A non-Differentiable Wavelet Algorithm for Transient Analysis

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Abstract – **An alternative method for power system transient analysis, based on the Haar wavelet series approximation, is proposed. The various types of components, such as resistor, inductor, and capacitor are modeled just once. The system model is set up assembling component models in accordance with the system configuration. The method explores properties of the Haar wavelet operation matrix, so its computational efficiency is very high. A concise description of the method and some basic applications are presented. The obtained results are like to EMTP outputs.**

Keywords: Transient analysis, wavelet series, state equations, EMTP program.

I. INTRODUCTION

Transient analysis assumes important role on power system planning, project and operation, as well as on power quality issue. Many computer-based techniques have been developed to solve transient problems. They are generally classified into time-domain and frequency-domain methods.

Time-domain methods are the most widely used on power systems. This is due to, mainly, the simplicity to simulate nonlinear systems. However, they can originate sustained numerical oscillations and, in general, they are dependents upon the integration-step size.

Methods of both class present advantages and disadvantages depending on the case, nevertheless they have a common drawback: inability to handle short-time transients mixed with low frequency signals.

Since the new electrical environment is full of complex transient phenomena, it is impossible for the power engineer to monitor, analyze and mitigate these phenomena with traditional tools. The advent of modern techniques and tools has become an indispensable tool than a sophisticated academic toy [1]. Attempts to the limitations of the traditional tools, recently methods based on the wavelet theory have been proposed to solve power quality problems, including transient analysis [2, 3, 4, 5].

Wavelets have been the focus of considerable research over the last years. Although an immature area of study in power system, wavelet transforms have already proved efficiency to detect, localize, identify and classify the disturbances [6, 7, 8].

We believed that, in a near future, the flexibility and adaptability of this new tool could be utilized in an **integrated wavelet system** to solve many power quality problems (Fig. 1).

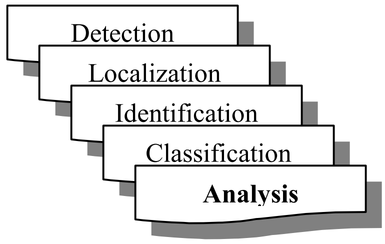


Fig. 1. Integrated wavelet system.

To fully realize this potential, additional research is needed in the development and optimization of the wavelet based methods for transient analysis.

II. WAVELET BASED TRANSIENT ANALYSIS

Lee & Meliopoulos [4] developed a method for power system transient analysis that is based on the wavelet companion equivalent circuit of each circuit element by applying wavelet series expansion on the integral-differential equations.

By combining the element wavelet equivalents and network topology, the wavelet equivalent network is build. The procedure results in a set of algebraic equations. The solution is in terms of the wavelet coefficients of the voltages at the nodes of the network.

The method has an elegant mathematical formulation and presents similarity with traditional transient analysis methods. However, the computation effort for integration and differentiation operations on the wavelet basis increase rapidly as the number of levels increase, besides these operations are processed by rudimentary methods (trapezoidal rule and the finite difference approximation).

Similarly to time domain methods, another disadvantage is the matrix inversion operation on solving the network. When the number of levels is high or the system dimension is large, the effort computational is high.

The authors also affirm that their method is valid for any set of orthogonal wavelets, but this is not completely true, because the wavelet basis must be orthogonal and differentiable.

Chen & Hsiao [2] developed a solution method to solve the state equations of lumped and distributed-parameter linear systems based on the Haar wavelet. The main characteristic of this technique is to convert the differential equations into algebraic equations, and hence the solution procedures are either reduced or simplified [2]. This method presents three important advantages:

- simple and compact mathematical formulation for the wavelet;
- the method does not use the derivative of wavelet basis;
- the Haar transform coefficient matrix of the integrals of the Haar basis (**P**$_m$ matrix) is obtained in a simple and fast way.

The exploration of the properties of the **P**$_m$ matrix allows the development of a fast and efficient algorithm, which demand very low computational effort. The disadvantage of this method is the mathematical formulation, in terms of state equations, that is not usual for power systems transient analysis.

