Analysis of a Transmission Line Model using an Equivalent Impedance Test Concept

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Abstract - **The objective of this work is to define an equivalent impedance test of the "exact" modes of transmission lines and to apply it to a line model written in the modal domain. Initially, the equivalent impedance test concept will be presented. Then, we will calculate the equivalent impedance of the "Quasi-modes" of the transmission line represented by p-circuits. The equivalent impedance test will be applied to two transmission lines, with different lengths, represented by p-circuits. The comparison between the "exact" result and the "Quasi-Modes" model will be analyzed in the frequency domain**.

Keywords : Transmission Line Models, Frequency Domain, Modal Domain, Equivalent Impedance.

I. INTRODUCTION

In one of his works, Budner developed a transmission line model taking as a base the fundamental equations of the line decomposed in its modes. With a base in the voltage and current equations of the line, the equations were written in such a way that the same equations represented a two-port network whose parameters were the admittance parameters. Using the Fourier inverse transformation Budner presented, in time domain, the currents and voltages in line modes. The conversion from the frequency domain to the time domain, of the effect of elements of admittance matrices, was made with the use of convolution integrals. The parameters which evaluate the effect of elements of admittance matrices, in time domain, in a convolution sense were denominated "weight functions" [1].

In this work we will write the "exact" modal equations of a transmission line and obtain the equivalent impedance tests of each mode. For each mode two equivalent impedance tests will be calculated : the impedance calculated at one line terminal, with the other terminal opened, and the impedance calculated at one line terminal, with the other terminal short-circuited.

The equivalent impedance test of the "exact" modes will be compared with the equivalent impedance test obtained with the line represented through a transmission line model, the Quasi-Modes Model.

We will carry out comparative tests with a model which use π circuits. The equivalent impedance test of the "exact" modes will be compared with the equivalent impedance test of the "Quasi-modes" of the transmission line represented through π-circuits (each π representing 10 km of transmission line). Examples of a 500 km line and a 100 km line will be presented.

II.THE EQUIVALENT IMPEDANCE TEST OF EACH TRANSMISSION LINE MODE

Consider a multiphase transmission line. It is well known that the n-phase equations of a transmission line with n-phases are solved by transforming the n-coupled equations into the n-uncoupled equations [1,2,3,4,5,6].

Manipulating the n-uncoupled voltage and current equations, we can obtain two equivalent impedance tests, defined as the impedances calculated at one terminal, with the other terminal either with zero current (opened) or zero voltage (short-circuited). Therefore, for a k mode, we can write:

$$Z_{open(k)} = \sqrt{\frac{z_k}{y_k}}\text{cotanh}(\sqrt{z_k y_k}\, l) \qquad (1)$$

$$Z_{close(k)} = \sqrt{\frac{z_k}{y_k}}\tanh(\sqrt{z_k y_k}\, l) \qquad (2)$$

In the equations shown previously, z_k is the per unit length longitudinal impedance of the k mode and y_k is the per unit length transversal admittance of the k mode. $Z_{open(k)}$ and $Z_{close(k)}$ are the two equivalent impedance test, defined as the impedances calculated at one terminal, with the other terminal either with zero current (opened) or zero voltage (short-circuited).

III. THE EQUIVALENT IMPEDANCE TEST OF EACH TRANSMISSION LINE MODE REPRESENTED THROUGH π-CIRCUITS

It is verified that in several works [1,2,3,7] π-circuits are used to represent each one of the line modes, as shown in Fig. 1.

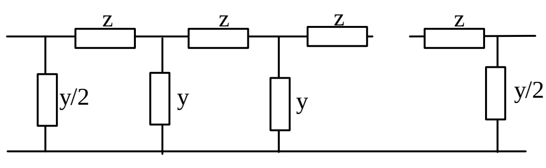

Fig. 1. Mode *m* represented through π-circuits model

Fig. 1 shows the m-mode of transmission line, with length *l*, which is represented by n π-circuits. Therefore, in Fig. 1, we have:

$$z = z_m(l/n) \qquad (3)$$

$$y = y_m(l/n) \qquad (4)$$

Where:

