

Influence of Earth Conductivity and Permittivity Frequency Dependence in Electromagnetic Transient Phenomena

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Abstract – In this article the quasi-modes model is used to observe the influence, in Electromagnetic Transient Phenomena, of considering a more accurate representation of soil, taking into account the earth conductivity and permittivity frequency dependence.

For an actual 440 kV three-phase transmission line the soil behavior is represented through an unique real value of conductance and through a more accurate model, considering its electromagnetic behavior.

Keywords: Soil model, Line parameters, Frequency dependence.

I. INTRODUCTION

The soil model presented satisfies the physical coherence conditions in what concerns the relations between the conductivity (σ) and the permittivity (ϵ) in the frequency domain. Some examples of ground parameters are presented. The effect of the soil behavior in some transmission line transients, and its influence in the overvoltages obtained, are shown.

For an actual 440 kV three-phase transmission line the soil behavior is represented through an unique real value of conductance and through a more accurate model of its electromagnetic behavior, considering the earth conductivity and permittivity frequency dependence.

In some cases, a proper earth model can lead to very different results than the ones obtained with simple real conductivity value, as shown. The influence of the frequency dependence of the soil parameters, in some line transient phenomena, is presented. In the cases where the transients have high homopolar component, the effect of using a more accurate soil model can be very important.

II. SOIL ELECTROMAGNETIC BEHAVIOR

One essential aspect of grounding systems study and simulation is the adequate soil modeling.

Except for very high electric fields, which originate significative soil ionization, soil electromagnetic behavior is essentially linear, but with electric conductivity, s , and electric permittivity, e , strongly frequency dependent. The magnetic permeability, m , is, in general, almost equal to vacuum magnetic permeability, m_0 . For slow variation of electromagnetic entities, it may occur an hysteresis type behavior. For direct current or very slow variations of electromagnetic entities, it may occur humidity migration

phenomena, including electroosmosis and effects of temperature heterogeneity, which can not be dealt with only by means of local soil parameters.

For fast transients, namely those associated to lightning, the soil behavior is important in a reasonably wide frequency range, typically [0 , 2 MHz].

Field measurements have some difficulties. After several trials with quite different soils, field and laboratory systematic measurements, namely varying water content of samples, we have developed a measurement procedure, considering the methods to collect soil samples, to measure parameters in function of frequency, and to obtain a physically consistent model of soil electromagnetic behavior. As an example of sample collection and measurement aspects to cope with: it is necessary to assure the maintenance of soil structure and humidity, in samples, and to minimize effects of local soil heterogeneity. Three main procedures have been developed, for compact soils (namely those including clay), for pulverulent soils (including sand), and for rock. Basic description of such procedures is presented in [1-5]. Such procedures have been applied, with good results, to a large number of sites and soil types, including remote sites.

The field measurements of real soil have inherent dispersion. A purely mathematical fitting may lead to physical inconsistent models, with quite wrong results obtained with such models, e.g. by Fourier methods. It is adequate to have a robust validation criteria of soil models, covering real soil characteristics.

In [1-5] several soil electric models have been presented and justified, which :

- Cover a large number of soil measured parameters, with good accuracy, and within the range of confidence of practical field measurement.
- Satisfy coherence conditions.

In this paper the electric soil parameters are applied (s , w , e), in function of frequency, considering a particular set of the models described in [1-5]. The parameters of such model were chosen according a minimum difference criteria, for field measured electric parameters, in function of frequency, for 68 ground samples in eight Sites, in Brazil, covering very different soil types and geological structures. The agreement of obtained models with measured parameters is within or near the confidence range of field measurement values. The measurements

were done in a frequency range [100 Hz, 2 MHz]. In each Site, the maximum distance among ground points at which samples were collected is less than 500 m.

II.1. Soil Models

The model which have been used in presented results are some of models described in [1-5].