The method by Lee & Meliopoulos have referenced as WBTA (wavelet-based transient analysis) and the one by Chen & Hsiao as HWM (Haar wavelet method).

III. THE PROPOSED METHOD

The study of the methods above described conduced to the development of an alternative method for transients analysis. This new methodology combines advantages of those methods: Haar wavelet basis and **P**$_\mathbf{m}$ matrix from HWM method and mathematical formulation from WBTA method. We have named it **Haar-based transient analysis** or **HBTA**.

The choice of the wavelet base function defines the characteristics and the properties of the representation of the signals on the wavelet domain [9]. Therefore, although the proposed method uses nodal admittance matrix instead of forming the state equations, it provides the same representation as HWM method.

A. Haar Series Expansions

The Haar wavelet is the oldest and most basic of the wavelets. Alfred Haar constructed it in 1910 and according to [10], the wavelet theory is a generalization of his work.

Haar showed that the translates and scalings of the Haar base function form an orthogonal basis on the interval [0,1). Then, any function $f(\cdot)$ that is finite and is square integrable in this interval can be represented into Haar series [10]:

$$f(x) = \sum_{j=1}^{m} a_j h_j(x) \tag{1}$$

where

- x is a normalized variable defined as $x = t/T$;
- t is the independent variable and $[0,T)$;
- a_j are the Haar coefficients of $f(x)$;
- $h_j(x)$ are the Haar base functions;
- m is the size of Haar basis ($m=2^{M-1}$);
- M is the number of levels ($M \geq 1$).

The Haar basis is a regular basis and it is only real orthogonal wavelet basis with compact support and symmetric. However, it is not continuous [11].

B. Operational Properties of the Haar wavelets

In the study of transient analysis, it is frequently required to take the derivative or integral of functions. The differentiation of Haar basis results in impulse functions, which are rather difficult to deal with. The integration of Haar basis results in triangular functions, which should be represented into Haar series [12]:

$$\int_0^x \mathbf{h}_m(\tau)d\tau \cong \mathbf{P}_m \mathbf{h}_m(x) \tag{2}$$

where **P**$_m$ is a m-square matrix which can be obtained by the following equation [12]:

$$\mathbf{P}_m = \frac{1}{2m}\begin{bmatrix} 2m\mathbf{P}_{m/2} & -\mathbf{h}_{m/2} \\ \mathbf{h}_{m/2}^{-1} & \mathbf{0} \end{bmatrix}, \quad \mathbf{P}_1 = \left[\frac{1}{2}\right] \tag{3}$$

where **h**$_m(x)$ is the Haar basis:

$$\mathbf{h}_m(x) = \left[h_1(x)\ h_2(x)\ \cdots\ h_m(x)\right]^{\mathrm{T}}. \tag{4}$$

The **P**$_m$ matrix is named **Haar wavelet operational matrix**. According to [12], this matrix provides an important connection between the Haar wavelet transform and dynamic system analysis.

The core of the idea starts from the representation into Haar series of the **derivative of the voltage** $v(x)$ at each node n (except at the reference node) of the electrical circuit:

$$\frac{d}{dx}v_n(x) = \sum_{j=1}^{m} u_{n,j}h_j(x), \quad 0 \leq x \leq 1. \tag{5}$$

Integrating (5) yields:

$$v_n(x) = \sum_{j=1}^{m}\left[\sum_{l=1}^{m} u_{n,l}p_{l,j}\right]h_j(x) \tag{6}$$

where $p_{l,j}$ is an element of the **P**$_m$ matrix.

C. Haar Domain Equivalents Circuits

Similarly to WBTA method, each electrical circuit component has a representation into Haar domain.