z_m is the per unit length longitudinal impedance of the m-mode

y_m is the per unit length transversal admittance of the m-mode

l is length line

n is number of π-circuits

In several conditions, namely if the transposition procedure allows to assume, with a small acceptable error, a behavior equivalent to an ideally transposed line, the Clarke transformation separate (with a small error) the "exact" line modes [7, 8, 9]. In other conditions, if an ideal line transposition cannot be assumed, the Clarke transformation leads to "Quasi-modes", that can be treated like modes, with an acceptable small error. The Clarke transformation is real, frequency independent, and can be easily simulated in time domain (what does not occur, in general, with "exact" modes). Within this hypothesis (exact or approximated) the 1, 2 and 3 modes can be considered to be α, β and Zero components of Clarke Transformation. When we use Clarke transformation, Clarke matrix is used as the unique transformation matrix for the entire frequency range. This procedure is useful to represent line modes in software, for instance EMTP, that simulate electromagnetic transitory phenomena in electric power systems in time domain [7, 8, 9].

In the "Quasi-modes" represented through π-circuits we can calculate two equivalent impedance tests, defined as the impedances calculated in one terminal, with the other terminal either with zero current (opened) or zero voltage (short-circuited).

IV. RESULTS OBTAINED

We will show the equivalent impedance test of the 1 "exact" mode of the transmission line and the equivalent impedance test of the α "Quasi-mode" represented through π-circuits. These comparisons will be made in two lines (500 km and 100 km) represented through 1 π-circuit for each 10 km. We will consider the transmission line in two situations : line with one of the ends opened and line with one of the ends in short-circuit.

Fig. 2 shows a hypothetical three-phase transmission line, without transposition, used to illustrate the application of an equivalent impedance test concept. With an acceptable small error, we can consider that the 1 "exact" mode corresponds to the α "Quasi-mode".

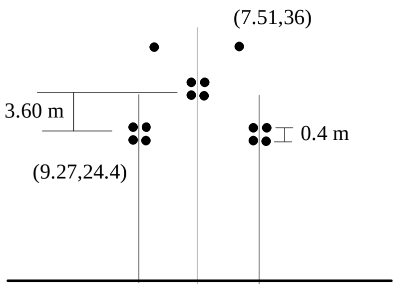

Fig. 2. Three-phase transmission line

In the transmission line shown in Fig. 2, we have:

- phase conductors: grosbeak
- ground wires: EHSW – 3/8"
- earth resistivity: 1000 Ω.m
- line length: 500 km and 100 km

The results are numerical and are numerically evaluated by MATLAB and a FORTRAN program.

A. Line, with 500 km, with one of the ends open

Figs. 3-8 show the comparison between the equivalent impedance test of the 1 "exact" mode and the equivalent impedance test of the α "Quasi-mode" calculated at one terminal, with the other terminal also with zero current, represented through 50 π-circuits (line with 500 km), for several frequency ranges.

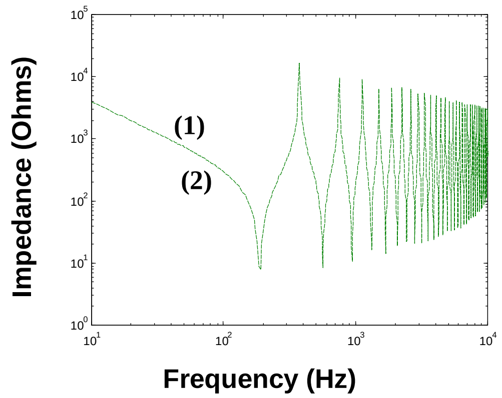

Fig. 3. Absolute value of $Z_{open(1)}$ (1) and of $Z_{open\pi(\alpha)}$ (2)

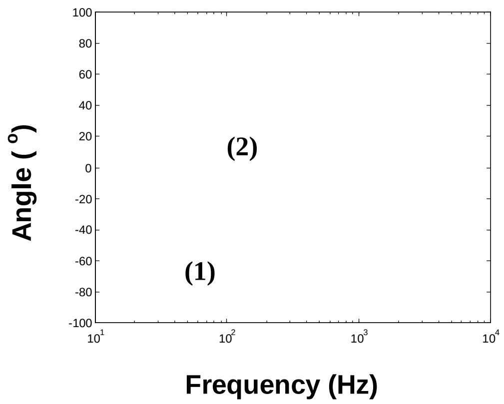

Fig. 4. Angle of $Z_{open(1)}$ (1) and of $Z_{open\pi(\alpha)}$ (2)