With the exception indicated below, the models, whose results are presented, are a sum of minimum phase shift parcels, \mathbf{W}_j , which apply to the immittance type magnitude (in complex or tensorial formulation of alternating magnitudes)

$$\mathbf{W} = \mathbf{s} + i \mathbf{w} \mathbf{e} \quad (w = 2 \pi f, \text{ being } f \text{ the frequency}) \quad (1)$$

being $i = +\sqrt{-1}$ and

$$\mathbf{W} = \sum_{j=0}^m \mathbf{W}_j \quad (2)$$

All submodels used for \mathbf{W}_j are particular conditions of Type 3 model described in [1], presented below.

Apart from slow phenomena and hysteresis type phenomena, soil behavior is, typically, of minimum phase shift type. For a great number of soils, on frequency range [0, 2 MHz], in a first approach, it is

$$\sigma = \mathbf{a} + \mathbf{b} \cdot \omega^\alpha \quad \text{and} \quad \omega \epsilon = \mathbf{c} \cdot \omega^\alpha \quad (3)$$

being \mathbf{a} , \mathbf{b} , \mathbf{c} , a constant parameters (frequency independent).

For some soils, a similar behavior occurs, but for a smaller frequency range, e.g. [0, 100 kHz], and, for higher frequency, the behavior is different, namely with a lower $\omega \epsilon$ increase, or till a $\omega \epsilon$ decrease, when frequency increases.

In order to analyze the frequency behavior of \mathbf{s} , \mathbf{e} , it is convenient to consider complex formulation of electromagnetic entities, and to consider $\mathbf{s} + i \mathbf{w} \mathbf{e}$ as an immittance. In fact, apart geometric factors, $\mathbf{s} + i \mathbf{w} \mathbf{e}$ may be associated to the admittance of a volume element δv .

Type 3 model can be described as presented below, for which :

$$\mathbf{W}_j(\omega) = k \cdot \left\{ \frac{\mathbf{b}^a}{\mathbf{a}} {}_2F_1 \left[1, a, 1+a, \frac{i\mathbf{b}}{\mathbf{w}} \right] - \frac{\mathbf{a}^a}{\mathbf{a}} {}_2F_1 \left[1, a, 1+a, \frac{i\mathbf{a}}{\mathbf{w}} \right] \right\} \quad (4)$$

representing ${}_2F_1[\dots, \dots, \dots]$ the hypergeometric function, with four arguments, ${}_2F_1$, according notation of [6].

This submodel has four independent parameters (\mathbf{k} , α , \mathbf{a} , \mathbf{b}).

Considering, in this model (4), $\mathbf{a} = 0$, the model becomes :

$$\begin{aligned} \mathbf{W}_j(\omega) &= k \frac{\mathbf{b}^a}{\mathbf{w}} {}_2F_1 \left[1, a, 1+a, \frac{i\mathbf{b}}{\mathbf{w}} \right] \\ &= k_1 {}_2F_1 \left[1, a, 1+a, \frac{i\mathbf{b}}{\mathbf{w}} \right] \end{aligned} \quad (5)$$

Considering, in the model (4), $\mathbf{a} = 0$, $\mathbf{b} \rightarrow \infty$ and $\alpha_j = \alpha$, the model becomes :

$$\mathbf{W}_j(\omega) = K_j \left[1 + i \tan \left(\frac{\pi}{2} \alpha_j \right) \right] \cdot \omega^{\alpha_j} \quad (6)$$

A parcel \mathbf{W}_j as indicated in (6) is equivalent to parcel $\mathbf{b} \cdot \omega^\alpha$ of σ and to $\omega \epsilon = \mathbf{c} \cdot \omega^\alpha$, as indicated in (1) and (3),

doing $\mathbf{b} = K_j$, $\mathbf{c} = K_j \cdot \tan \left(\frac{\pi}{2} \alpha_j \right)$ and $\alpha_j = \alpha$, with the

condition $\frac{\mathbf{c}}{\mathbf{b}} = \tan \left(\frac{\pi}{2} \alpha \right)$. This condition has been verified in soil measurements, within measurement accuracy and soil heterogeneity effects.

