Current and Voltage Dependent Sources in EMTP-based Programs

Benedito Donizeti Bonatto *
Department of Electrical and Computer Engineering
The University of British Columbia
2356 Main Mall, Vancouver, B.C., V6T 1N4, Canada
benedito.bonatto@ieee.org

Hermann Wilhelm Dommel
Department of Electrical and Computer Engineering
The University of British Columbia
2356 Main Mall, Vancouver, B.C., V6T 1N4, Canada

Abstract - **This paper presents the fundamental concepts for the implementation of current and voltage dependent sources in EMTP-based programs. These current and voltage dependent sources can be used to model many electronic and electric circuits and devices, such as operational amplifiers, etc., and also ideal transformers. As long as the equations of the dependent sources are linear, they could be added directly to the network equations, but the matrix will then become unsymmetric. Another alternative discussed here in more detail is based on the compensation method, which can also handle nonlinear effects with a Newton-Raphson algorithm. Nonlinear effects arise with the inclusion of saturation or limits in the dependent sources. The implementation of independent sources, which can also be connected between two ungrounded nodes, is also presented. A practical example is given to illustrate the solution methods.**

Keywords: EMTP, current controlled voltage source (CCVS), current controlled current source (CCCS), voltage controlled voltage source (VCVS), voltage controlled current source (VCCS), dependent source, compensation method.

I. INTRODUCTION

EMTP-based programs are widely used in the electric power industry and in universities for the analysis of power system transients. Since the publication of [1], many have contributed to the development of models, which have been documented in [2] and elsewhere.

As far as the authors know, dependent sources of all types have not been implemented in any EMTP-based programs, leading to the motivation for the development of this work. Dependent sources expand the capabilities of EMTP-based programs for modeling many electric and electronic circuits and devices. With a voltage controlled voltage source, for example, it becomes easy to simulate operational amplifiers. These can then be used to set up control circuits with analog-computer block diagrams, which is an alternative to the state-space representation of control circuits and their simultaneous solution with the power network discussed in [4].

If the equations of the dependent sources are linear, they can be added directly to the system of the nodal equations used in EMTP-based programs, if a linear equation solver for unsymmetric matrices is used. Another approach is based on the compensation method, which is chosen here because nonlinear equations can easily be handled as well.

This paper provides the fundamental equations for the implementation of dependent sources, as well as of ungrounded independent sources, in EMTP-based programs.

II. COMPENSATION METHOD

The compensation method has long been used in EMTP-based programs for solving the equations of nonlinear elements with the Newton-Raphson iterative method. If the nonlinear elements are not too numerous, this approach confines the iterations to a relatively small system of equations, compared to the nodal equations for the entire system.

When there are M nonlinear elements in a circuit, the following system of equations, (1) to (6), allows the simultaneous solution of the nonlinear equations with the rest of the linear network [2],[3], which is then represented by its M-phase Thevenin equivalent circuit, as illustrated in Fig.1:

$$-\left[v_{OPEN}\right]+\left[r_{THEV}\right]\cdot\left[i\right]+\left[v\right]=0 \qquad (1)$$

where:

$$[v_{OPEN}]=\begin{bmatrix} v_{OPEN_1} \\ v_{OPEN_2} \\ \vdots \\ v_{OPEN_M} \end{bmatrix} \qquad (2)$$

$$[r_{THEV}]=\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1M} \\ r_{21} & r_{22} & \cdots & r_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ r_{M1} & r_{M2} & \cdots & r_{MM} \end{bmatrix} \qquad (3)$$

