A NEW MODEL OF DOUBLE THREE-PHASE TRANSMISSION LINE INCLUDING FREQUENCY DEPENDENCE - A TRANSIENT STUDY

A. J. Prado * — afonsojp@uol.com.br
J. Pissolato Filho — pisso@dsce.fee.unicamp.br
UNICAMP – Universidade Estadual de Campinas, Campinas, SP

M. C. Tavares — cristina@sel.eesc.sc.usp.br
Universidade de São Paulo, São Carlos, SP

C. M. Portela — portelac@ism.com.br
COPPE – Universidade Federal do Rio de Janeiro, Rio de Janeiro, RJ

BRAZIL

*Rua Lopes Fragoso, 68; Valparaíso - SP; zip code: 16880-000; phone: +17 2231405 ; fax: +19 37883860 **(author contact)**

Abstract - **This article presents a model to represent double three-phase transmission lines, including the frequency dependence of longitudinal parameters in mode domain. For the typically important frequency range of switching electromagnetic transients, from 10 Hz to 10 kHz, the proposed model uses a single real phase-mode transformation matrix. The model includes the frequency dependence of line parameters in digital programs, such as ATP, EMTP, EMTDC and MICROTRAN, using ideal transformers. This model is applied for three types of line transposition and non-transposed line, called the Quasi-Mode transmission line model. It is employed for an actual double three-phase transmission line in Microtran program. Some transient simulations are performed, such as energization and fault analysis.**

Keywords: frequency dependence, double three-phase transmission line, mode domain, real transformation, Microtran.

I - INTRODUCTION

Most digital simulator programs of the electromagnetic transients work in time domain ([1], [3]). This choice facilitates the simulation of conditions easily expressed in time domain, e. g. switching of a circuit breaker, and, also, of some non-linear elements. However, one of the great difficulties in studying electromagnetic transients is the correct representation of the transmission lines. In general, the transmission lines can not be expressed directly in time domain, because the longitudinal parameters of transmission lines depend on the frequency ([8]). This fact creates one full impedance matrix for each frequency value. If this representation uses a modal transformation, the longitudinal parameters are modeled in mode domain. In this case, the mode impedance matrices are diagonal and frequency dependence is more easily introduced.

The proposed model presents a real single transformation matrix for transmission line electromagnetic transients analysis. For this, the transformation matrix uses geometrical properties of the line and is associated with Clarke transformation. The geometrical properties are introduced by sum and difference of pairs of phase currents or transversal phase voltage, called media-antimedia transformation matrix. This transformation matrix creates two uncoupled single three-phase circuits, when it is applied to the double three-phase transmission line. Afterwards, the Clarke transformation matrix is applied to produce the mode or Quasi-mode impedance matrices ([2], [5], [6], [7], [9]).

In this paper, three types of line transposition are considered. Ideally, a transposition section has a small value which permits the use of a medium impedance, or admittance, value for a cycle of transposition. The three types of transposition line analyzed are : the complete transposition (ideal case), the rotational transposition (similar to a six-phase line case), the operational transposition (similar to some practical cases). Some applications of this proposed model are presented, using Microtran program.

II - LINE LONGITUDINAL IMPEDANCE MATRIX, Z, AND TRANSVERSAL ADMITTANCE MATRIX, Y, PER UNIT LENGTH

The generic structure of double three-phase transmission line, with a symmetry vertical plane, is shown in Figure 1.

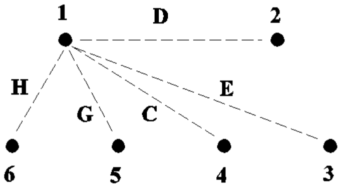

Figure 1 – Schematic representation of mutual elements in matrices Z and Y.

The ground wires are assumed implicit in matrices referred to phases. The phase longitudinal impedance matrix, per unit length, has the form below. Similar basic shape applies, also, to matrix Y.

$$Z = \begin{vmatrix} A & D & E & C & G & H \\ D & A & H & G & C & E \\ E & H & B & J & L & M \\ C & G & J & I & N & L \\ G & C & L & N & I & J \\ H & E & M & L & J & B \end{vmatrix} \qquad (1)$$