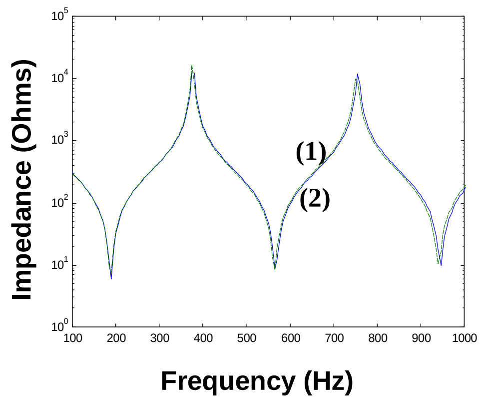

Fig. 5. Absolute value of $Z_{open(1)}$ (1) and of $Z_{open\pi(\alpha)}$ (2)

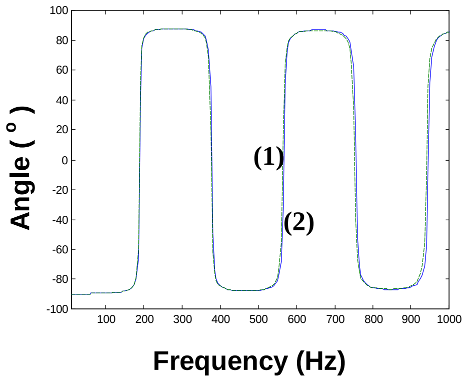

Fig. 6. Angle of $Z_{open(1)}$ (1) and of $Z_{open\pi(\alpha)}$ (2)

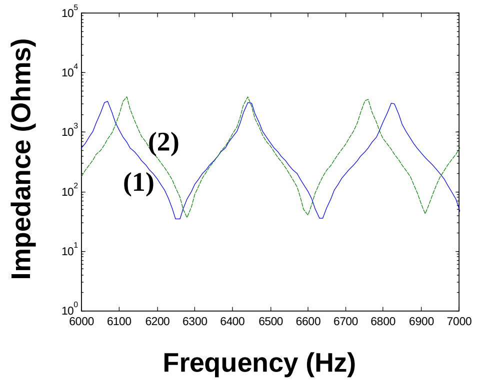

Fig. 7. Absolute value of $Z_{open(1)}$ (1) and of $Z_{open\pi(\alpha)}$ (2)

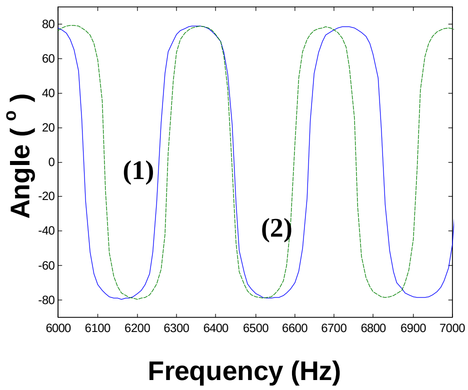

Fig. 8. Angle of $Z_{open(1)}$ (1) and of $Z_{open\pi(\alpha)}$ (2)

Figs 5 and 6 show that at low frequencies the equivalent impedance test of the α "Quasi-mode" represented through 50 π-circuits is similar to the equivalent impedance test of 1 "exact" mode.

Figs. 7 and 8 show that at intermediary frequencies the equivalent impedance test of α "Quasi-mode" represented through 50 π-circuits is similar to the equivalent impedance test of the 1 "exact" mode, but presents a phase shift. This phase shift is associated to a difference in phase velocity, in wave length and in propagation delay along the line (interpreted in correlation with phase, group and signal velocities, and with frequency-time transformation behavior).

B. Line, with 100 km, with one terminal opened

Figs. 9-14 show the comparison between the equivalent impedance test of the 1 "exact" mode and the equivalent impedance test of the α "Quasi-mode" calculated at one terminal, with the other terminal also with zero current, represented through 10 π-circuits (line with 100 km), for several frequency ranges.