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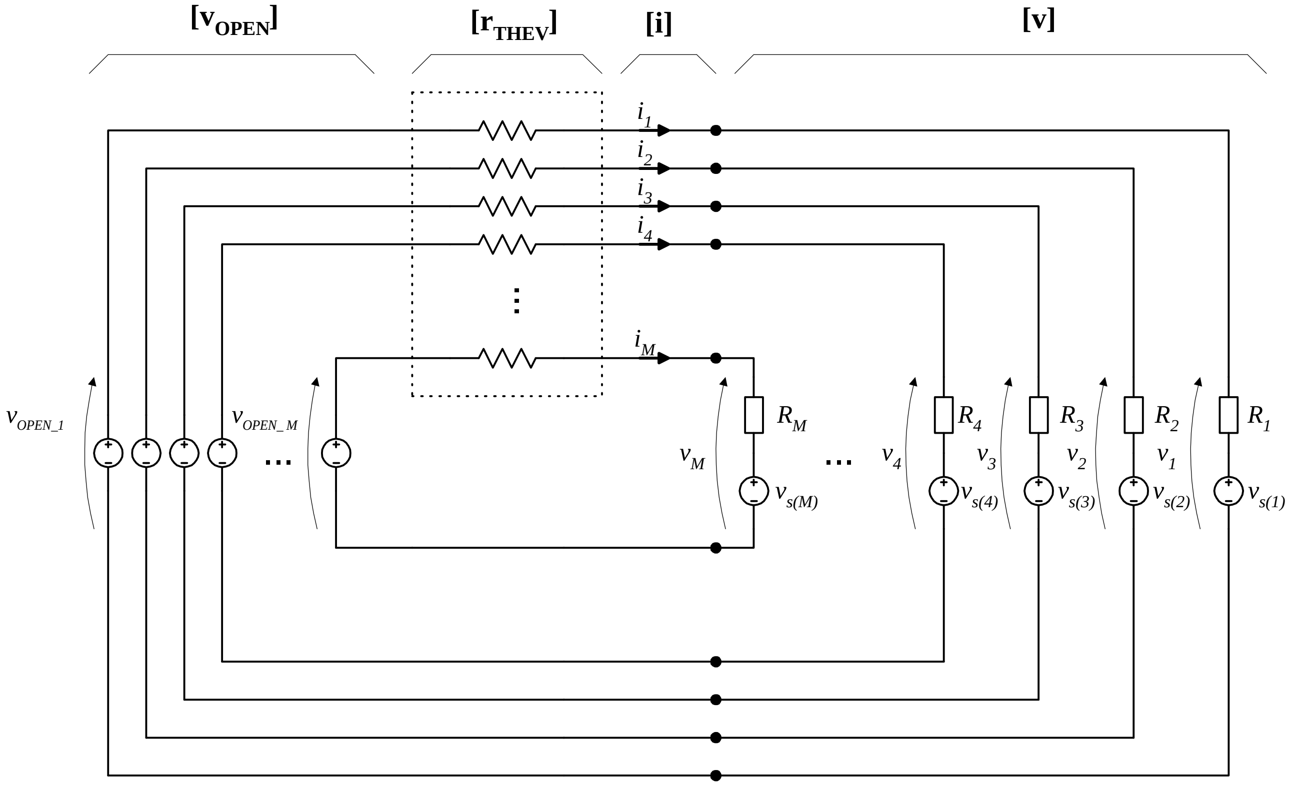

Fig. 1 – M-phase Thevenin equivalent circuit.

$$[i] = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_M \end{bmatrix} \quad (4)$$

$$[v] = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_M \end{bmatrix} \quad (5)$$

and (6) are the branch equations of the nonlinear elements:

$$v_k = f_k([v],[i],t,etc....) \quad k = 1,...M \quad (6)$$

If the branch equations in (6) are linear, as in the case of dependent sources, they can be represented in the form of a voltage source behind an impedance, as illustrated in Fig. 2, or in the form of a current source in parallel with an impedance, as shown in Fig. 3. In this paper, it is assumed that the branch impedances are not coupled, and that they are resistive (R_k).

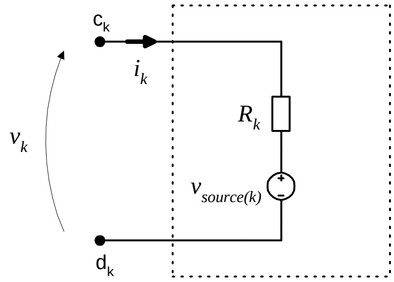

Fig. 2 –Representation of branch equation k as a voltage source in series with a resistance.

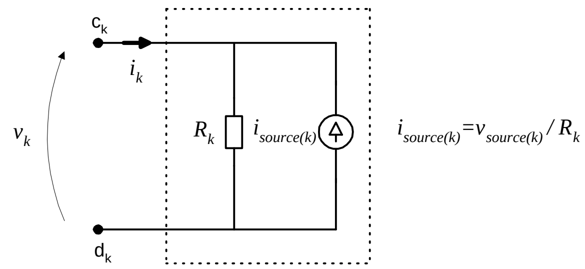

Fig. 3 –Representation of branch equation k as a current source in parallel with a resistance.