<u>Capacitance</u>: The current/voltage relationship for the capacitor is:

$$i(t) = C\frac{d}{dt}v(t), \quad v(0) = 0, \quad 0 \leq t < T. \tag{7}$$

Upon substitutions of the variable t with the normalized variable x and of (5) into (7), we obtained the Haar equivalent for the capacitor:

$$i(x) = \frac{C}{T}\sum_{j=1}^{m} u_j h_j(x) \tag{8}$$

<u>Resistance</u>: The current/voltage relationship for the resistor is:

$$i(t) = \frac{1}{R}v(t), \quad 0 \leq t < T. \tag{9}$$

Upon substitutions of the variable t with the normalized variable x and of (6) into (9), we obtained the Haar equivalent for the resistor:

$$i(x) = \frac{1}{R}\sum_{j=1}^{m}\left[\sum_{l=1}^{m} u_j p_{l,j}\right] h_j(x) \tag{10}$$

Inductance: The current/voltage relationship for the inductor is:

$$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau\,, \quad i(0) = 0, \quad 0 \le t < T. \tag{11}$$

Upon substitutions of the variable t with the normalized variable x and of (6) into (11), we obtained the Haar equivalent for the inductor:

$$i(x) = \frac{T}{L}\sum_{j=1}^{m}\left[\sum_{l=1}^{m} u_j p^2{}_{l,j}\right] h_j(t) \tag{12}$$

D. Haar Based Transient Analysis

The construction of Haar equivalent of the whole circuit follows the same methodology of [13].

Initially, we represented the vector of the derivative of the voltage at all nodes of the circuit $\mathbf{v}(x)$ into Haar series:

$$\frac{d}{dx}\mathbf{v}(x) = \mathbf{U}\mathbf{h}_m(x) \tag{13}$$

where

$$\mathbf{v}(x) = \begin{bmatrix} v_1(x) \\ v_2(x) \\ \vdots \\ v_n(x) \end{bmatrix} \text{ and } \mathbf{U} = \begin{bmatrix} u_{1,1} & u_{1,2} & \cdots & u_{1,m} \\ u_{2,1} & u_{2,2} & \cdots & u_{2,m} \\ & \vdots & & \\ u_{n,1} & u_{n,2} & \cdots & u_{n,m} \end{bmatrix} \tag{14}$$

The vector $\mathbf{v}(x)$ is obtained integrating (13):

$$\mathbf{v}(x) = \mathbf{U}\mathbf{P}_m\mathbf{h}_m(x). \tag{15}$$

In general, the current/voltage relationship for a device k of the circuit can be expressed as:

$$\mathbf{i}^k(x) = \mathbf{b}^k(x) + \mathbf{A}^k\mathbf{v}^k(x) + \mathbf{B}^k\frac{d}{dx}\mathbf{v}^k(x) + \mathbf{C}^k\int_0^x \mathbf{v}^k(\lambda)d\lambda \tag{16}$$

where

$\mathbf{A}^k$, $\mathbf{B}^k$ and $\mathbf{C}^k$ are q-square matrices associated to the resistive, capacitive and inductive elements, respectively;

q is the number of the device terminal;

$\mathbf{i}^k(x)$ and $\mathbf{v}^k(x)$ are qx1 vectors of the terminal node currents and voltages for device k, respectively.

The term $\mathbf{b}^k(x)$ is a qx1 vector associated to the current sources and it can be represented into Haar series:

$$\mathbf{b}^k(x) = \mathbf{E}^k\mathbf{h}_m(x) \tag{17}$$

where $\mathbf{E}^k$ is a qxm matrix.

We can also express the vector $\mathbf{v}^k(x)$ in terms of the vector of voltages at all nodes of the circuit:

$$\mathbf{v}^k(x) = \mathbf{D}^k\mathbf{v}(x) \tag{18}$$

where $\mathbf{D}^k$ is the device connectivity matrix.