With the modal transformation, this impedance matrix is changed into a diagonal matrix (similar transformation applies to Y). The mode impedance matrix is:

$$Z_{md} = T_{FM} * Z * T_{FM}^{-1} \qquad (2)$$

The transformation (T_{FM}) can be considered the multiplication of two transformation matrices. One is based on the line geometrical properties, called media-antimedia transformation. This transformation separates the circuits of double three-phase line in two uncoupled "lines". Each of these two uncoupled "lines" can be treated in mode domain, with routine procedures applicable to three-phase lines ([2], [5], [6]). With an eventual small error, instead of an exact mode manipulation of each of these two "lines", Quasi-modes can be considered. One way of considering the mode transformation is to consider Clarke transformation. It has the important advantage of being real and frequency independent, and, in most cases, introduces quite small errors, if Quasi-mode are treated as exact modes. Some details of these transformation are described in sections III and IV.

III - MEDIA-ANTIMEDIA TRANSFORMATION

The media-antimedia transformation is a linear transformation where there are no mathematical approximations. It depends on the vertical symmetrical plane of the line. The main aim of this transformation is to uncouple the two circuits of a double three-phase line. This transformation uses the geometrical properties of the line and it is obtained from sum and difference of pairs of phase currents or transversal phase voltage ([1], [2], [5], [6]). Figure 2 shows the symmetrical axis for double three-phase lines.

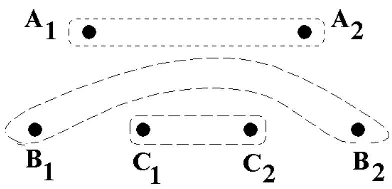

Figure 2 – Symmetrical axis for double three-phase lines.

The media-antimedia transformation matrix is:

$$T_{ma} = \begin{vmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\ 0 & 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ 0 & 0 & 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{vmatrix} \qquad (3)$$

The T_{ma} matrix creates a new impedance matrix: the media-antimedia impedance matrix (Z_{ma}).

$$Z_{ma} = T_{ma} * Z * T_{ma}^{-1}$$

The media-antimedia is:

$$Z_{ma} = \begin{vmatrix} Zm & 0 \\ 0 & Za \end{vmatrix} \qquad (4)$$

This matrix can be used to separate double three order matrices, such as the media impedance matrix (Z_m) and the antimedia impedance matrix (Z_a).

The Z_m matrix is:

$$Zm = \begin{vmatrix} A + D & E + H & G + C \\ E + H & B + M & L + J \\ G + C & L + J & I + N \end{vmatrix} \qquad (5)$$

The Z_a matrix is:

$$Za = \begin{vmatrix} A - D & E - H & G - C \\ E - H & B - M & L - J \\ G - C & L - J & I - N \end{vmatrix} \qquad (6)$$

These two matrices (and, in a similar way, two corresponding Y matrices) correspond to two uncoupled three-phase "lines", as indicated above. Similar transformations apply, also, to the transversal impedance matrix, per unit length.

IV - CLARKE TRANSFORMATION

With the approach mentioned above, Clarke transformation can be applied to each of the two uncoupled three-phase "lines". The Clarke transformation uses linear combinations among the elements of the media and antimedia matrices and this transformation substitutes the exact mode transformation ([2], [5], [6], [7], [8], [9]).

The Clarke matrix is:

$$T_{Cl} = \begin{vmatrix} -1/\sqrt{6} & 2/\sqrt{6} & -1/\sqrt{6} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{vmatrix} \qquad (7)$$

Each three-phase line created by the T_{ma} transformation matrix will be used to obtain a modal impedance matrix. However, the double three-phase line is represented by a six-order impedance matrix, and it is necessary to generate a Clarke Transformation matrix of order six. This six order transformation matrix (T_{Cl6}) is used to obtain the real transformation matrix (T_{FM}). The sixth order Clarke matrix is:

$$T_{Cl6} = \begin{vmatrix} T_{Cl} & 0 \\ 0 & T_{Cl} \end{vmatrix} \qquad (8)$$

The single real modal transformation is analyzed in the next section.

