Calculation of Lightning-Induced Overvoltages using MODELS

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Abstract - **The paper presents a method for calculation of lightning-induced overvoltages (LIO). The modeling of the lightning channel as well as the coupling to overhead lines is formulated analytically and implemented in MODELS in ATP. The analytical approach is possible as long as the overhead line and the ground is assumed lossless and the transmission line model is used for the lightning channel with constant current velocity. A large number of overhead lines can be included in an ATP data case, and each line can consist of several conductors. The paper presents calculation results of LIOs in a larger low voltage system with and without a neutral wire. The dependency of stroke location, overhead line configuration and the low voltage in the system is studied as well as the effect of arresters.**

Keywords: Lightning, induced-volages, low-voltage, protection, MODELS, ATP-EMTP

I. INTRODUCTION

The calculation of lightning-induced overvoltages (LIO) in overhead lines was analytically formulated by Rusck [1]. Assuming the transmission line (TL) model [2] for the lightning channel and an infinite long overhead line and ignoring loss effects he established well-known engineering equations for LIOs. Today, more sophisticated calculation models exist [3], but the approach proposed by Rusck is still believed to give acceptable results [4, 5].

An advantage with Rusck's formulation is that it is analytical. This results in fast computation and makes it simple to study the effect of specific parameters and as well as induced voltages in larger systems. This paper extends the results from Rusck, using the Agrawal coupling model [6], which is based on measurable electromagnetic field quantities. A finite length of the overhead line is taken into account as well as the option of multiple phases. The lossy ground effect, handled in [7, 8], is ignored. This is reasonable for high conducting ground and line lengths shorter than about 1 km. The equations for induced voltage are completely analytical.

A model of a lossless overhead line excited by electromagnetic fields from a lightning channel has been established in the ATP [9], using the MODELS language [10]. In this model the lightning current parameters, the overhead line length geometry and orientation is user selectable. The graphical tool ATPDraw [11] is used to visualise the electrical network.

II. MODELLING

In this section models for induced voltage calculations are presented. The basic assumptions are:

- The overhead line can be treated as loss-less.
- The electrical field is assumed to propagate unaffected by the ground.
- The Agrawal coupling model is used.
- The transmission line (TL) model is used for the lightning channel, assuming a pure step current at ground.
- The electrical field from the lightning leader is constant, and the resulting induced voltage is zero.

These assumptions enable a completely analytical solution. The actual lightning current shape is taken into account by a convolution integral of the inducing voltages. If $g_0(t)$ is the step response of a current I_0, then the response of a time varying current $i(t)$ is:

$$g(t) = \int_0^t \frac{g_0(t-\tau)}{I_0} \cdot \frac{\partial i(\tau)}{\partial t} \cdot d\tau \qquad (1)$$

The configuration of the system is shown in Fig. 1. The end A of the overhead line is oriented so that $x_A > x_B$.

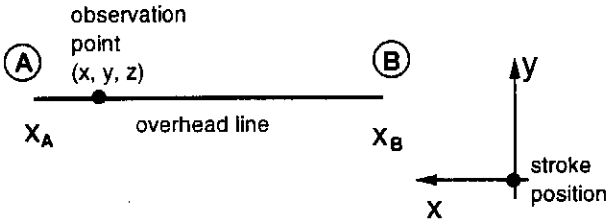

Fig. 1. Co-ordinate system and configuration.

The overhead line can further be modelled electrically as shown in Fig. 2.

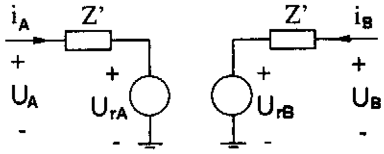

Fig. 2. EMTP model of overhead line.

where

$$U_{rA}(t) = U_{indA}(t) + U_B(t-\tau) + Z' \cdot i_B(t-\tau)$$
$$U_{rB}(t) = U_{indB}(t) + U_A(t-\tau) + Z' \cdot i_A(t-\tau) \qquad (2)$$

Z' is the overhead line's characteristic impedance and τ is the travelling time ($\tau = L/c$ where L is the line length and c is the speed of light).