Considering, is this model (6), $\alpha_j = 0$, the model becomes :

$$\mathbf{s} \text{ constant, } \omega \mathbf{e} \text{ null ("pure" conductor)} \quad (7)$$

Considering, is the model (6), $\alpha_j \rightarrow 1$, the model becomes :

$$\mathbf{s} \text{ null, } \mathbf{w} \mathbf{e} \text{ proportional to } \mathbf{w}, \mathbf{e} \text{ constant ("pure" dielectric)} \quad (8)$$

Within the range [0, 2 MHz], for all soil samples modeled in this paper, it is accurate enough to consider two parcels, for $\mathbf{s} + i \mathbf{w} \mathbf{e}$, one constant (in most cases real), and another of type (4) or of type (5), frequency dependent. In a few cases, there is a net hysteresis effect, that can be modeled with an imaginary part of constant parcel. For all samples, \mathbf{a} is the dominant parameter of the relative shape of frequency dependent parcel, \mathbf{W}_j , of $\mathbf{s} + i \mathbf{w} \mathbf{e}$. For $\mathbf{a} = 0$ such parcel corresponds to a "pure" conductor (\mathbf{s} frequency independent, \mathbf{e} null). For $\mathbf{a} = 1$, such parcel corresponds to a "pure" dielectric (\mathbf{s} null, \mathbf{e} constant). In all samples, for frequency dependent parcel, it is $0 < \mathbf{a} < 1$.

II.2. Statistical Distribution of Soil Parameters

In order to allow a direct interpretation of statistical distribution of main electric parameters of ground, in a way independent of model details, the following parameters were chosen, according the models adopted, independently, for the 68 soil samples, satisfying physical coherence conditions:

$$\begin{aligned} \mathbf{S}_0 &= \mathbf{s} (100 \text{ Hz}) & \mathbf{s} \text{ at } 100 \text{ Hz.} \\ \mathbf{D}_r &= \mathbf{D} \mathbf{S}_1 = & \\ & \mathbf{s} (1 \text{ MHz}) - \mathbf{s} (100 \text{ Hz}) & \mathbf{s} \text{ increase between} \\ & & 100 \text{ Hz and } 1 \text{ MHz.} \end{aligned}$$

$$\begin{aligned} \mathbf{D}_i &= \mathbf{D}(\mathbf{w} \mathbf{e})_1 = & \\ & \mathbf{w} \mathbf{e} (1 \text{ MHz}) - \mathbf{w} \mathbf{e} (100 \text{ Hz}) & \mathbf{w} \mathbf{e} \text{ increase between} \\ & & 100 \text{ Hz and } 1 \text{ MHz.} \end{aligned}$$

\mathbf{a} parameter of frequency dependent parcel of $\mathbf{s} + i \mathbf{w} \mathbf{e}$.

It was verified that, for these samples, the two parcels of $\mathbf{s} + i \mathbf{w} \mathbf{e}$, one constant, the other frequency dependent, are statistically independent. This fact, and the fact that no significative correlation exists between the pair $[\mathbf{D}_i, \mathbf{a}]$, but it exists between the pair $[\mathbf{D}_r, \mathbf{a}]$, arises the hypothesis that:

- The constant and frequency dependent parcels of $\mathbf{s} + i \mathbf{w} \mathbf{e}$ are related to quite distinct aspects of physical ground behavior.
- The frequency dependent parcel is mainly associated to a dielectric physical process, with related dissipative effects. Such dissipative effects are quite different of conductive behavior associated to constant parcel.

In Figure 1 we represent the probability density, \mathbf{p} , of parameters \mathbf{S}_0 , \mathbf{D}_r , \mathbf{D}_i , \mathbf{a} , considered separately, and, in Figure 2, the probability density, \mathbf{p} , of parameters $[\mathbf{D}_i, \mathbf{a}]$,

considered together, with Weibull approximations based in the 68 soil samples.