V - TRANSFORMATION MATRIX

The multiplication between the media-antimedia matrix (T_{ma}) and the six order Clarke matrix (T_{Cl6}) creates the transformation matrix (T_{FM}). Similar procedures apply to matrix Y. The T_{FM} matrix is:

$$T_{FM} = \begin{vmatrix} -1/2\sqrt{3} & -1/2\sqrt{3} & 1/\sqrt{3} & -1/2\sqrt{3} & -1/2\sqrt{3} & 1/\sqrt{3} \\ 1/2 & 1/2 & 0 & -1/2 & -1/2 & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & 1/\sqrt{6} & 1/\sqrt{6} & 1/\sqrt{6} & 1/\sqrt{6} \\ -1/2\sqrt{3} & 1/2\sqrt{3} & 1/\sqrt{3} & 1/2\sqrt{3} & -1/2\sqrt{3} & -1/\sqrt{3} \\ 1/2 & -1/2 & 0 & 1/2 & -1/2 & 0 \\ 1/\sqrt{6} & -1/\sqrt{6} & 1/\sqrt{6} & -1/\sqrt{6} & 1/\sqrt{6} & -1/\sqrt{6} \end{vmatrix} \qquad (9)$$

This transformation matrix creates two impedance matrices which are three order matrices, called the media matrix and antimedia matrix. Each of these two groups of three order matrices is associated with a group of three modes. With the approximation indicated above, instead of exact modes, in each group, Quasi-modes can be obtained, using Clarke transformation. In some conditions, namely with some transposition assumptions, such modes are exact. The conversion of phase magnitudes in Quasi-modes (or eventually exact modes), according to transformation defined above, can be done with ideal transformers, and, so, can be included in a common transient simulation programs, working in time domain ([2], [7], [9]).

Figure 3 shows the ideal transformers for antimedia α mode. The transformation matrix (T_{FM}) can be modeled in a time-domain program like

MICROTRAN through ideal transformers.

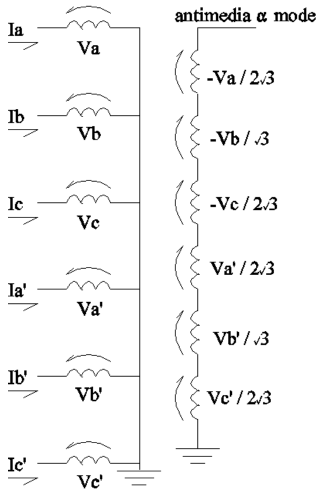

Figure 3 – Transformation for antimedia α mode.

VI - REAL CASE AND LINE TRANSPOSITIONS

A real case, which is used in this article, is a real double three-phase transmission line. The line voltage (RMS) is 440 kV and the line length is 160 km. The line structure is shown in Figure 4.

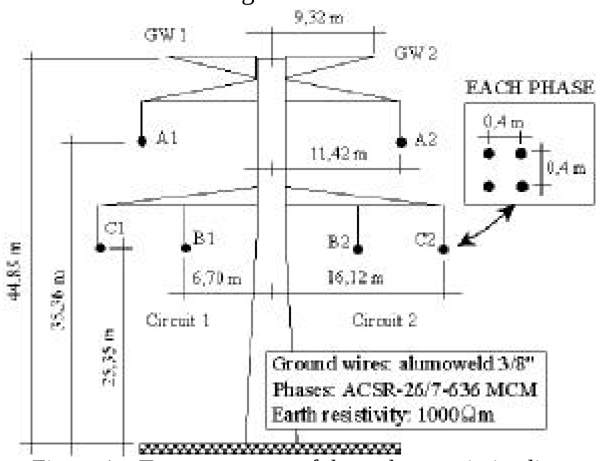

Figure 4 – Tower structure of the real transmission line.

In the assumption of an ideal transposition of phases, in the sense that a transposition cycle length is reasonably shorter than a quarter wave length, in the dominant frequency range of phenomena to study, the matrices Z and Y, averaged in a transposition cycle, have, in general, several additional symmetry properties, which depend on the type of transposition used in the line. With such assumption, with some types of transposition, the use of Clarke transformation, in conjunction with media-antimedia transformation (including some eventual variants) leads to exact modes. Such property is related, in general, to some degeneracy conditions, in which there are several coincident eigenvalues, and, so, some freedom in choosing some eigenvectors (any linear combination of eigenvectors with the same eigenvalue is, also, an eigenvector with the same eigenvalue). Although the assumption of ideal transposition is not deductively justified for a large frequency spectrum, in usual conditions, the assumption of ideal transposition leads to reasonably accurate results for most common transient studies.