Several phases are handled straightforward by using matrix expressions in eq. (2).

The impact of the incident field from a return stroke is embedded in the terms U_{indA} and U_{indB}, called the *inducing voltages*. These two terms become equivalent source terms. Loss effects on the electrical field due to propagation above a lossy ground can be taken into account by modifying these terms [8]. The model in Fig. 2 is implemented in ATP-EMTP without modifying the source code as in [12].

When the ground is assumed lossless the inducing voltage at terminal A can be written:

$$U_{indA}(t) = 2 \cdot U_{xA}(t) + U'^{i}_{A}(t) - U'^{i}_{B}(t-\tau) = \int_{x_B}^{x_A} E_x\left(x, y, z, t - \frac{x_A - x}{c}\right) \cdot dx - \int_0^z \left(E_z(x_A, y, z, t) - E_z(x_B, y, z, t-\tau)\right) \cdot dz \qquad (3)$$

where $E_x(x,y,z,t)$ and $E_z(x,y,z,t)$ are the incident horizontal and vertical field, respectively and U_x and U^i are the induced voltage terms set up by these fields. The height of the overhead line, z, is further assumed to be much less than the distance y and the length of the overhead line L, and the vertical field is thus assumed to be constant between the ground and the line. The horizontal electrical field in the integrand in (3) is zero when $t \cdot c < \sqrt{y^2 + x^2} + x_A - x$ which is the distance from the lightning stroke to the point of observation, (x,y,0) and further along the line to the endpoint A.

Using the TL model for the lightning channel, the inducing voltage at terminal A is:

$$U_{indA}(t) = U_0(x_A, t) \cdot \begin{cases} 0 & \text{for } t \in [0, t_A] \\ f(x_A, t) + 1 & \text{for } t \in (t_A, t_B] \\ f(x_A, t) - f(x_A - L, t - L/c) & \text{for } t \in (t_B, \infty) \end{cases} \qquad (4)$$

where

$$u_0(x,t) = \frac{60 \cdot I_0 \cdot z \cdot \beta \cdot (c \cdot t - x)}{y^2 + \beta^2 \cdot (c \cdot t - x)^2} \quad [V] \qquad (5)$$

$$f(x,t) = \frac{x + \beta^2 \cdot (c \cdot t - x)}{\sqrt{(v \cdot t)^2 + (1 - \beta^2) \cdot (y^2 + x^2)}} \qquad (6)$$

$$t_A = \frac{\sqrt{x_A^2 + y^2}}{c}, \quad t_B = \frac{L + \sqrt{x_B^2 + y^2}}{c} \quad \text{and} \quad \beta = {}^{v}\!/_{c} \qquad (7)$$

v and I_0 are the lightning current wave velocity and step amplitude, respectively.

For times less than t_B the expression in (4) gives the same result as Rusck's model [1, eq. (105)] (called U_l) even though a different coupling model is used and only the field from the return stroke is considered in (4). Rusck's expression is equivalent since he assumes an infinitely long line, which causes the static contribution from the charged leader to vanish.

The inducing voltage at terminal B is found by substituting $-x_B$ for x_A in (4).

A model called INDUS2 using the expressions in eq. (2) to calculate the LIOs in a 2-phase overhead line, is written in the MODELS language and shown in appendix 1. Combining this model with Fig. 2, assuming two overhead line conductors and using the graphical pre-processor ATPDraw, gives an equivalent circuit shown in Fig. 3. The overhead line is modelled with a lumped resistive matrix, representing the lossless characteristic impedances. The model INDUS2 measures the terminal voltage of two phases at each end of the line and calculates the inducing voltage terms and reflections at the line terminals. The current into the line terminal is calculated internally, and (2) is efficiently reformulated into:

$$\begin{aligned} U^{j}_{rA}(t) &= U_{indA}(t) + 2 \cdot U^{j}_{B}(t-\tau) - U^{j}_{rB}(t-\tau) \\ U^{j}_{rB}(t) &= U_{indB}(t) + 2 \cdot U^{j}_{A}(t-\tau) - U^{j}_{rA}(t-\tau) \end{aligned} \qquad (8)$$

where j is the phase number (1..2).