III. DEPENDENT SOURCES

This section presents the necessary equations for implementing current and voltage dependent sources in EMTP-based programs. The following assumptions are made:

1. A Thevenin equivalent circuit can be calculated where the dependent source is to be connected, and also where the controlling current or controlling voltage is to be measured. In cases where this calculation fails, the connection of large resistors in parallel may make a Thevenin equivalent circuit possible.
2. Proper precautions are taken to handle extremely large numbers and zero values.

The following models are derived: Current Controlled Voltage Source (CCVS), Current Controlled Current Source (CCCS), Voltage Controlled Voltage Source (VCVS) and Voltage Controlled Current Source (VCCS). In all cases, the equations from the Thevenin equivalent circuit are the same, namely, for the controlling branch

$$\begin{aligned}&-v_{OPEN_j} + r_{j1} i_1 + \ldots \\ &\ldots + r_{jj} i_j + r_{jk} i_k + \ldots + r_{jM} i_M + v_j = 0\end{aligned} \tag{7}$$

and for the dependent source branch

$$\begin{aligned}&-v_{OPEN_k} + r_{k1} i_1 + \ldots \\ &\ldots + r_{kj} i_j + r_{kk} i_k + \ldots + r_{kM} i_M + v_k = 0\end{aligned} \tag{8}$$

where:

v_{OPEN_k} = voltage v_k for $i_k = 0$ (open circuit).

r_{kk} = Thevenin resistance (self resistance of branch k).

r_{kj} = Thevenin resistance (coupling or mutual resistance between branches k and j).

A. Current Controlled Voltage Source (CCVS)

Assume that the controlling current is measured through a branch between nodes a and b in a circuit, such that v_j is its branch voltage and i_j is its branch current, i.e.,

$$v_j = v_a - v_b \tag{9}$$

$$i_j = i_{ab} \tag{10}$$

and that the dependent source, CCVS, is connected between nodes c and d with branch voltage

$$v_k = v_c - v_d \tag{11}$$

and branch current

$$i_k = i_{cd} \,. \tag{12}$$

Then the necessary equations for the implementation of this current controlled voltage source are (7) and (8) as well as:

$$v_j = R_{in} i_j \tag{13}$$

$$v_k = \Omega i_j + R_{out} i_k \tag{14}$$

where:

R_{in} = Input resistance of branch j.

R_{out} = Output resistance of the dependent source in branch k.

Ω = Gain over the controlling or measured current, applied as dependent source at branch k.

From (7), (8), (13) and (14), one can also obtain the following equations:

$$\begin{aligned}&-v_{OPEN_j} + r_{j1} i_1 + \ldots \\ &\ldots + \left(r_{jj} + R_{in}\right) i_j + r_{jk} i_k + \ldots + r_{jM} i_M = 0\end{aligned} \tag{15}$$

$$\begin{aligned}&-v_{OPEN_k} + r_{k1} i_1 + \ldots \\ &\ldots + \left(r_{kj} + \Omega\right) i_j + \left(r_{kk} + R_{out}\right) i_k + \ldots + r_{kM} i_M = 0\end{aligned} \tag{16}$$

For an ideal current controlled voltage source, $R_{in} = 0$ and $R_{out} = 0$, from which results:

$$\begin{aligned}&-v_{OPEN_j} + r_{j1} i_1 + \ldots \\ &\ldots + r_{jj} i_j + r_{jk} i_k + \ldots + r_{jM} i_M = 0\end{aligned} \tag{17}$$

$$\begin{aligned}&-v_{OPEN_k} + r_{k1} i_1 + \ldots \\ &\ldots + \left(r_{kj} + \Omega\right) i_j + r_{kk} i_k + \ldots + r_{kM} i_M = 0\end{aligned} \tag{18}$$

B. Current Controlled Current Source (CCCS)

The necessary equations for the implementation of a current controlled current source are (7) and (8) as well as:

$$v_j = R_{in} i_j \tag{19}$$

$$v_k = R_{out} \mathrm{B} i_j + R_{out} i_k \tag{20}$$

where:

B = Gain over the controlling or measured current, applied as dependent source at branch k.