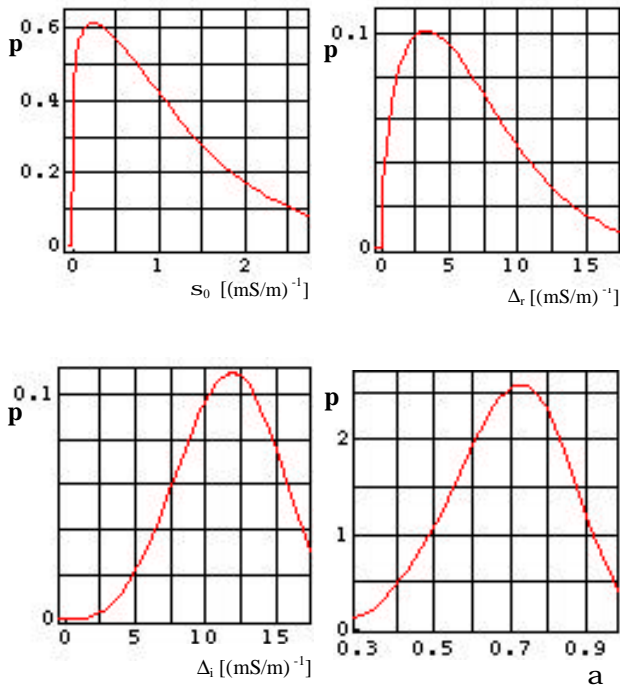


Figure 1- Probability density, p , of parameters σ_0 , Δ_r , Δ_i , α , considered separately, with Weibull approximations based in the 68 soil samples. Scales of p applicable to σ_0 , Δ_r , Δ_i are graduated in $(\text{mS/m})^{-1}$.

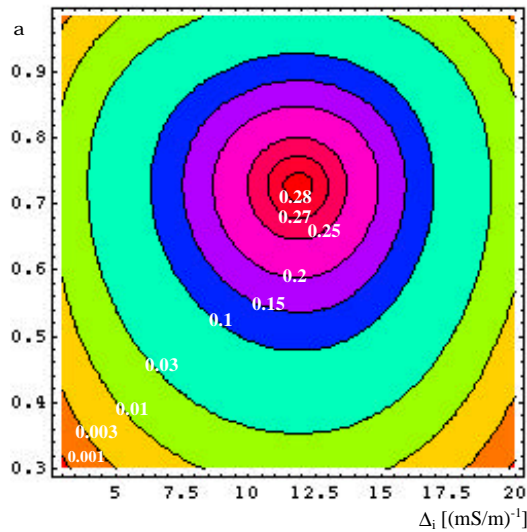


Figure 2- Probability density, p , of parameters $[\Delta_i, a]$, considered together, with Weibull approximations based in the 68 soil samples and without correlation between Δ_i and a . Values of p , in white, are expressed in $(\text{mS/m})^{-1}$.

II.3. Soil Parameters Applied

The soil parameters used in these examples were obtained from the experiments described in [3], and are presented below :

- Soil 1 : $s + i \omega e = A + B \omega^\alpha$
being A, B, α constants, and
 $A = 84.16 \mu\text{S/m}$
 $B = [0.057849 + 0.12097 i] (\mu\text{S/m}) s^\alpha$

$$\alpha = 0.71603$$

- Soil 2 : $s = A$; $\omega e = 0$

which results in $\rho : 11882 \Omega.\text{m}$, constant.

The conductivity of the studied soils were chosen to be equal at low frequency, in order to compare the obtained results, taking into account that traditional measurement of soil resistivity is done at low frequency. Soil 1 considers two parcels W_j as described in II.3, namely one constant parcel, A , of the type (7) and a frequency dependent parcel, $B \cdot \omega^\alpha$, of the type (6). In soil 2, only the first parcel, A , is considered.

III. CALCULATING THE LINE PARAMETERS

In order to implement the soil model, the line parameters were calculated using the approximated formula which include the earth effect in longitudinal impedance as equivalent to have an ideal ground surface at a depth D' (complex) below physical ground surface [7].