A) Complete transposition

For this transposition, each phase of both circuits occupies all phase positions on the tower. All diagonal elements of each of matrices Z and Y are equal, and all non-diagonal elements of each of matrices Z and Y are equal. The mode media matrix is:

$$Z_{m\alpha\beta0} = \begin{vmatrix} A-D & 0 & 0 \\ 0 & A-D & 0 \\ 0 & 0 & A+5D \end{vmatrix} \qquad (10)$$

The mode antimedia matrix is:

$$Z_{a\alpha\beta0} = \begin{vmatrix} A-D & 0 & 0 \\ 0 & A-D & 0 \\ 0 & 0 & A-D \end{vmatrix} \qquad (11)$$

In the mode impedance matrices, there are six exact modes: five with the same eigen-value and one different mode. The different mode is the media homopolar mode (m0).

B) Rotational transposition

The rotational transposition is what should be the recommended transposition for six-phase transmission line. Each phase moves to the adjacent phase position without changes on the relative positions among the phases. Figure 5 shows coupling impedances for this case.

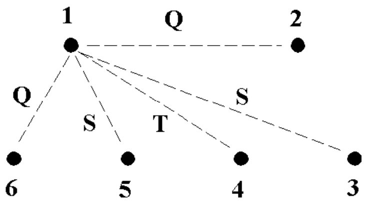

Figure 5 – Coupling impedances for rotational transposition. (average values for one phase)

If the phase impedance value is represented by A (average value), the mode media matrix will be:

$$Z_{m\alpha\beta0} = \begin{vmatrix} A-Q-S+T & 0 & 0 \\ 0 & A+Q-S-T & 0 \\ 0 & 0 & A+2Q+2S+T \end{vmatrix} \qquad (12)$$

The mode antimedia matrix is:

$$Z_{a\alpha\beta0} = \begin{vmatrix} A+Q-S-T & 0 & 0 \\ 0 & A-Q-S+T & 0 \\ 0 & 0 & A-2Q+2S-T \end{vmatrix} \qquad (13)$$

There are two sets of two modes with the same eigen-value: by one side, the mα mode and the aβ mode, by other side the mβ mode and the aα.

C) Operational transposition

For this type of line transposition, each three-phase circuit is ideally transposed, creating, for typical frequency values, only one coupling impedance within a circuit and only one coupling impedance among the circuits. Figure 6 shows these coupling impedances and the generic structure of this transposition type. The average phase impedance value is represented by A . The coupling impedances are represented by R (coupling impedance within a circuit) and P (coupling circuit among the circuits).

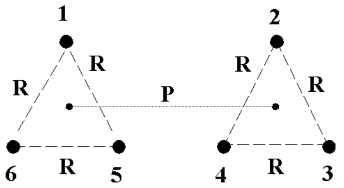

Figure 6 – Coupling impedances for operational transposition.

The media mode matrix is:

$$Z_{m\alpha\beta0} = \begin{vmatrix} A-R & 0 & 0 \\ 0 & A-R & 0 \\ 0 & 0 & A+3P+2R \end{vmatrix} \quad (14)$$

The antimedia mode matrix is:

$$Z_{a\alpha\beta0} = \begin{vmatrix} (3A-8P+5R)/3 & 0 & 4(P-R)/3\sqrt{2} \\ 0 & A-R & 0 \\ 4(P-R)/3\sqrt{2} & 0 & (3^{A}-P-2R)/3 \end{vmatrix} \quad (15)$$

Three modes have the same eigen-value ($m\alpha$, $m\beta$ and $a\beta$) and three modes are individualized. There is a coupling impedance between the $a\alpha$ Quasi-mode and the a0 Quasi-mode and these are not exact modes. If the coupling impedance is not considered, the result will be a diagonal matrix. When the proposed model is applied to the operational transposition, it is called the Quasi-mode transmission line model, because it does not obtain exact modes.

D) Non-transposed line

For non-transposed line, the coupling impedances have different values. This case can be represented by Figure 1. The equations (5) and (6) represent media and antimedia matrices for this case. If the Clarke transformation is applied to these matrices, the new impedance matrices will be non-diagonal ones. By using a simple representation, the new media matrix is:

$$Z_{m\alpha\beta0} = \begin{vmatrix} m\alpha & m\alpha\beta & m\alpha0 \\ m\alpha\beta & m\beta & m\beta0 \\ m\alpha0 & m\beta0 & m0 \end{vmatrix} \quad (16)$$

The new antimedia matrix is:

$$Z_{a\alpha\beta0} = \begin{vmatrix} A\alpha & a\alpha\beta & a\alpha0 \\ a\alpha\beta & a\beta & a\beta0 \\ a\alpha0 & a\beta0 & a0 \end{vmatrix} \quad (17)$$

The new impedance matrices are symmetrical and non-diagonal matrices. There are coupling impedances among the Quasi-modes.