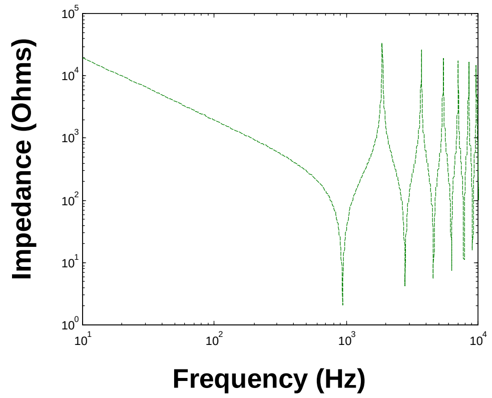

Fig. 9. Absolute Value of $Z_{open(1)}$ (1) and of $Z_{open\pi(\alpha)}$ (2)

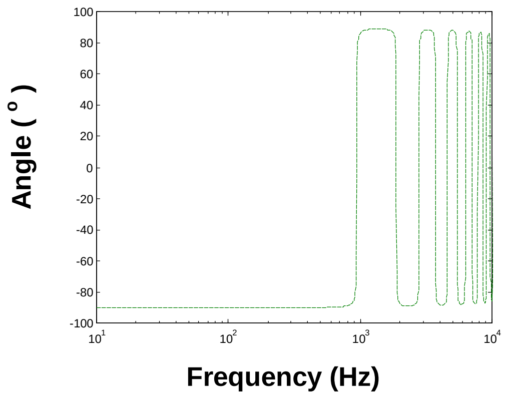

Fig. 10. Angle of $Z_{open(1)}$ (1) and of $Z_{open\pi(\alpha)}$ (2)

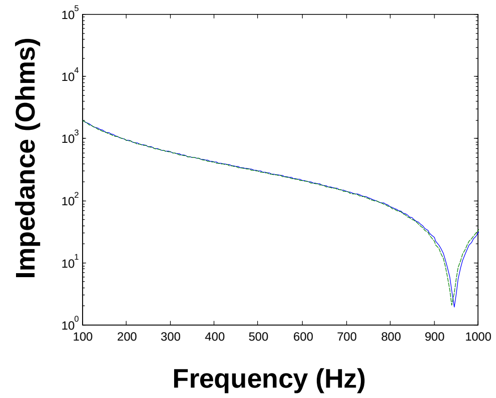

Fig. 11. Absolute Value of $Z_{open(1)}$ (1) and of $Z_{open\pi(\alpha)}$ (2)

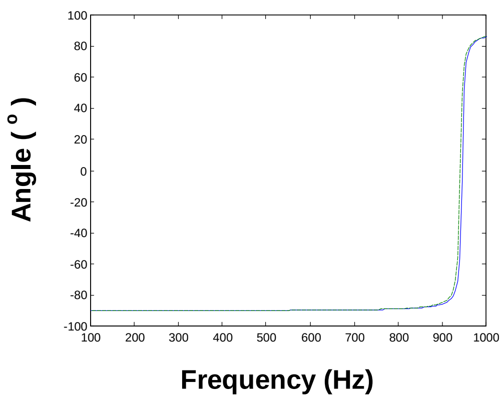

Fig. 12. Angle of $Z_{open(1)}$ (1) and of $Z_{open\pi(\alpha)}$ (2)

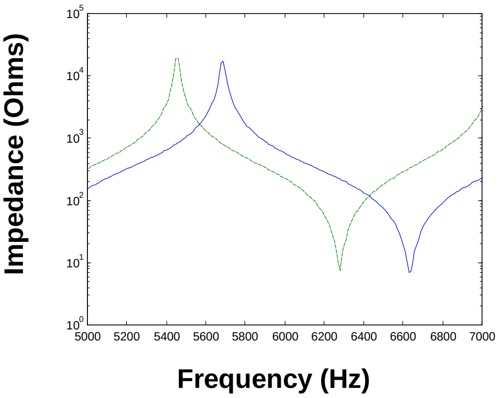

Fig. 13. Absolute value of $Z_{open(1)}$ (1) and of $Z_{open\pi(\alpha)}$ (2)

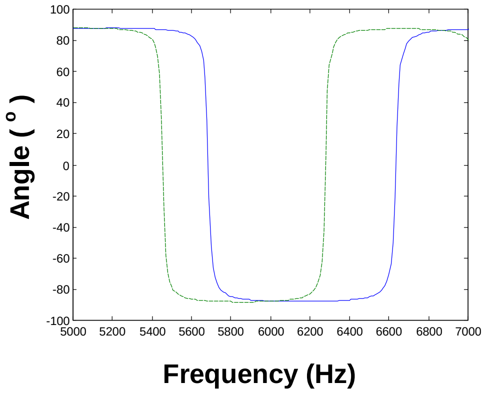

Fig. 14. Angle of $Z_{open(1)}$ (1) and of $Z_{open\pi(\alpha)}$ (2)

C. Line, with 500 km, with one of the ends in short circuit

Figs. 15-20 show the comparison between the equivalent impedance test of the 1 "exact" mode and the equivalent impedance test of the α "Quasi-mode" calculated at one terminal, with the other terminal also with zero voltage, represented through 50 π-circuits (line with 500 km), for several frequency ranges.