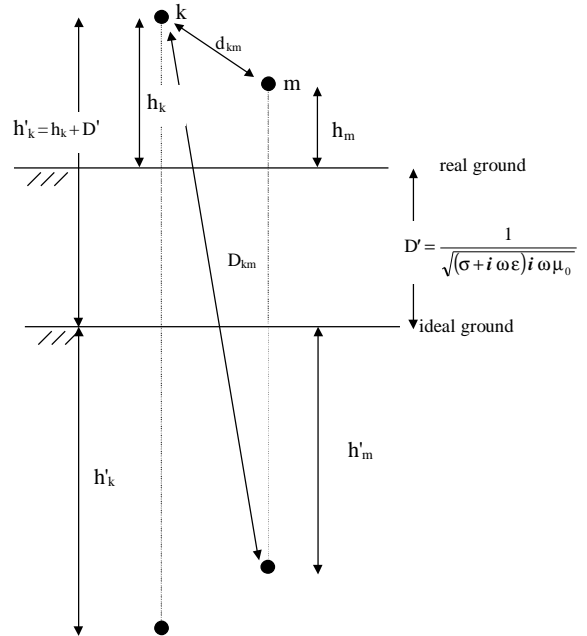


Figure 3 - Conductors \underline{k} and \underline{m} position supposing the earth at a complex depth D' .

The transmission line longitudinal parameters are formed by :

$$Z_{km} = Z_{km}^0 \quad k, m = 1, 2, \dots, n \quad (9)$$

where :

Z_{km} – longitudinal impedance matrix element, per unit length;

n – total number of conductors

and

$$Z^0 = Z_{\text{int}} + Z_{\text{ext}} \quad (10)$$

where

Z_{int} – conductor internal impedance, per unit length;

Z_{ext} – conductor external impedance, per unit length;

and

$$Z_{\text{ext}} = i \frac{\omega \mu_0}{2\pi} \ln \frac{D_{km}}{d_{km}} \quad k, m = 1, 2, \dots, n \quad (11)$$

where

D_{km} and d_{km} are defined schematically in Fig. 3, being :

$$D' = \frac{1}{\sqrt{(\sigma + i\omega\epsilon) i\omega\mu_0}} \quad (12)$$

For the self terms ($k = m$)

$$D_{km} = 2h'_k \quad (13)$$

$$d_{km} = r_k \quad (14)$$

and

$$Z_{int} = R_{int} + i X_{int} \quad (15)$$

where

R_{int} – internal conductor resistance

X_{int} – internal conductor reactance

In Figs. 4-5 the per unit longitudinal parameters for the transposed line using both soil models are presented.

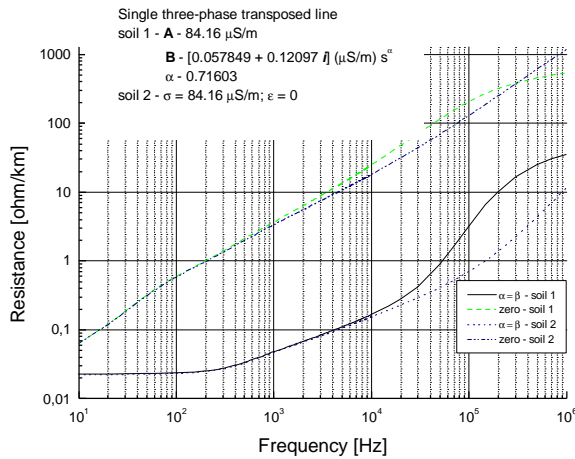


Figure 4 – Resistance per unit length comparing both soil models.