For the non-transposed line, the considered Quasi-modes are not exact modes. However, the error of treating Quasi-modes as exact modes may be acceptable in most typical applications ([2], [5], [6]).

VII - SYNTHETIC CIRCUITS

The Quasi-mode model transformed the double three-phase line into six uncoupled mode circuits (with an eventual error when Quasi-modes are not exact modes). This transformation is applied through ideal transformers in programs such as EMTP, ATP, EMTDC and MICROTRAN. For this, the six uncoupled mode lines are represented by synthetic circuits. These synthetic circuits are modified π-circuits which have the following structure:

a) RL series circuit, for low frequencies;

b) RL parallel circuits, for frequency dependence of longitudinal parameters when it added with RL series circuit;

c) two electrical branches (C/2), for transversal parameters.

For each mode, the number of RL parallel circuits depends on the analyzed frequency range. The number of these circuits depend on the desirable accuracy too. These elements are obtained, e.g., considering chosen frequency intervals and, in each one, taking as basis geometric mean frequency. Figure 7 shows one π-circuit unit. The maximum line length represented correctly by this π-circuit depends on important frequency range for the phenomena to be simulated.

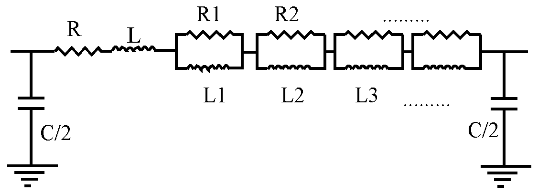

Figure 7 - π-circuit unit.

This model uses the modified π-circuits to introduce the frequency dependence. For good accuracy in the next tests, the used number of RL parallel circuits was 5 parallel circuits for 0 mode and 4 parallel circuits for the other modes.

VIII - TESTS WITH SIGNAL PROPAGATION

When phase voltages or phase currents are multiplied by the transformation matrices, then mode voltages and mode currents are obtained. If phase magnitudes correspond to a single mode, then only one mode is non zero. For example:

$$\begin{vmatrix} V_{A1} \\ V_{B1} \\ V_{C1} \\ V_{A2} \\ V_{B2} \\ V_{C2} \end{vmatrix} = \begin{vmatrix} -0,5 \\ 1 \\ -0,5 \\ -0,5 \\ 1 \\ -0,5 \end{vmatrix} \Longrightarrow \begin{vmatrix} V_{m\alpha} \\ V_{m\beta} \\ V_{m0} \\ V_{a\alpha} \\ V_{a\beta} \\ V_{a0} \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} \quad (18)$$

Proportional signals can test the transformation matrix representation through the ideal transformers. If the propagated signals have the same proportion of the input signals, the transformation matrix (T_{FM}) is properly represented through ideal transformers. For this kind of test, the values of Table I can be used.

Table I - Signal values for the propagation tests.

	A1	B1	C1	A2	B2	C2
m α	-0.5	1	-0.5	-0.5	1	-0.5
m β	0.5	0	-0.5	0.5	0	-0.5
m 0	0.5	0.5	0.5	0.5	0.5	0.5
a α	-0.5	-1	-0.5	0.5	1	0.5
a β	0.5	0	-0.5	-0.5	0	0.5
a 0	0.5	-0.5	0.5	-0.5	0.5	-0.5

For example, Figure 8 shows the media α mode voltage test. It uses the signal values of the Table I first line and the reception terminal circuit is open. The line transposition is the operational transposition. The signals are rectangular signals.

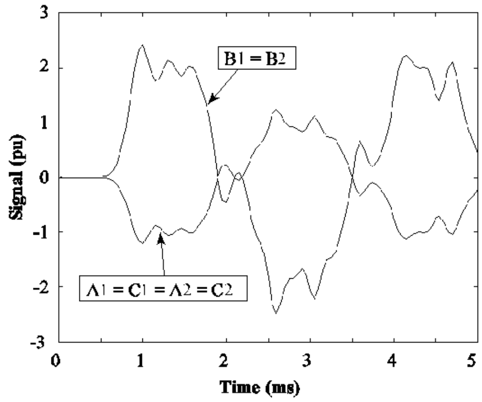

Figure 8 – Propagation test results for media α mode.