The timestep between measured and calculated quantities must be taken into account, as shown in the appendix.

In this paper the same inducing voltage is assumed in all line conductors, but this could easily be extended since the inducing voltage is proportional to the line height, z.

The circuit shown in Fig. 3 can be connected to any component in the ATP, and several line segments are allowed.

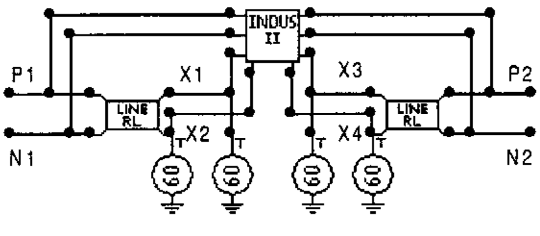

Fig. 3. Equivalent circuit of 2-phase overhead line, excited by nearby lightning. Modelled in ATPDraw.

Below, an example of the electric circuit part of the two-phase model in Fig. 3 is listed in ATP file format.

```
/BRANCH
51X1     P1                300.
52X2     N1                200.            500.
51P2     X3                300.
52N2     X4                200.            500.
/SOURCE
60X1
60X2
60X3
60X4
```

A further simplification of this model is to rewrite it into a *type94* Norton-transmission component [10].

III. CALCULATIONS

Fig. 4 shows the configuration of the low-voltage network used as an example in this paper. The network consists 5 overhead line segment, in an H-shape. Four lines are 250 m long and the one line in the middle is 500 m. The height of the line is in all cases 6 m. The same voltage is assumed to be induced in all three phase-conductors and they are represented by one conductor with characteristic impedance 300 Ω. The mutual impedance between the equivalent phase conductor and the neutral conductor is 200 Ω, and the characteristic impedance of the neutral is 500 Ω. Loads are attached at each point (1-6).

Measurements on distribution transformers and low voltage power installations (LVPI) networks and developed models in the frequency range 10 kHz to 1 MHz are presented in [13, 14]. The simplified results for LVPI networks are summarised in tab. 1. In point 1 a 3-leg distribution transformer is modelled with a zero sequence inductance of $Z_T = 10\ \mu H$. In point 2-6 low-voltage power installations (LVPI) are modelled with a simple representation according to tab. 1 and fig. 4. The transformer has grounding impedance of 5 Ω, while the LVPI networks have 50 Ω each.

The configuration investigated in this paper is a simplification with the purpose to illustrate the LIO level in low-voltage systems, and how the lightning parameters and loads or terminations influence it.

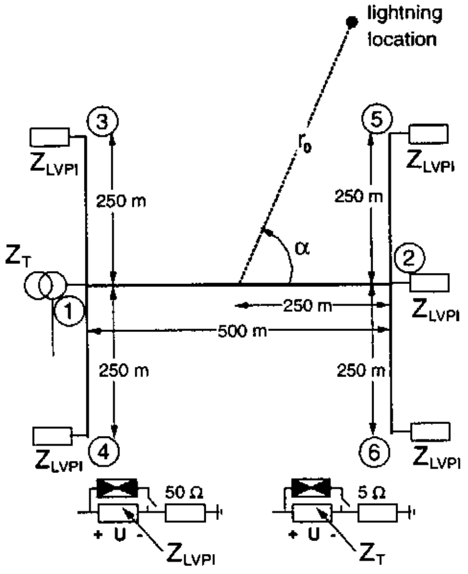

Fig. 4. Low-voltage system configuration. Overhead line system, and electrical load at the bottom (left: loads, right: transformer).