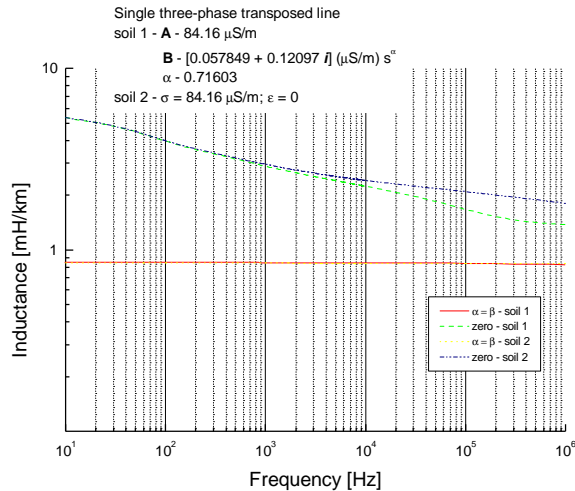


Figure 5 – Inductance per unit length comparing both soil models.

In Table 1 and 2 some values of resistance and inductance per unit length are presented, for the transposed line.

The difference between line parameters for the two soil models is important, namely for homopolar mode (e.g., 37 % in the longitudinal resistance per unit length, at 10 kHz). For fast transients, for which important frequency range may include frequencies above 10 kHz, the difference between the two soil models may be important also for non-homopolar modes. E. g. , for 100 kHz, there is an order of magnitude difference in resistance per unit length, between the two soil models, for non-homopolar

modes. Typical cases in which frequency range above 10 kHz is important are : transients originated by lightning; front of wave aspects of transients associated with short-circuits, which may be quite important in what concerns insulation coordination.

Table 1 – R and L per unit length values – transposed line – soil 1.

Freq [Hz]	$R_a(\Omega/\text{km})$	$L_a(\text{mH}/\text{km})$	$R_h(\Omega/\text{km})$	$L_h(\text{mH}/\text{km})$
10	0.02249	0.849511	0.06364	5.35
100	0.02319	0.849057	0.60150	3.97
1000	0.04700	0.844898	3.69759	2.89
10000	0.16296	0.840029	24.9454	2.24

Table 2 – R and L per unit length values – transposed line – soil 2.

Freq [Hz]	$R_a(\Omega/\text{km})$	$L_a(\text{mH}/\text{km})$	$R_h(\Omega/\text{km})$	$L_h(\text{mH}/\text{km})$
10	0.02249	0.849511	0.06316	5.35
100	0.02319	0.849056	0.58836	3.99
1000	0.04693	0.844895	3.37578	2.96
10000	0.15442	0.839967	18.2182	2.41

IV. SINGLE THREE-PHASE LINE APPLICATION

In Fig. 6 the data of the three-phase line used to illustrate the model are presented.

The line parameters were calculated in the range of 10 Hz to 10 kHz. As it is a single line, to represent its modes (exact ones for transposed line and quasi-modes for non-transposed line) it was applied Clarke's transformation matrix, as explained in [8-10]. With the longitudinal impedance and transversal admittance in mode domain, the synthetic circuits were calculated, composing one cascade of π -circuits for each mode, each representing 10 km length.

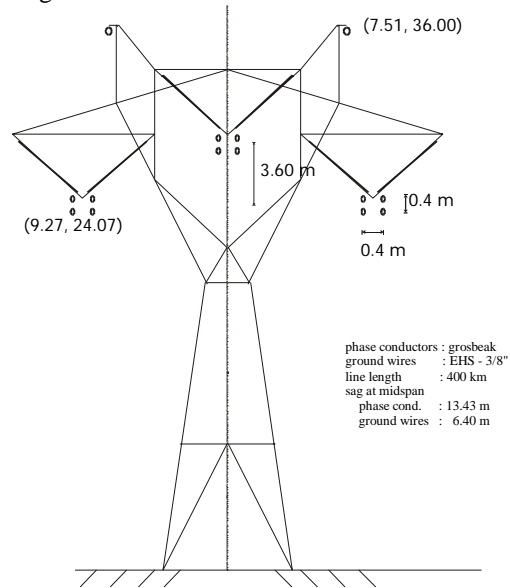


Figure 6 - Schematic representation of the 440 kV three-phase line.

The line was supposed transposed. Some transient studies were performed in order to analyze the models, as presented.