From the equations above and from (7) and (8), one can also obtain the following equations:

$$\begin{aligned}&-v_{OPEN_j} + r_{j1} i_1 + \ldots \\ &\ldots + \left(r_{jj} + R_{in}\right) i_j + r_{jk} i_k + \ldots + r_{jM} i_M = 0\end{aligned} \tag{21}$$

$$-\frac{v_{OPEN_K}}{R_{out}}+\frac{r_{k1}}{R_{out}}i_1+\ldots$$
$$\ldots+\left(\frac{r_{kj}}{R_{out}}+\mathrm{B}\right)i_j+\left(\frac{r_{kk}}{R_{out}}+1\right)i_k+\ldots+\frac{r_{kM}}{R_{out}}i_M=0 \quad (22)$$

For an ideal current controlled current source, $R_{in}=0$ and $R_{out}\to\infty$, resulting in:

$$\begin{aligned}&-v_{OPEN_j}+r_{j1}i_1+\ldots\\&\ldots+r_{jj}i_j+r_{jk}i_k+\ldots+r_{jM}i_M=0\end{aligned} \quad (23)$$

$$\mathrm{B}i_j+i_k=0 \quad (24)$$

C. Voltage Controlled Voltage Source (VCVS)

The necessary equations for the implementation of a voltage controlled voltage source are (7) and (8) as well as:

$$v_j=R_{in}i_j \quad (25)$$

$$v_k=\mathrm{A}v_j+R_{out}i_k=\mathrm{A}R_{in}i_j+R_{out}i_k \quad (26)$$

where:
$\mathrm{A}=$ Gain over the controlling or measured voltage, applied as dependent source at branch k.

From the equations above and from (7) and (8), one can also obtain the following equations:

$$\begin{aligned}&-\frac{v_{OPEN_J}}{R_{in}}+\left(\frac{r_{j1}}{R_{in}}\right)i_1+\ldots\\&\ldots+\left(\frac{r_{jj}+R_{in}}{R_{in}}\right)i_j+\frac{r_{jk}}{R_{in}}i_k+\ldots+\frac{r_{jM}}{R_{in}}i_M=0\end{aligned} \quad (27)$$

$$\begin{aligned}&-v_{OPEN_j}+\frac{v_{OPEN_K}}{\mathrm{A}}+\left(r_{j1}-\frac{r_{k1}}{\mathrm{A}}\right)i_1+\ldots\\&\ldots+\left(r_{jj}-\frac{r_{kj}}{\mathrm{A}}\right)i_j+\left(r_{jk}-\frac{r_{kk}+R_{out}}{\mathrm{A}}\right)i_k+\ldots\\&\ldots+\left(r_{jM}-\frac{r_{kM}}{\mathrm{A}}\right)i_M=0\end{aligned} \quad (28)$$

If $\mathrm{A}\to\infty$, $R_{in}\to\infty$, and $R_{out}\to 0$ for an ideal voltage controlled voltage source, we obtain:

$$i_j=0 \quad (29)$$

$$\begin{aligned}&-v_{OPEN_j}+r_{j1}i_1+\ldots\\&\ldots+r_{jj}i_j+r_{jk}i_k+\ldots+r_{jM}i_M=0\end{aligned} \quad (30)$$

Equations (29) and (30) can be used to model ideal operational amplifiers.

D. Voltage Controlled Current Source (VCCS)

The necessary equations for the implementation of a voltage controlled current source are (7) and (8) as well as:

$$v_j=R_{in}i_j \quad (31)$$

$$v_k=R_{out}\Gamma v_j+R_{out}i_k \quad (32)$$

where:
$\Gamma=$ Gain over the controlling or measured voltage, applied as dependent source at branch k.