Tab. 1. Models of LVPI-networks used in the calculations, Z_{LVPI}.

Type	small	medium	large
TN	10 uH	5 uH	2 uH
IT	10 uH 50 nF	5 uH 100 nF	2 uH 200 nF

Fig. 5 shows how the LVPI network loads are connected to the overhead line in the IT- and TN-system. In Figs. 8-11 the voltage *across* the LVPI loads, Uj, are denoted Pj-Nj with j=1..6. In Fig. 12-13 the maximum of this voltage is shown.

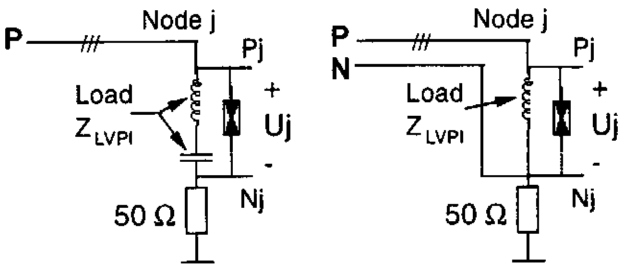

Fig. 5. Overhead line connection to LVPI loads in the IT-system (left) and the TN-system (right).

The metal oxide arrester used in the investigation has a rated voltage of 440 V and an energy capacity of 650 J. The current-voltage characteristic used by ATP [9] is shown in Fig. 6, with the data for a standard 8/20 μs impulse shown as circles.

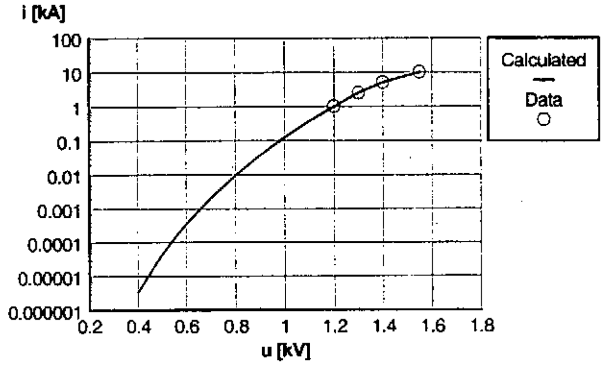

Fig. 6. Current-voltage characteristic for MOV.

The Heidler model is used to represent the lightning current in all the calculations with front time constant τ_1=2 μs, decay constant τ_2=50 μs and slope parameter m=5. The amplitude is 30 kA and the time to crest is just above 4 μs. Eq. (1) is used to take this current shape into account. The velocity of the lightning current in the lightning channel according to the TL model is v=1.1e8 m/s.

Fig. 7 shows how this system is modelled in ATPDraw, using the developed model. Tab. 2 shows the geometry of the system and the settings of the three co-ordinates (Y, XA, XB) for each of the five line segments.

Tab. 2. Geometry of the five line segments. Ref. Fig. 7.
Line length L=XA-XB, XA>XB.

Lightning location: Angle α=90° and distance r_0=100 m.			
Line	Y	XA	XB
1-2	100	250	-250
1-3	250	100	-150
1-4	250	350	100
2-5	250	150	-100
2-6	250	-100	-350