IV.1. Frequency Analysis

A frequency scan analysis was performed for both models where the sending terminal had a 1 V source and the receiving end was opened. The relations between the line ends were analyzed in the range of 10 Hz to 10 kHz.

In Fig. 7 the zero sequence response is presented for the transposed line.

The results for both soil models are discussed below :

- The positive sequence response was similar for both models.
- The zero sequence response for the frequency dependant soil model is more damped than the one which uses only constant conductance.

In Table 3 and 4 some values of zero sequence response are presented, for the transposed line.

The difference between the zero sequence response for the two soil models is important, e.g. 14 % at 1000 Hz.

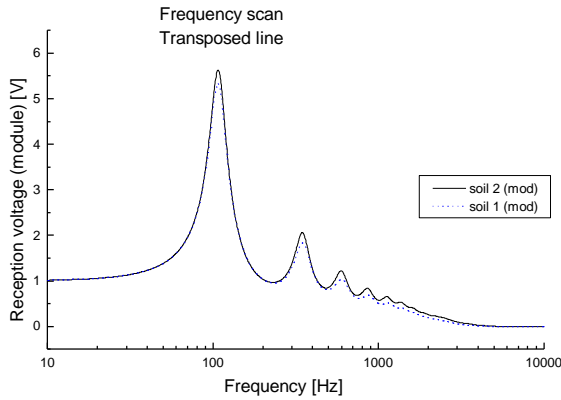


Figure 7 - Zero sequence - Transposed line.

Table 3 – Zero sequence response – transposed line – soil 1.

Freq [Hz]	Module (V)	Phase (°)
106.66	5.33197	-81.336
107.15	5.33508	-83.375
107.64	5.33165	-85.423
1000.0	0.50442	10.068

Table 4 – Zero sequence response – transposed line – soil 2.

Freq [Hz]	Module (V)	Phase (°)
106.66	5.62118	-81.054
107.15	5.62774	-83.207
107.64	5.62660	-85.372
1000.0	0.57741	-0.8830

IV.2. Mode Analysis

The following test performed with both models was to verify the natural mode behavior for a single three-phase transmission line, supposing it ideally transposed and non-transposed.

The simulation consisted of applying a 1 V step of 1 ms to verify the model behavior to transients in the frequency range of the normal switching phenomenon. In Fig. 8 the diagram of the studied system is shown.

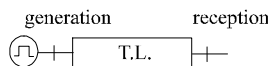


Figure 8 - Simulated system for mode analysis.

To represent the modes the step voltages were input as described in Tab. 5. The reception end was opened.

In Fig. 9 some results of the mode analysis are presented.

Analyzing the results it can be seen that :

- Modes alpha and beta had very similar results for

both models, as could be seen from the previous results;

- The homopolar mode presents some differences, which can imply in different overvoltages if the phenomenon has high contribution of this mode.

Table 5 - Steps to represent the modes.

Mode	Phase	Voltage (V)
alpha	a	- 0.5
	b (central)	+ 1.0
	c	- 0.5
beta	a	+ 1.0
	b	0.0
	c	- 1.0
zero	a	1.0
	b	1.0
	c	1.0