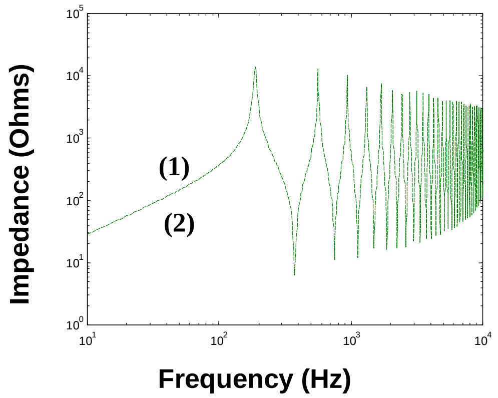

Fig. 15. Absolute value of $Z_{close(1)}$ (1) and of the $Z_{close\pi(\alpha)}$ (2)

Figs. 17 and 18 show that at low frequencies the equivalent impedance test of the α "Quasi-mode" represented through 50 π-circuits is similar to the equivalent impedance test of the 1 "exact" mode.

Figs. 19 and 20 show that in intermediary frequencies the equivalent impedance test of α "Quasi-mode" represented through 50 π-circuits is similar to the equivalent impedance test of the 1 "exact" mode, but presents a phase shift. This phase shift is associated to a difference in phase velocity, in wave length and in propagation delay along the line (interpreted as indicated above).

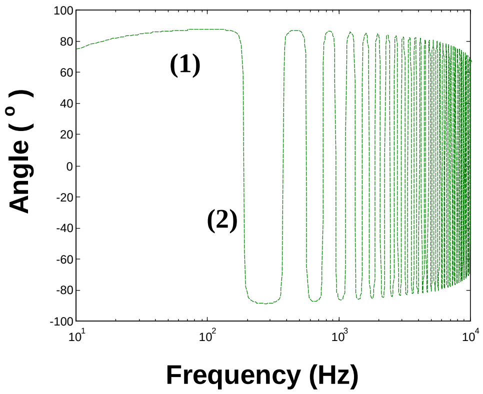

Fig. 16. Angle of $Z_{close(1)}$ (1) and of $Z_{close\pi(\alpha)}$ (2)

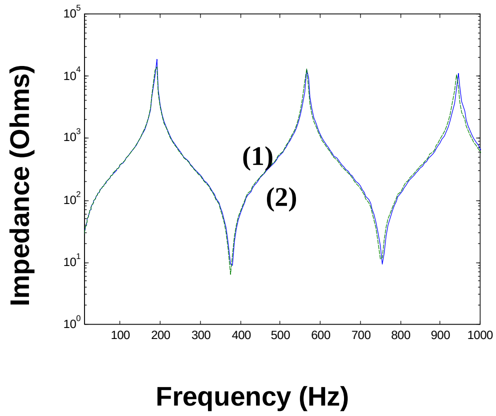

Fig. 17. Absolute value of $Z_{close(1)}$ (1) and of $Z_{close\pi(\alpha)}$ (2)

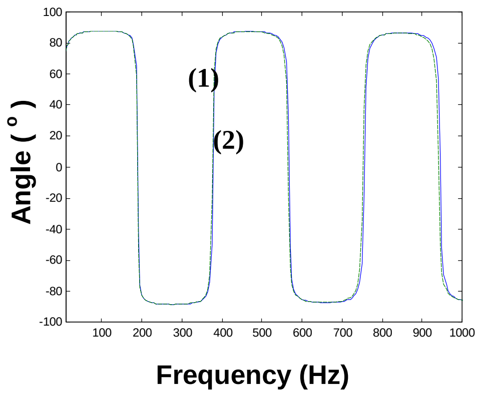

Fig. 18. Angle of the $Z_{close(1)}$ (1) and of the $Z_{close\pi(\alpha)}$ (2)