Upon substitution of (17) and (18) into (16), we have:

$$\mathbf{i}^k(x) = \mathbf{E}^k\mathbf{h}_m(x) + \mathbf{A}^k\mathbf{D}^k\mathbf{v}(x) + \mathbf{B}^k\mathbf{D}^k\frac{d}{dx}\mathbf{v}(x) + \mathbf{C}^k\mathbf{D}^k\int_0^x \mathbf{v}(\tau)d\tau. \tag{19}$$

Now, upon substitution of (13) and (15) into (19), we have:

$$\mathbf{i}^k(x) = \mathbf{E}^k\mathbf{h}_m(x) + \mathbf{A}^k\mathbf{D}^k\mathbf{U}\mathbf{P}_m\mathbf{h}_m(x) + \mathbf{B}^k\mathbf{D}^k\mathbf{U}\mathbf{h}_m(x) + \mathbf{C}^k\mathbf{D}^k\mathbf{U}\mathbf{P}_m^2\mathbf{h}_m(x). \tag{20}$$

The vector of currents at all nodes of the circuit for device k, $\mathbf{i}^{kF}(x)$, can also be expressed in terms of $\mathbf{i}^k(x)$:

$$\mathbf{i}^{kF}(x) = \mathbf{D}^{k^{\mathrm{T}}}\mathbf{i}^k(x) \tag{21}$$

where $\mathbf{D}^{k^{\mathrm{T}}}$ is the transposition of $\mathbf{D}^k$.

Substituting (20) into (21) and applying Kirchhoff's current Law, yields:

$$\sum_k\left\{\mathbf{D}^{k^{\mathrm{T}}}\left(\mathbf{E}^k + \mathbf{A}^k\mathbf{D}^k\mathbf{U}\mathbf{P}_m + \mathbf{B}^k\mathbf{D}^k\mathbf{U} + \mathbf{C}^k\mathbf{D}^k\mathbf{U}\mathbf{P}_m^2\right)\right\}. \tag{22}$$

This new methodology also results in a set of algebraic equations. For convenience, (22) must be restructured to be solved by a direct method. After $\mathbf{U}$ is obtained, it can be used in (15) to find $\mathbf{v}(x)$.

The HBTA method can be easily implemented according to the following algorithm:

1. Build Haar basis.
2. Compute $\mathbf{P}_m$ and $\mathbf{P}_m^2$ matrices.
3. Build and solve (22).
4. Solve (15).

IV. VALIDATION

In this section, the HBTA method is applied to basic circuit configurations. The validation is made by comparing its results with the ones obtained from EMTP.

This paper uses Microtran as the EMTP software package (we have named it MT), Fortran90 programming language and circuit configurations from [13].

Circuit A. Consider the following electrical circuit:

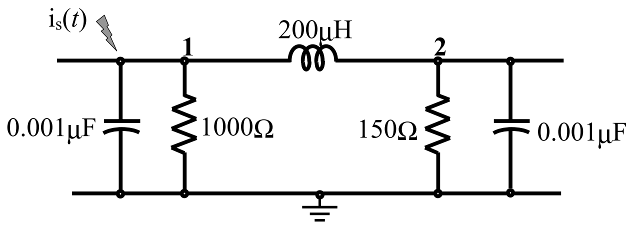


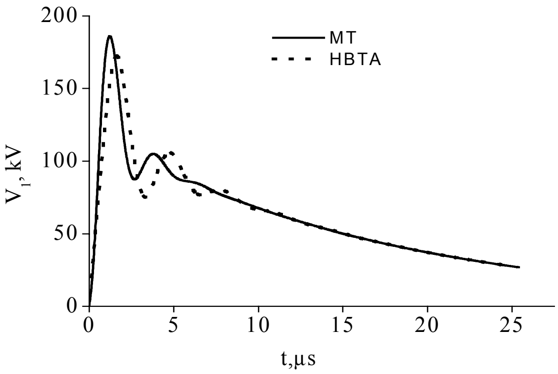
Fig. 2. Electrical circuit A.