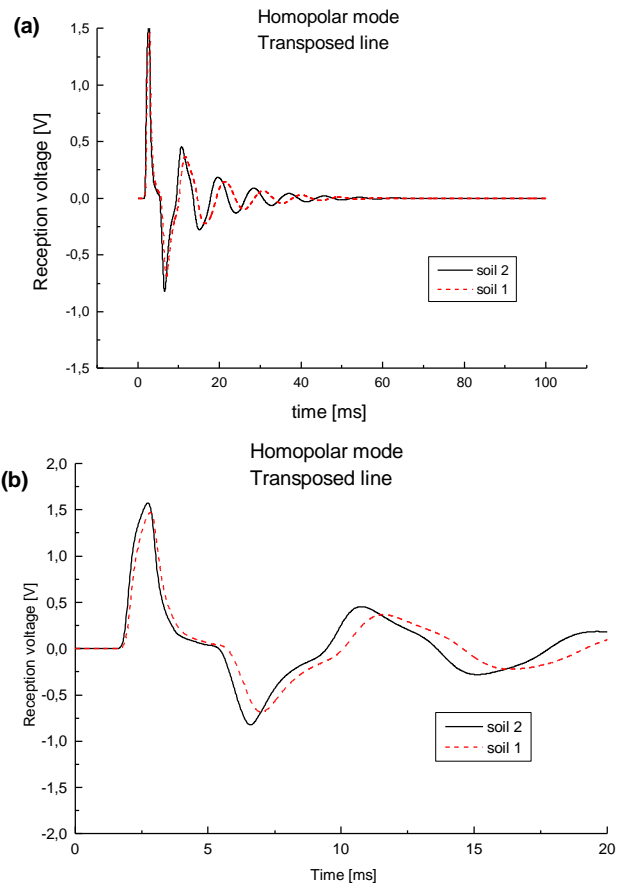


Figure 9 - Step response for mode homopolar - Transposed line - (a) – complete simulation; (b) – 20 ms simulation detail.

V. CONCLUSIONS

We have presented some basic aspects of soil modeling, and shown that :

- It is essential to choose a soil model that satisfies the physical coherence conditions in what concerns the relations between the conductivity (σ) and the permittivity (ϵ) in the frequency domain.
- The soil behavior is, typically, of minimum phase shift type.
- The usual assumptions about ground electric permittivity, e. g. of the order of 10 to 30 times vacuum electric permittivity, and frequency

independent, is too far from reality, for most soils.

- Soil parameters σ and ϵ are strongly frequency dependent. For most electrical engineering applications, $s + i\omega e$ may be considered the sum of two statistically independent parcels, one constant, and the other frequency dependent. Such two parcels are associated to distinct physical aspects.
- The frequency dependent parcel has a statistical dispersion, among different soil types and conditions, much lower than the constant parcel.
- The frequency dependent parcel may be defined by two parameters, which may be considered statistically independent.
- There are important differences, sometimes of order of magnitude, among the induced voltages, electric field in ground, transferred voltages, according to the soil models, till among models with similar behavior at low frequency, and due to aspects usually not considered, as it is the case of soil parameters' dependence with frequency.

So, it is essential, for most applications concerning grounding systems, or involving electromagnetic phenomena affected by ground, to model adequately the ground behavior, including several aspects not considered in common practice.

For transmission lines, according to specific conditions, and the phenomena being studied, it may be quite important to model correctly the soil, considering frequency dependence of $s + i\omega e$.

We have presented some illustrative results for a 440 kV three-phase transmission line. The soil behavior is represented through two alternative soil models. In the first soil model we have considered an accurate soil representation, satisfying coherence conditions and with $s + i\omega e$ frequency dependent. In the second soil model, we have considered a constant, frequency independent, conductance and $\omega\epsilon$ much lower than σ .

In some cases, an adequate earth model can lead to results quite different from those obtained with the usual procedure of considering the parameter σ of soil frequency independent and parameter ϵ frequency independent with a relatively small value. The conditions in which such difference can be important include the following examples :

- Switching conditions in which an important homopolar component may occur, either due to spread of switching of the three poles, or to fault conditions, and in which the important frequency spectrum is not restricted to extremely low frequencies (< 1 kHz), and includes frequencies till about 10 kHz.
- Network sustained operation, faults and maneuvers in which it occurs conditions near resonance, for the homopolar component, for frequencies not restricted to extremely low frequencies (< 1 kHz), and, e. g. , for frequency between 1 and 10 kHz.
- Fast transients, for which important frequency range may include frequencies above 10 kHz. In this case, the difference between an accurate soil model and usual assumptions may be important also for non-

homopolar modes. Typical cases in which frequency range above 10 kHz is important are: transients originated by lightning; front of wave aspects of transients associated with short-circuits, which may be quite important in what concerns insulation coordination.

VI. ACKNOWLEDGEMENT

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