulator", *Proceedings of the „5x5" Session*, Strathclyde University, 1989, Glasgow.

[14] R. Mienski, „The simulator of the industrial power network for short-circuit calculation", *Proceedings of V International Symposium „Short-Circuit Currents in Power System"*, pp. 1.29.1-1.29.3, September 1992 , Warsaw.

[15] A. Kanicki, W. Mielczarski, R. Mieński, „Electrical network simulator", *Proceedings of the First European Conference on Power Systems Transients*, pp.180-183, June 1993, Lisbon.

[16] J.D. Ainsworth, „The Phase Locked Oscillator - A New Control System for Controlled Static Converters", *IEEE Transaction on Power Apparatus and Systems*, vol. PAS-87, March 1968, pp.859-865.