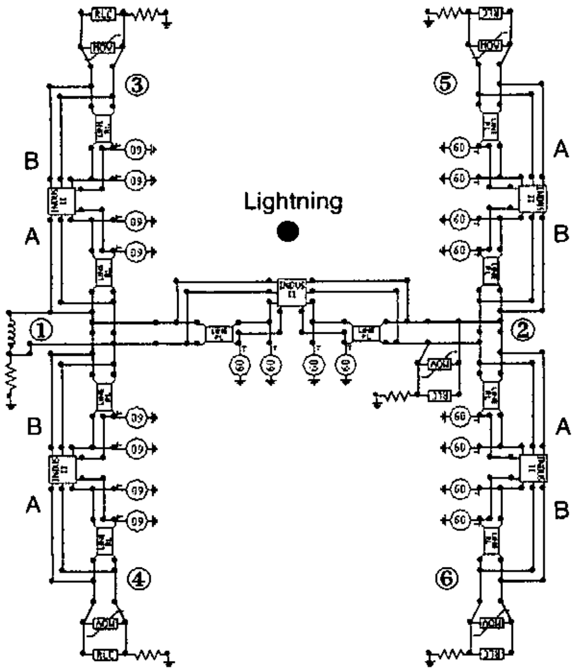

Fig. 7. System model in ATPDraw, using MODELS.

Fig. 8 shows the lightning induced voltages across loads at point 1, 2, and 5, with no arresters installed in the system. The loads (LVPI networks) are of TN type small (inductance of 10 μH). Fig. 9 shows the same simulation, but with arresters installed at all loads 2-6. The voltages at the other points 3, 4 and 6 are always lower then at point 5.

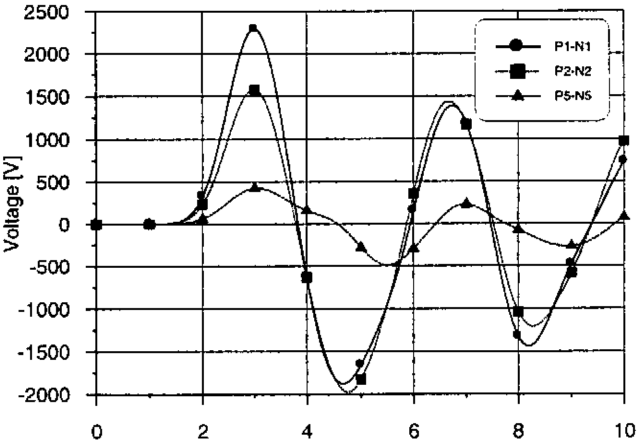

Fig. 8. LIO across loads. TN-system. No arresters. Lightning location: Angle α=90° and distance r_0=100m.

The voltage is highest across the transformer equivalent, reaching just above 2 kV, and this voltage is unaffected by arresters in the system. At load position 2, the voltage is limited by the surge arrester, and at the other positions 3-6 the voltage is below the protective level of the arrester. The voltage across the transformer in Fig. 8 is higher than across load in position 2 due to the lower grounding impedance of the transformer.

Figs. 10 and 11 shows the same calculation as Figs. 8-9, but now for an IT system (without a neutral conductor). A capacitance of 50 nF is added in series with all loading inductances, representing a typical small IT system installation. The connection to the neutral conductor is removed, and the coupling between the neutral conductor and the phase conductors is set to zero. The transformer is still modelled as an inductance of 10 μH, since its low-voltage neutral is assumed grounded. An ungrounded neutral will give even larger voltages.

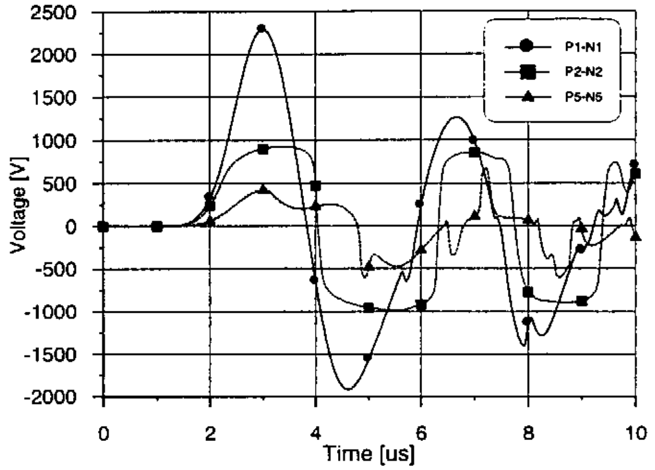

Fig. 9. LIO across loads. TN-system. With arresters. Lightning location: Angle α=90° and distance r_0=100m.

