Approximations introduced by lumped resistances in a transmission line model

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Abstract – **Using lumped resistances causes a discretization error that is analyzed in this paper when the lumped resistances are small compared with the characteristic impedance of the line. The error depends on the variation of the current along the line. This variation is a function of the frequency and the impedance of the network connected to the line. The greatest deviation for a line model with three lumped resistances occurs in general for frequencies where the length of the line corresponds to twice the wavelength of the line multiplied by a positive integer. Examples gave an apparent resistance (i.e. the distributed resistance giving the same losses) between zero and twice the correct value. Reasonably accuracy is in most cases to be expected for frequencies up to 75% of the value where twice the wavelength corresponds to the length of the line. The apparent resistance may, however, deviate significantly from the correct value even at low frequencies. A significant improvement may then be achieved by increasing the number of lumped resistances from 3 to 5.**

Keywords – Transmission line model, losses, lumped resistances, accuracy, frequency domain analysis, time domain responses

I. INTRODUCTION

An accurate model for transmission lines with losses must take the frequency dependence of the series resistance and inductance into account. There exist models that do this in a satisfactory way. The use of such models is, however, somewhat limited due to the required input data. The user knows in most cases the resistance and inductance at power frequency only. Those data correspond to a lossless line model plus a constant (i.e. frequency independent) resistance. An accurate line model with this resistance distributed along the line does not introduce any significant simplification compared with a line model that takes the frequency dependency of the series impedance into account.

An alternative line model [1] has therefore been introduced (e.g. in EMTP) where the series resistance is represented by three lumped resistances, one at each end corresponding to ¼ of the total resistance, and one in the middle resistance corresponding to ½ of the total resistance. The model corresponds then to two lossless line segments and three lumped resistances, except that the additional nodes introduced by the resistances are eliminated. The elimination has the advantage that no numerical problem occurs when the resistance approaches zero.

The model with the lumped resistances is probably the most commonly applied model when losses are taken into account, mainly because it is very easy to use. It has further the advantage that it does not increase the total computation time significantly.

The model gives unreasonable results unless the lumped resistances are small compared with the characteristic impedance of the line. Most users are aware of this, but it is not uncommon to observe results that ignore this limitation.

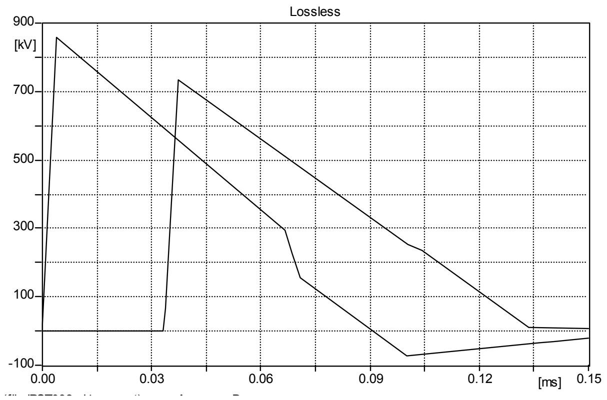
The influence of lumping the resistances may be significant even when the total resistance is small compared with the characteristic impedance. This paper analyses the effect of lumping the resistance. A major part of this analysis is performed in the frequency domain where the current distribution along the line is determined assuming that the resistance is approaching zero.

II. INCREASED PEAK VOLTAGE DUE TO LUMPED RESISTANCE MODEL

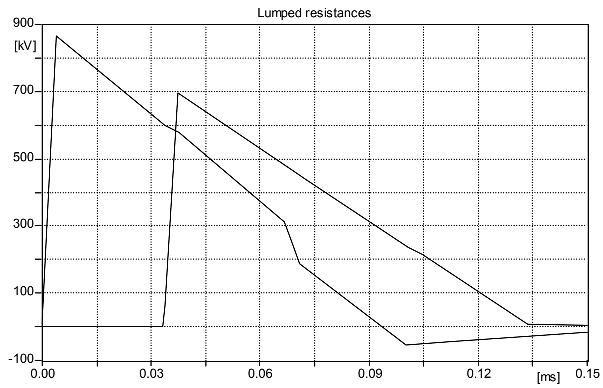
The line model with lumped resistances may give an increased voltage compared with a lossless model. This is shown in Fig. 1 where a 10 km single-phase line with 400 Ω characteristic impedance is inserted in a line with 300 Ω characteristic impedance. That line is assumed lossless and infinite long. A current is injected at one end of the 10 km line. The peak value of the current is 5 kA and it has linear increase and decrease with front time 4 μs and 50 μs time to half value.