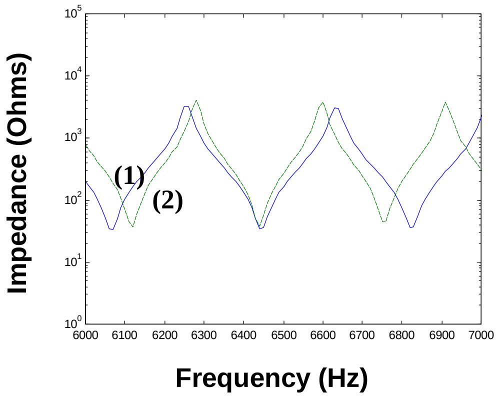

Fig. 19. Absolute value of $Z_{close(1)}$ (1) and of $Z_{close\pi(\alpha)}$ (2)

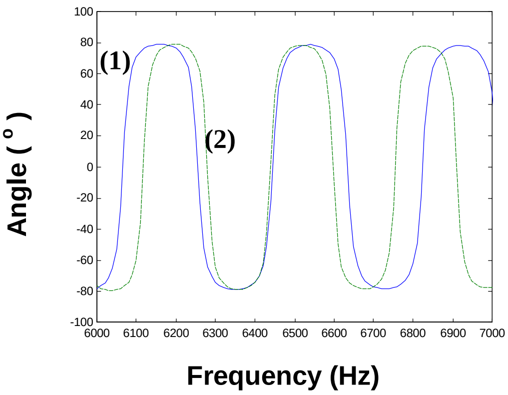

Fig. 20. Angle of $Z_{close(1)}$ (1) and of $Z_{close\pi(\alpha)}$ (2)

D. Line, with 100 km, with one of the ends in short circuit

Figs. 21-26 show the comparison between the equivalent impedance test of the 1 “exact” mode and the equivalent impedance test of the α “Quasi-mode” calculated at one terminal, with the other terminal also with zero voltage, represented through 10 π-circuits (line with 500 km), for several frequency ranges.

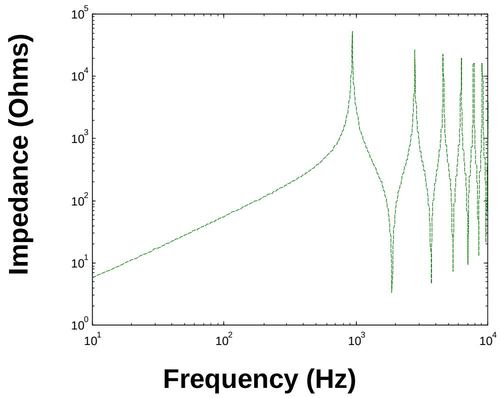

Fig. 21. Impedance of $Z_{close(1)}$ (1) and of $Z_{close\pi(\alpha)}$ (2)

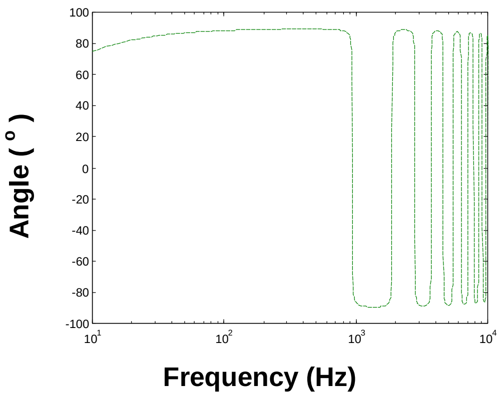

Fig. 22. Angle of $Z_{close(1)}$ (1) and of $Z_{close\pi(\alpha)}$ (2)

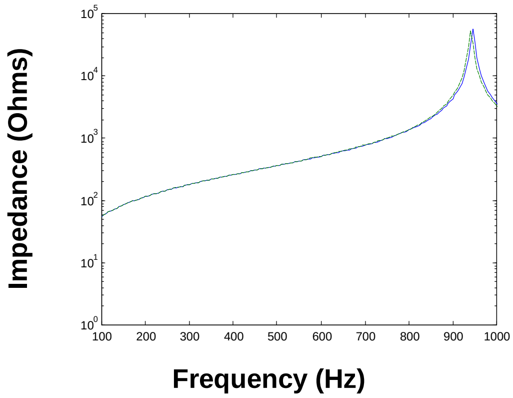

Fig. 23. Impedance of $Z_{close(1)}$ (1) and of $Z_{close\pi(\alpha)}$ (2)