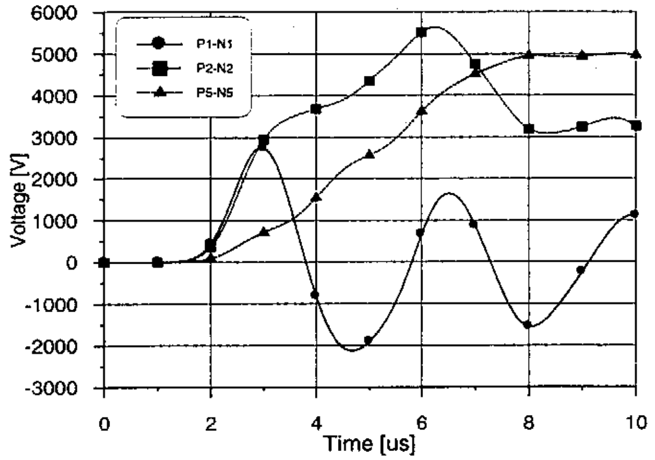

Fig. 10. LIO across loads. IT-system. No arresters. Lightning location: Angle α=90° and distance r_0=100m.

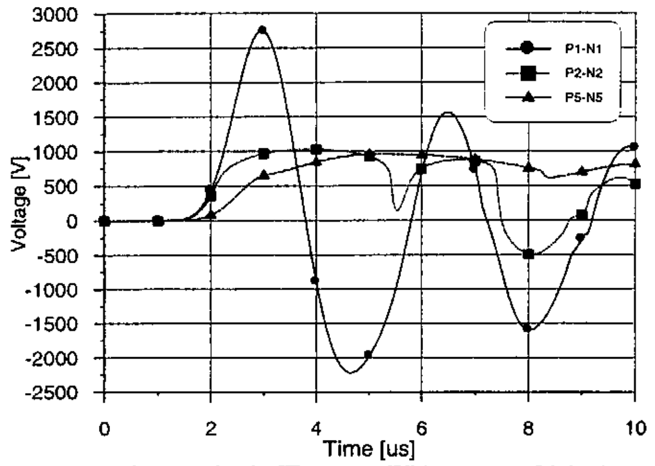

Fig. 11. LIO across loads. IT-system. With arresters. Lightning location: Angle α=90° and distance r_0=100m.

Figs. 12 and 13 show the maximum induced voltage across the load (Z_T and Z_{LVPI}) in the network in Fig. 4 (with a lightning distance r_0=500) as a function of angle α. All the load types shown in tab. 1 are investigated. The transformer in point 1 is in all cases modelled with Z_T = 10 µH. No arresters are installed in the system. The markers on the curves indicate where in the network (point 1-6) the maximum voltage occurs.

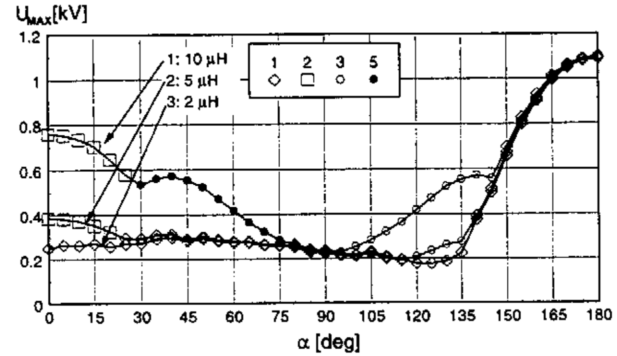

Fig. 12. Maximum LIO in TN system, dependency on LVPI model. Distance r_0=500 m.