Initially, the circuit is not energized. At time, t=0, a lightning strike $i_s(t)$ occurs at node 1, on the interval $0 \le t < 25.6\mu s$:

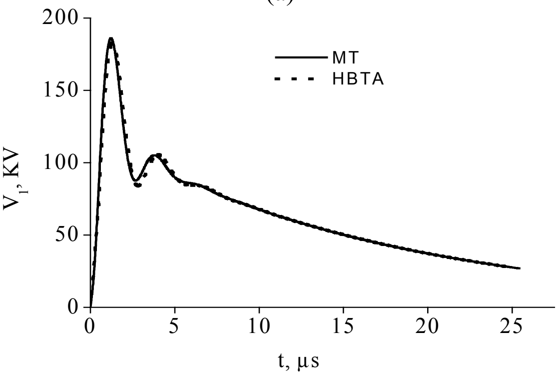
$$i_s(t) = i_0(e^{-\alpha_1 t} - e^{-\alpha_2 t}) \tag{23}$$

where α_1=0.06 10^6 s^{-1}, α_2=1.2 10^6 s^{-1} and i_0=1 kA.

Figs. 3 and 4 show the voltage waveforms at nodes 1 and 2 via HBTA (6 and 7 levels of resolution) and MT (128 samples) methods, respectively.

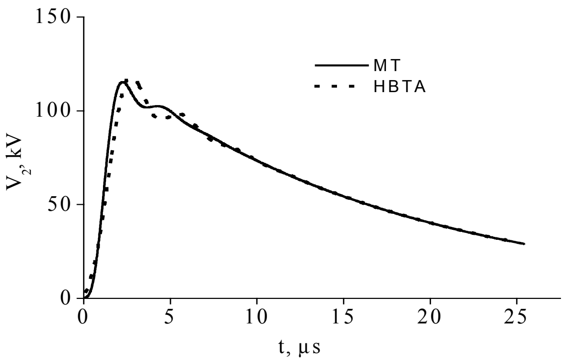


(a)

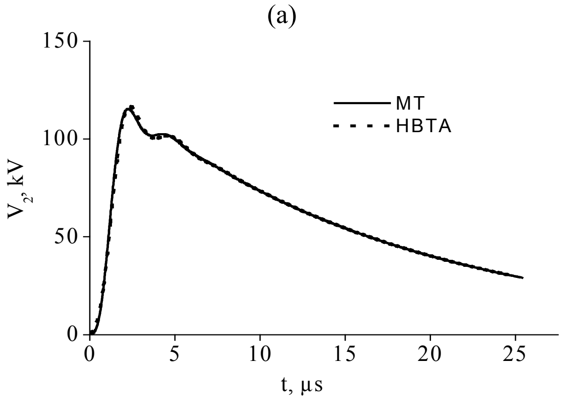


(b)

Fig. 3. Voltage at node 1 from circuit A via MT and HBTA methods with a) 6 and b) 7 levels of resolution.



(a)



(b)

Fig. 4. Voltage at node 2 from circuit A via MT and HBTA methods with a) 6 and b) 7 levels of resolution.

The accuracy of the proposed method is determined by comparing of the average normalized error and the maximum normalized error between the HBTA and MT methods [13]. Tab. 1 shows these error indices for this example.

Tab. 1. Errors of HBTA method x MT method (circ. A).

M	V_1 (KV)		V_2 (KV)	
	Average	Maximum	Average	Maximum
6	0.1874	0.5793	0.0688	0.2638
7	0.0378	0.1526	0.0146	0.0657
8	0.0002	0.0006	0.0002	0.0008

Circuit B. Consider the following electrical circuit:

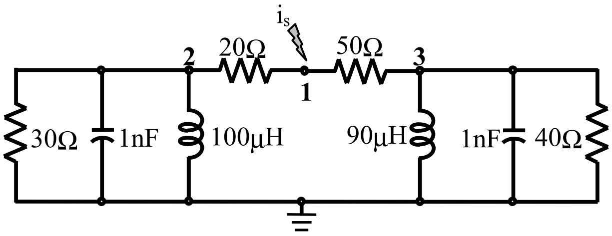


Fig. 5. Electrical circuit B.

The system was also not energized, when a lightning strike occurred at node 1, now on the interval $0 \le t < 12.8\mu s$.

Fig. 6 shows the voltage waveforms at nodes 1, 2 and 3 by MT method and the approximation achieved by HBTA method, now using 8 levels of resolution. In this case, the HBTA method shows to be as accurate as the MT method (the curves are superimposed).