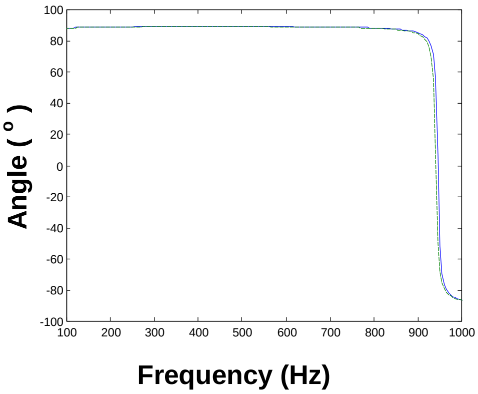

Fig. 24. Angle of $Z_{close(1)}$ (1) and of $Z_{close\pi(\alpha)}$ (2)

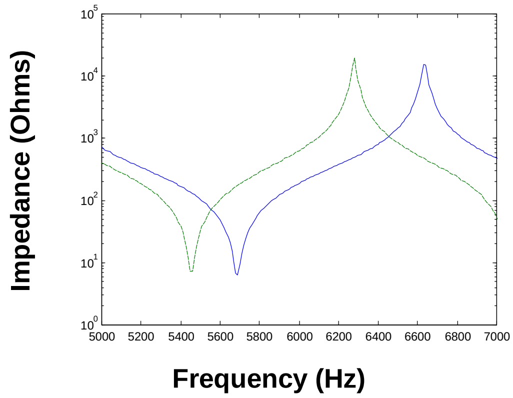

Fig. 25. Impedance of $Z_{close(1)}$ (1) and of $Z_{close\pi(\alpha)}$ (2)

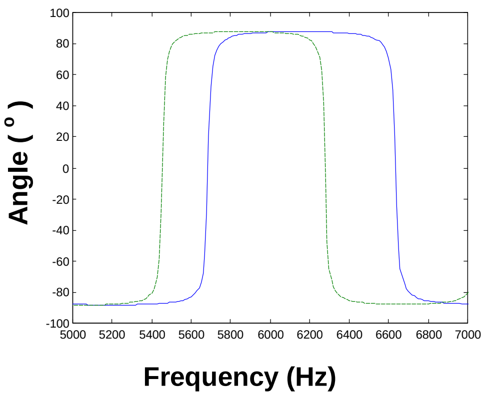

Fig. 26. Angle of $Z_{close(1)}$ (1) and of $Z_{close\pi(\alpha)}$ (2)

All results presented show a phase shift between the equivalent impedance test of the 1 “exact” mode and the equivalent impedance test of α “Quasi-mode”. This phase shift is caused, mainly, by use of the π-circuits to represent the hyperbolic equations. The use of the π-circuits causes a phase shift in the 1 and 2 modes. The use of the Clarke transformation matrix instead of the eigenvectors matrix does not cause a significant phase shift in the modes. The Clarke matrix causes a small phase shift for the 1 mode [10].

V. CONCLUSIONS

The Equivalent Impedance Test concept can be applied to analyze the transmission line characteristics and, as an extension, to analyze the performance of a transmission line model.

Using the Equivalent Impedance Test Concept to analyze the model behavior of the transmission line modeled through the "Quasi-Modes" and represented by a cascade of π-circuits, the following conclusion can be taken, analyzing α "Quasi-mode":

- A transmission line can be represented through a cascade of π-circuit, if the number of π-circuits is adequate for the line length. The maximum line length represented by each π-circuit is defined by the frequency range for which it is desired to have an accurate modeling, and, in some extent, of required accuracy. Typically, for the shape of frequency spectrum of most common switching transients, the important spectrum is about 0-5 kHz. To have reasonable accuracy, it can be assumed that for a higher frequency of a dominant spectrum the electric line length represented by a π does not exceed about one radian. This condition gives about 10 km for 5 kHz, for non-homopolar modes, and a little more for homopolar modes (the applicable limit depends, mainly, of soil parameters, and also, of line arrangement).

- When the π–circuit is used to represent every 10 km of the line, it can be observed that for frequencies below 1 kHz the "Quasi-Mode" impedance α behavior is practically identical to the "exact" mode 1 behavior. For frequencies higher than 1 kHz, a phase shift appears, which can be verified through the two Equivalent Impedances Tests. This phase spread is mainly due to the use of the π–circuit cascade instead of the hyperbolic line equations [10].

VI. ACKNOWLEDGEMENT

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