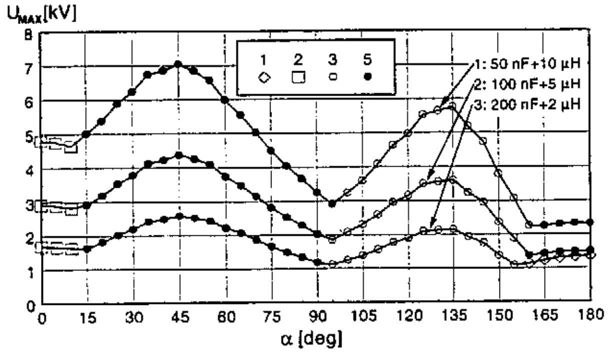

Fig. 13. Maximum LIO in IT system, dependency on LVPI model. Distance r_0=500 m.

Figures 12 and 13 show that the amplitude of induced voltages in a TN and IT system strongly dependent of the nature and size of the LVPI. In general larger installations result in lower overvoltages. The voltages induced in an IT-system are substantially higher than in a TN-system.

IV. DISCUSSION

The MODELS language makes it possible to interface induced voltages calculations with the ATP, and thus to investigate induced overvoltages in a larger and practical system. The following main simplifications are applied:

- The transmission line model is used for the lightning channel. This is reasonable for the first few microseconds.
- The same voltage is assumed induced in all phase conductors and only the common mode system is studied. This is also reasonable for the first few microseconds, but is doubtful for strong unsymmetry in load configurations.
- Lossy ground effects are ignored. It is reasonable to assume the overhead line to be lossless for lengths shorter than 1 km, but ignoring the propagation effect on the electromagnetic fields is more doubtful. Ignoring the lossy ground effects for nearby lightning (<1 km) could be reasonable.
- The lightning-induced voltage has a frequency content below 500 kHz. This justifies the simple load models, and the assumption is reasonable.
- The static voltage from the cloud and lightning leader is ignored, and this is reasonable if the power system has a connection to ground, and the distance from the stroke location to the line is above ca. 100 m.

Completely analytical expressions are established, and the calculation speed is limited by the convolution integrals performed in MODELS, taking the lightning current shape into account. The development of a *type94* component would simplify the equivalent circuit further.

A main motivation factor initiating this work was the discussion regarding IT- versus TN- systems and the assumed vulnerability of IT-system to lightning-induced effects. The IT-system is the common standard in Norway and overhead lines, without a ground or neutral wire, are in frequent use, particularly in rural areas. The calculations show that the IT-system in general results in much higher voltage across loads (and in general phase-to-ground) than the TN-system. This is mainly caused by:

- Lower terminating impedances of overhead lines in the TN-system, due to electrical loads.
- The presence of a ground-wire in the overhead line in TN-systems.
- The grounding of the distribution transformer's LV-neutral in TN-systems.

The capacitive behaviour of IT-system power installations up to at least 100 kHz is the most important factor. In general larger installations result in lower overvoltages.

V. CONCLUSION

An analytical model for calculation of lightning-induced voltages in overhead lines has been developed based on a few specific assumptions. In order to be able to calculate the voltages in a complex power system the model is implemented in the MODELS language of ATP-EMTP. In this way no external calculation program is needed, and multiple overhead lines can be included with no additional limits than given by the ATP program. The calculation speed is mostly limited by the convolution integral required to take a specific lightning current shape at ground into account. Development of a type 94 component would further simplify the connection between the induced voltage calculation and the electric circuit.

Based on the listed assumptions, the developed model is believed to be accurate for the first few microseconds, or up to the first peak in the lightning-induced voltage. Most likely, the model gives the best results for a distance between lightning and an overhead lines in the medium-close range of 100-1000 m.