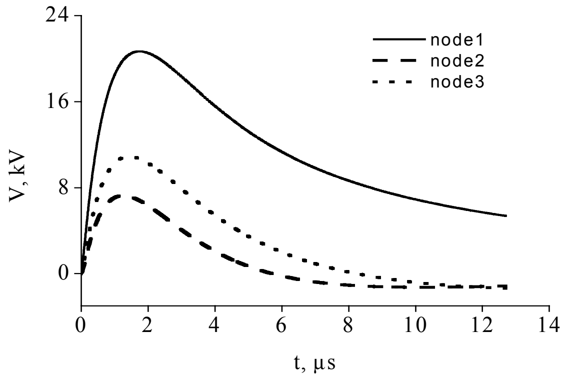


Fig. 6. Voltages at nodes 1, 2 and 3 from circuit B via MT HBTA methods with 8 levels of resolution.

Tab. 2 shows the error indices between the HBTA and MT methods for this example.The error indices presented at Tabs. 1 and 2 validate the proposed methodology for transient analysis since that they are within acceptable limits.

The proposed method provides excellent results, even if a small number M is used. For M=8, the accuracy in Haar domain is always the same as that of time domain. The evaluated curves will be closer and closer to the MT method, if a larger M is assigned [2]. Therefore, high precision can be ensured by HBTA method.

However, when the number of levels is high or the system dimension is large, the computational effort grows [5]. Therefore, similarly to the WBTA and HWM methods, the HBTA method provides trade-offs between solution accuracy and computational speed.

Tab. 2. Errors of HBTA method x MT method (circ. B).

M	V_1 (KV)		V_2 (KV)		V_3 (KV)	
	Aver.	Max.	Aver.	Max.	Aver.	Max
6	0.0101	0.0357	0.0372	0.1372	0.0259	0.0906
7	0.0043	0.0298	0.0145	0.1025	0.0114	0.0785
8	0.0002	0.0005	0.0001	0.0004	0.0002	0.0010

V. HBTA VERSUS HWM AND WBTA

In order to evaluate the performance between the HBTA, WBTA and HWM methods, a simple circuit is used, with the same conditions from circuit B.

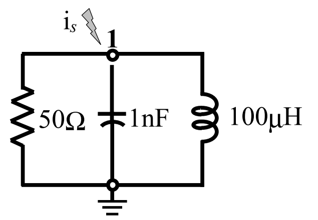


Fig. 7. Electrical circuit C.

Fig. 9 shows the voltage waveform at node 1 via HBTA, WBTA and HWM methods, with M=6, 7 and 8.

The error indices for M=8 are presented at Tab. 3.

Tab. 3. Wavelet-based methods x MT method (circ. C).

	HWM	HBTA	WBTA
Average Error	0.00020	0.00020	0.00846
Maximum Error	0.00099	0.00078	0.08623

Since the error indices are very small, they can be better visualized by error analysis presented in Fig. 8.

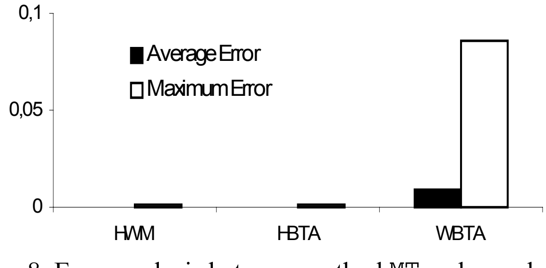


Fig. 8. Error analysis between method MT and wavelet-based methods from circuit C.

Fig. 10 presents the same analysis for circuit A. In this case, we have only evaluated the average error.

It is noted that the accuracy of the wavelet based methods depends decisively on the number of wavelet levels used [2, 3, 4, 5].