From the equations above and from (7) and (8), one can also obtain the following equations:

$$\begin{aligned}&-\frac{v_{OPEN_J}}{R_{in}}+\left(\frac{r_{j1}}{R_{in}}\right)i_1+\ldots\\&\ldots+\left(\frac{r_{jj}+R_{in}}{R_{in}}\right)i_j+\frac{r_{jk}}{R_{in}}i_k+\ldots+\frac{r_{jM}}{R_{in}}i_M=0\end{aligned} \quad (33)$$

$$\begin{aligned}&-\Gamma v_{OPEN_j}+\frac{v_{OPEN_K}}{R_{out}}+\left(\Gamma r_{j1}-\frac{r_{k1}}{R_{out}}\right)i_1+\ldots\\&\ldots+\left(\Gamma r_{jj}-\frac{r_{kj}}{R_{out}}\right)i_j+\left(\Gamma r_{jk}-\frac{r_{kk}+R_{out}}{R_{out}}\right)i_k+\ldots\\&\ldots+\left(\Gamma r_{jM}-\frac{r_{kM}}{R_{out}}\right)i_M=0\end{aligned} \quad (34)$$

For an ideal voltage controlled current source, $R_{in}\to\infty$ and $R_{out}\to\infty$, resulting in:

$$i_j=0 \quad (35)$$

$$\begin{aligned}&-\Gamma v_{OPEN_j}+\Gamma r_{j1}i_1+\ldots\\&\ldots+\Gamma r_{jj}i_j+\left(\Gamma r_{jk}-1\right)i_k+\ldots+\Gamma r_{jM}i_M=0\end{aligned} \quad (36)$$

IV. IDEAL TRANSFORMERS

Even though an ideal transformer model has already been implemented in EMTP-based programs with a special connection of 8 resistances and an extra node [2], it can also be implemented as a special dependent source. The necessary equations for the implementation of an ideal transformer are (7) and (8) as well as:

$$\frac{v_j}{v_k} = \frac{1}{n} \tag{37}$$

$$\frac{i_j}{i_k} = -n \tag{38}$$

where:

$n = \frac{1}{a} = \frac{n_k}{n_j} =$ reciprocal of the turns ratio of the ideal transformer.

From the equations above and from (7) and (8), one can easily obtain:

$$i_j + n i_k = 0 \tag{39}$$

$$\begin{aligned} &-v_{OPEN_j} + \frac{v_{OPEN_K}}{n} + \left(r_{j1} - \frac{r_{k1}}{n} \right) i_1 + \ldots \\ &\ldots + \left(r_{jj} - \frac{r_{kj}}{n} \right) i_j + \left(r_{jk} - \frac{r_{kk}}{n} \right) i_k + \ldots \\ &\ldots + \left(r_{jM} - \frac{r_{kM}}{n} \right) i_M = 0 \end{aligned} \tag{40}$$

Equations (39) and (40) can be used to model an ideal transformer in EMTP-based programs.

V. INDEPENDENT SOURCES

It may be useful in a circuit or device model to have an independent current or independent voltage source connected between two ungrounded nodes. This can be accomplished by the same technique used for the implementation of dependent sources, but using only one equation in this case.

A. Independent Current Source

Assuming that the independent current source is connected between nodes c and d with branch voltage

$$v_k = v_c - v_d \tag{41}$$

and branch current

$$i_k = i_{cd} \tag{42}$$

then the necessary equations for the implementation of an independent current source are:

$$\begin{aligned} &-v_{OPEN_k} + r_{k1} i_1 + \ldots \\ &\ldots + r_{kk} i_k + \ldots + r_{kM} i_M + v_k = 0 \end{aligned} \tag{43}$$

$$v_k = R_{out} i_{source} + R_{out} i_k \tag{44}$$

where:

$i_{source} =$ independent current source at branch k, which can be a linear or nonlinear function of time, etc..

From the equations above, one can also obtain the following equation:

$$\begin{aligned} &-\frac{v_{OPEN_K}}{R_{out}} + \frac{r_{k1}}{R_{out}} i_1 + \ldots \\ &\ldots + \left(\frac{r_{kk} + R_{out}}{R_{out}} \right) i_k + \ldots + \frac{r_{kM}}{R_{out}} i_M + i_{source} = 0 \end{aligned} \tag{45}$$

For the ideal current source, $R_{out} \to \infty$, resulting in:

$$i_k + i_{source} = 0 \tag{46}$$

Of course, there is a much easier way to represent an independent current source between nodes c and d directly in the nodal equations of the EMTP: inject the current source into node c and with a negative sign into node d [2].

B. Independent Voltage Source

The necessary equations for the implementation of an independent voltage source are:

$$\begin{aligned} &-v_{OPEN_k} + r_{k1} i_1 + \ldots \\ &\ldots + r_{kk} i_k + \ldots + r_{kM} i_M + v_k = 0 \end{aligned} \tag{47}$$

$$v_k = v_{source} + R_{out} i_k \tag{48}$$

where:

$v_{source} =$ independent voltage source at branch k, which can be a linear or nonlinear function of time, etc..