APPENDIX

```
MODEL INDUS2
CONST Tmax {VAL:500}    --number of time steps
      Im   {VAL:30.e3}  --current amplitude
      T1   {VAL:2.e-6}  --front time constant
      T2   {VAL:50.e-6} --decay time constant
      m    {VAL:5}      --slope factor
      c    {VAL:3.e8}   --speed of light
      v    {VAL:1.1e8}   --lightning velocity
      Io   {VAL:1}       --step current ampl.
      z    {VAL:6}       --line height
INPUT UAP,UBP,UAN,UBN    --terminal voltages
DATA Y,XA,XB             --orientation param.
OUTPUT UrAP,UrBP,UrAN,UrBN --type 60 sources
VAR UindA[0..1000],UindB[0..1000],dI[0..1000],
  Tr,Ti,I,e,dt,UrAP,UrBP,UrAN,UrBN,
  ta,tb,b,n,L,x,Ko,Ui,Tj
FUNCTION SQR(x):=x*x
FUNCTION F(x,tr):=
         (x+b*b*(c*tr-x))/
         sqrt(sqr(v*tr)+(1-b*b)*(x*x+y*y))
FUNCTION U0(x,tr):=
         60*Io*z*b*(c*tr-x)/
         (y*y+sqr(b*(c*tr-x)))
HISTORY
  UrAP {dflt:0}, UrBP {dflt:0}
  UrAN {dflt:0}, UrBN {dflt:0}
  UAP  {dflt:0}, UBP  {dflt:0}
  UAN  {dflt:0}, UBN  {dflt:0}
INIT
  dt:= timestep
  b:=v/c
  L:=XA-XB
  FOR Tj:=1 TO 2 DO
    if Tj=1 then
      x:=XA else
      x:=-XB
    endif
    ta:=sqrt(x*x+y*y)/c
    tb:=sqrt(sqr(x-L)+y*y)/c
    FOR Ti:=0 TO Tmax DO
      Tr:=Ti*dt
      if Tr>ta
      then
        if Tr>tb+L/c
        then
          Ui:=U0(x,Tr)*(f(x,Tr)-f(x-L,Tr-L/c))
        else
          Ui:=U0(x,Tr)*(f(x,Tr)+1)
        endif
      else
        Ui:=0
      endif
      if Tj=1 then
        UindA[Ti]:=Ui else
        UindB[Ti]:=Ui
      endif
    ENDFOR
  ENDFOR
  FOR Ti:=0 TO Tmax DO  --Heidler current:
    Tr:=Ti*dt
    IF (Ti=0) THEN dI[0]:=0
    ELSE
      e:=exp(-(T1/T2)*exp(ln(m*T2/T1)/m))
      I:=Im/e*exp(m*ln(Tr/T1))/
         (exp(m*ln(Tr/T1))+1)*exp(-Tr/T2)
      dI[Ti]:=I*((m/Tr)/
             (exp(m*ln(Tr/T1))+1)-1/T2)
    ENDIF
  ENDFOR
 --Convolution integral. Small Io required!!
  Ti:=Tmax
  WHILE Ti>1 DO
    FOR Tr:=1 TO Ti-1 DO
      UindA[Ti]:=UindA[Ti]+
                 UindA[Tr]*dI[Ti-Tr]*dt
      UindB[Ti]:=UindB[Ti]+
                 UindB[Tr]*dI[Ti-Tr]*dt
    ENDFOR
    Ti:=Ti-1
  ENDWHILE
--Possible to scale Uind
  Tr:=L/c
ENDINIT
EXEC
  UrAP:=UindA[t/dt]+
    2*delay(UBP,Tr-dt,1)-delay(UrBP,Tr,1)
  UrAN:=UindA[t/dt]+
    2*delay(UBN,Tr-dt,1)-delay(UrBN,Tr,1)
  UrBP:=UindB[t/dt]+
    2*delay(UAP,Tr-dt,1)-delay(UrAP,Tr,1)
  UrBN:=UindB[t/dt]+
    2*delay(UAN,Tr-dt,1)-delay(UrAN,Tr,1)
ENDEXEC
ENDMODEL
```

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