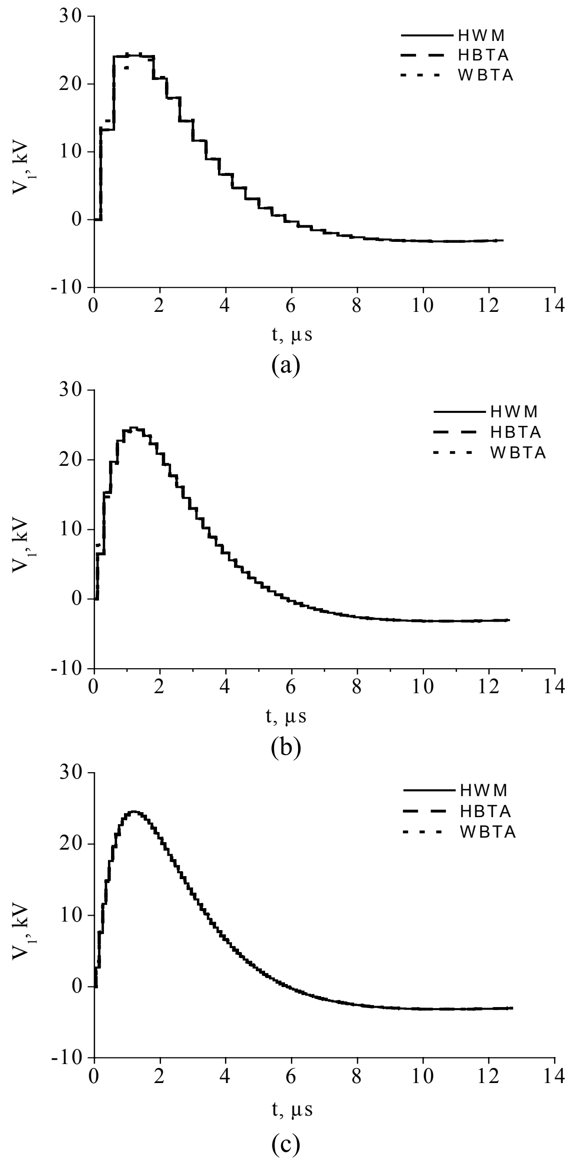


Fig. 9. Voltage at node 1 from circuit C via HBTA, WBTA and HWM methods with a) 6, b) 7 and c) 8 levels of resolution.

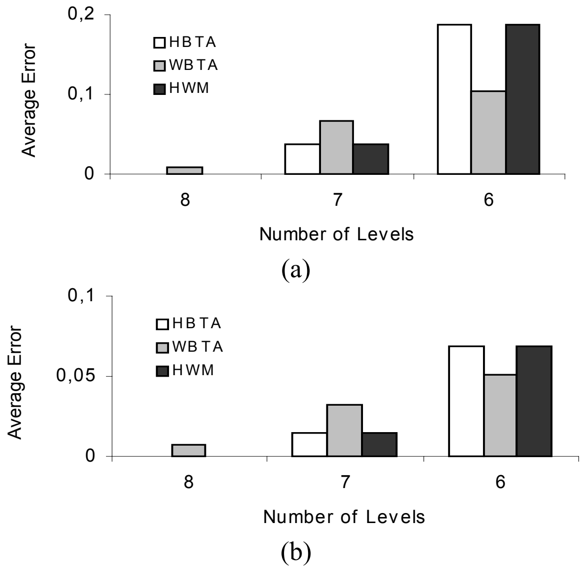


Fig. 10. Error analysis for voltages at a) node 1 and b) node 2 from circuit A.

As it would be obvious, the HBTA and HWM methods present similar error indices, since that both methods use the same wavelet basis and the same operational matrix. In all cases studied, the results presented by these methods were so much betters than those presented by WBTA method as large were it was the used number of levels. This is due to the rudimentary methods on integration and differentiation operations used by WBTA method.

For all wavelet methods, the maximum error always occurred at instants of abrupt variations (for example, initial instants of the disturbances).

About computer execution time, in all cases studied, the HWM and HBTA methods are faster than WBTA method. This is due to the advantages presented in section II. Therefore, for applications where computation speed is important, Haar wavelet based methods offers on of the best choices. However, all wavelet-based methods are always more slow than Microtran . The values from Tab. 4 reveal this conclusion.