In Figure 8, there is the same proportion between the final signals and the signals which were defined by first line of Table I. The used ideal transformers are a correct representation for the transformation matrix.

Another test is applied for the study of the synthetic circuits. If the same initial signal is applied to the modes, the propagated signals will be equal for modes which present equal impedance values for a defined transposition. For example, in Figure 9, it uses the same phase A_1 signal (rectangular with 0.5 pu), for the media α mode and antimedia β mode. The line transposition is the operational transposition.

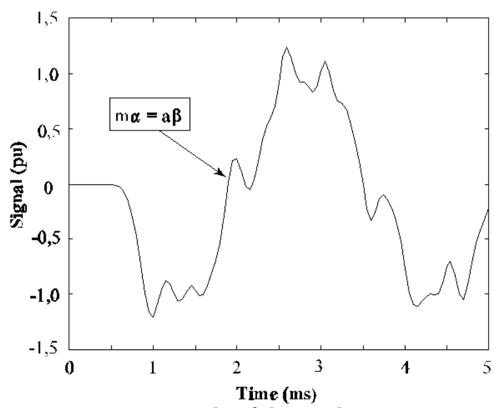

Figure 9 – Study of the synthetic circuits.

In this case, the used synthetic circuits are a correct representation for the operational transposition of the presented double three-phase transmission line.

IX - TRANSIENT SIMULATIONS

The first transient simulation is an energization test. The operational transposition presented in this paper is applied. The voltage is 440 kV (RMS) and the frequency is 60 Hz. The line energization starts with the peak voltage for the phase A. For phase B, the energization starts 3 ms after the energization of the phase A. This time difference is applied between the phase B and phase C. Figure 10 shows the voltages at the end line terminal. This is an opened terminal and its voltage is the double of the source voltage values.

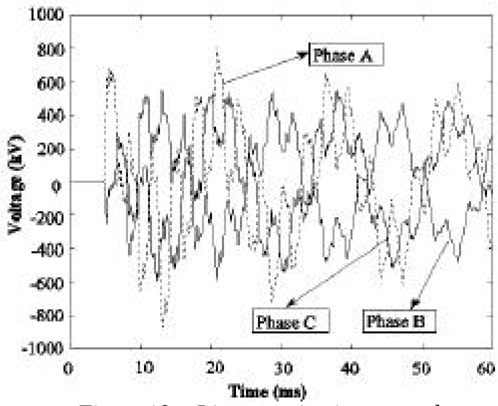

Figure 10 – Line energization example.

The other transient simulation is a single phase-to ground fault. This simulation begins with a steady state solution and a fault is applied on the phase A of circuit 1, when this phase reaches the voltage peak. The fault occurs on the line end. The initial voltage is 440 kV (RMS) and the frequency is 60 Hz. Figure 11 shows the test results for circuit 1 of the presented double three-phase line. The line is represented using both the Quasi-Mode Model and the frequency dependent model of the Microtran program (fdData), considering the operational transposition, in Microtran. Figure 12 shows the fault effects on the other circuit of the double three-phase transmission line.

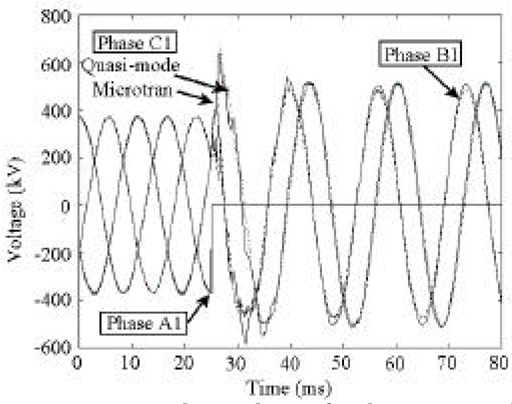

Figure 11 – Fault simulation for the operational transposition (circuit 1).

The fdData (Microtran) uses the exact eigenvectors matrix to produce a modal transformation matrix that is calculated for 10 kHz. The earth resistivity is 1000 Ω.m. The Quasi-mode model uses a single real modal transformation matrix that is frequency independent and the same value of the earth resistivity. The length line is 160 km.