From the equations above, one can also obtain the following equation:

$$\begin{aligned} &-v_{OPEN_K} + r_{k1} i_1 + \ldots \\ &\ldots + \left(r_{kk} + R_{out} \right) i_k + \ldots + r_{kM} i_M + v_{source} = 0 \end{aligned} \tag{49}$$

In an ideal voltage source, $R_{out} = 0$, resulting in:

$$\begin{aligned} &-v_{OPEN_K} + r_{k1} i_1 + \ldots \\ &\ldots + r_{kk} i_k + \ldots + r_{kM} i_M + v_{source} = 0 \end{aligned} \tag{50}$$

Another approach for voltage sources between ungrounded nodes frequently used in EMTP-based programs is the insertion of an ideal transformer between the two ungrounded nodes, with a voltage source to ground on the other side.

VI. POSSIBLE APPLICATIONS

1. Current and voltage sensors;
2. Operational amplifiers;
3. Ideal Transformers;
4. User-defined coupled branches in a circuit;
5. Modeling of electronic components, where the physical behavior would need to be represented by nonlinear equations;
6. *Instantaneous* solution of linear and nonlinear control systems;
7. User-defined linear and nonlinear functions;
8. User-defined modeling of linear and nonlinear devices, limited only by the creativity and ingenuity of the user.

Fig. 4 and Fig. 5 illustrate the solution method with an example of a noninverting amplifier circuit, which consists of a sinusoidal voltage source, an ideal operational amplifier and 2 resistors (R_f and R_g). The ideal operational amplifier was modeled using (29) and (30), whereas the sinusoidal voltage source and the resistors are part of the network, represented through a Thevenin equivalent circuit. If $R_f = 2R_g$, then $v_{output} = 3v_{input}$, as shown in Fig. 5.

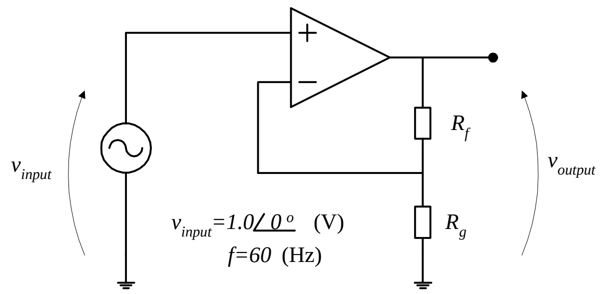

Fig. 4 – Circuit with ideal operational amplifier.

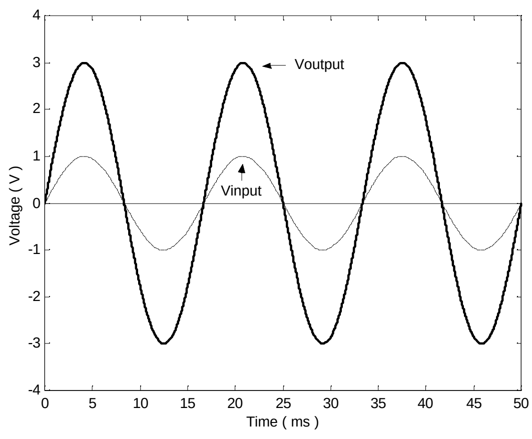

Fig. 5 – Simulation results of circuit with ideal operational amplifier (noninverting amplifier circuit).

Indeed, in theory this noninverting amplifier circuit should result in:

$$\frac{v_{output}}{v_{input}} = \left(1 + \frac{R_f}{R_g}\right) \tag{51}$$

VII. CONCLUSIONS

The implementation of current and voltage dependent sources in EMTP-based programs has been presented, as well as the implementation of independent sources which may be connected between two ungrounded nodes. It is based on the compensation method, which is already being used to solve nonlinear equations associated with nonlinear elements in electric or electronic circuits with Newton-Raphson iteration schemes. The method looks promising for future work in detailed modeling of circuits and devices. Future improvements will consider the implementation of saturation or limits for the elements or sources presented in this paper.

VIII. ACKNOWLEDGEMENTS

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