Tab. 4. Execution time of wavelet-based methods (circuit C).

M	MT	WBTA	HWM	HBTA
6	0.00794s	0.17s	0.05s	0.05s
7	0.00623s	0.77s	0.11s	0.39s
8	0.00615s	6.15s	0.60s	2.53s

VI. CONCLUSIONS

A new method for transient analysis has been presented. The method uses the simplest wavelet function, or, Haar wavelet. By combining the Haar wavelet and its operational matrix with traditional mathematical formulation for transient analysis, the proposed methodology results in an very simple and accurate method.

Several case studies have shown the feasibility and practicality of the method. To fully realize this potential, the method must be applied to, for example:

- larger circuits,
- three-phase circuits,
- distributed-parameter systems,
- non-linear systems.

Once that the same performance occurs, we believed that the proposed method will be a potential candidate to the integrated wavelet system (Fig.11):

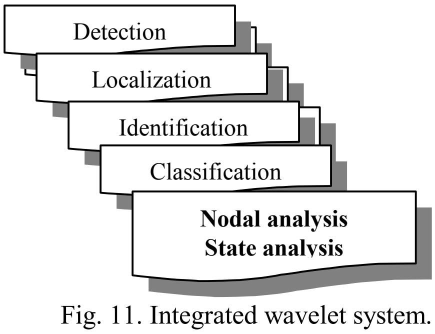


Fig. 11. Integrated wavelet system.

Now, the integrated wavelet system will can execute two types of analysis, using the **same wavelet**:

- nodal analysis → HBTA method,
- state analysis → HWM method.

VII. ACKNOWLEDGEMENTS

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VIII. REFERENCES

[1] P.F. Ribeiro and M. Samotyj, “Advanced techniques for voltage quality analysis unnecessary sophistication or indispensable tools”, *Proc. EPRI PQA’94*, October 1994.

[2] C.F. Chen and C.H. Hsiao, “Haar wavelet method for solving lumped and distributed-parameter systems”, *IEE Proc. Control Theory Appl.*, vol. 144, January 1997, pp. 87-94.

[3] G.T. Heydt and A.W. Galli, “Transient power quality problems analyzed using wavelets”, *IEEE Trans. on Power Delivery*, vol. 12, April 1997, pp. 908-915.

[4] A.P.S. Meliopoulos and C.H. Lee, “Wavelet based transient analysis”, *Proc. the Annual North American Power Symposium*”, pp. 339-346, October 1997, Laramie.

[5] T. Zheng et al., “Power system transient and harmonic studies using wavelet transform”, *IEEE Trans. on Power Delivery*, vol. 14, October 1999, pp. 1461-1468.

[6] S. Santoso et al., “Power quality disturbance waveform recognition using wavelet-based neural classifier- Part 1: Theoretical foundation”, *IEEE Trans. on Power Delivery*, vol. 15, January 2000, pp. 222-228.

[7] S. Santoso et al., “Power quality disturbance waveform recognition using wavelet-based neural classifier- Part 2: Application”, *IEEE Trans. on Power Delivery*, vol. 15, January 2000, pp. 229-235.

[8] S. Santoso et al., “Characterization of distribution power quality events with Fourier and wavelet transforms”, *IEEE Trans. on Power Delivery*, vol. 15, January 2000, pp. 247-254.

[9] R.K. Young, *Wavelet theory and its applications*, Kluwer Academic Publishers, Massachusetts, 1995.

[10] C.S. Burrus et al., *Introduction to wavelets and wavelet transforms*, Prentice Hall, New Jersey, 1998.

[11] I. Daubechies, *Ten lectures on wavelets*, SIAM, Philadelphia, 1992.

[12] C.F. Chen and C.H. Hsiao, “Wavelet approach to optimising dynamic systems”, *IEE Proc. Control Theory Appl.*, vol. 146, March 1999, pp. 213-219.

[13] C.H. Lee, *Wavelet-based transient analysis. Ph.D. thesis*, Georgia Institute of Technology, Atlanta, 1998.