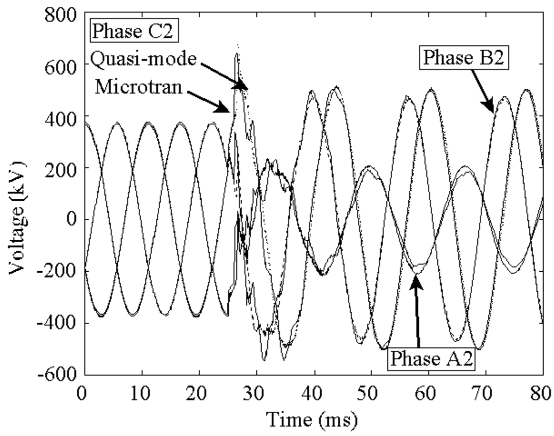

Figure 12 – Fault effects on circuit 2 for operational transposition.

X - CONCLUSION

A real double three-phase line is represented through the Quasi-mode model. This model represents the longitudinal parameters in mode domain and it uses a real single transformation matrix. The presented transformation matrix uses the geometrical properties of the double three-phase transmission line and it is associated with the Clarke transformation. This transformation matrix can be represented by ideal transformers, namely in the digital programs which work in time domain.

The frequency dependence in mode domain is introduced by modified π-circuits, where the frequency dependence of longitudinal parameters of the line is represented by RL series circuit added with RL parallel circuits.

Three different types of line transposition are analyzed : complete, rotational and operational. For complete and rotational transpositions, the proposed model obtains diagonal mode matrices. However, for operational transposition the antimedia impedance matrix is not a diagonal matrix, which means these obtained values can not be called modes. They are called Quasi-Modes. For this matrix, the mode coupling impedance between the α mode and the 0 mode is a negligible term. However, till in this case, at least for most applications, the Quasi-modes are a reasonable approximation of exact modes and allow to obtain results with small error margin. The non-transposed line is also analyzed and it produces full Quasi-Mode impedance matrices.

Some tests, such as line energization, propagation tests and fault, show that Quasi-mode model is appropriated to represent a double three-phase transmission line. This model can be implemented in digital programs which work in time domain.

The mathematical development of the presented model shows that some of the considered assumptions are "exact", without restrictions, and other, are either exact in some idealized assumptions, which can be accepted in most usual studies, or reasonable approximations, at least in some conditions.

XI - ACKNOWLEDGMENTS

The authors thank the support received from FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo) and CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior).

XII - REFERENCES

[1] H.W. Dommel, "Eletromagnetic Transients Program - Rule Book, Oregon, 1984.

[2] M.C. Tavares, "Linha de Transmissão Hexafásica - Características de Propagação e Comportamento da Linha a Freqüência Industrial e Transitórios Eletromagnéticos", M.Sc. Thesis, UFRJ/COPPE, Rio de Janeiro, Brazil, 1991.

[3] Microtran Power System Analysis Corporation, Transients Analysis Program Reference Manual, Vancouver, Canada, 1992.

[4] J.A. Brandão Faria and J. Briceño Mendez, "On the Modal Analysis of Asymmetrical Three-phase Transmission Lines using Standard Transformation Matrices, IEEE Winter Meeting, 97WMIEEE-PES, 1997.

[5] M.C. Tavares, J. Pissolato and C.M. Portela, "New Mode-domain Multiphase Transmission Line Model - Clarke Transformation Evaluation", 1998 *International Conference on Power Systems Technology (Powercon'98)*, pp. 860-864.

[6] M.C. Tavares, J. Pissolato and C.M. Portela, "Mode Domain Multiphase Transmission Line Model - Use in Transient Studies", *IEEE Transactions on Power Delivery*, vol. 14, October 1999, pp. 1533-1544.

[7] M.C. Tavares, J. Pissolato and C.M. Portela, "New Multiphase Mode Domain Transmission Line Model", *International Journal of Electrical Power and Energy Systems*, vol. 21, no. 8, November 1999.

[8] A. Morched, B. Gustavsen and M. Tartibi, "A Universal Model for Accurate Calculation of Electromagnetic Transients on Overhead Lines and Underground Cables", *IEEE Transaction on Power Delivery*, vol.14, no.3, pp. 1032-1038, 1999.

[9] M.C. Tavares, J. Pissolato and C.M. Portela, "Quasi-Modes Three-Phase Transmission Line Model - Transformation Matrix Equations" – *International Journal of Electrical Power & Energy Systems*, vol. 23/4, pp. 325-333, 2001.