The figure shows the voltage at both ends of the line. The resistance of the line is 4 Ω/km. No computation was made with the resistance distributed along the line. However, an additional computation was made with the lumped resistance model (3 lumped resistances) but with the line divided into 10 equal segments.

The voltage at the end (A) where the current is injected obtains its peak value after 4 μs and the peak value at the other end (B) is obtained after about 37 μs .



a) Lossless line model



b) Line model with lumped resistances

Fig. 1. Voltage response due to injected current at one end of a 10 km line

The following peak values were found:

Line model	Node A	Node B
Lumped resistances	866 kV	696 kV
Lumped resistances 10 line segments	858 kV	696 kV
Lossless	857 kV	734 kV

The increased peak voltage due to the lumped resistances is not very significant, but it shows that lumping the resistance may result in an increased peak voltage.

III. FREQUENCY DOMAIN ANALYSIS

The error caused by lumping the resistance depends on the variation of the magnitude of the current along the line. This variation is a function of the frequency and the external network connected to the line.

A very simple network is applied in this work in order to obtain results of general interest. One end of the line is assumed connected to an ideal voltage source U_o, and the other one to an impedance Z_{ex}. The current variation is calculated ignoring the resistance. This simplification is introduced since this paper focuses on cases where the lumped resistances are small compared with the characteristic impedance.

The current along the line can be expressed by the following equation:

$$I(x) = C \cdot \exp\left(-j\omega x / v\right) + D \cdot \exp\left(j\omega x / v\right) \tag{1}$$

where x is the position along the line and v is the speed of the traveling waves (i.e. the speed of light for an overhead line). C and D depend on the applied voltage U_o, the impedance Z_{ex}, the characteristic impedance of the line Z_{ch} and the length of the line. Details are presented in the appendix.

The losses along the line with the distributed resistance becomes:

$$P = r \cdot l \cdot I_D^2 \tag{2}$$

$$I_D^2 = \frac{1}{l} \int_0^l I(x) \cdot I(x)^* \, dx \tag{3}$$

r is the resistance per unit length and l is the length. I_D^2 is mean value of the square of the magnitude of the current.

Equation (1) gives:

$$I_D^2 = |C|^2 + |D|^2 + real \left[C \cdot D^* \cdot \frac{1 - \exp\left(-2j\omega l / v\right)}{j\omega l / v} \right] \tag{4}$$

The losses with the lumped resistances becomes:

$$P = r \cdot l \cdot I_L^2$$

where

$$I_L^2 = \frac{1}{4}\left[|I(0)|^2 + 2|I(l/2)|^2 + |I(l)|^2 \right] \tag{5}$$

I_L and I_D are proportional to U_o and depend otherwise on two parameters: ω l/v and Z_{ch}/Z_{ex}.

$Z_{ex} = Z_{ch}$ implies D = 0. The magnitude of the current is then constant (i.e. independent of the position) and $I_L^2 = I_D^2$ for all frequencies.

Analysis when $Z_{ex} \neq Z_{ch}$ has been made based on the following numerical values:

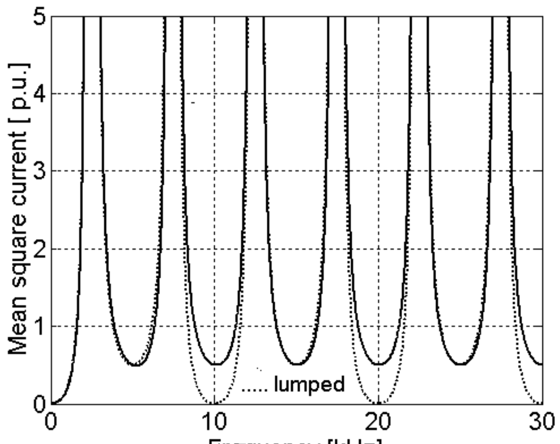
$Z_{ch} = 300\,\Omega$, v = c (speed of light) and l = 30 km. The results are presented here using U_o/Z_{ch} as p.u. reference for the current.

Fig. 2 shows I_D^2 and I_L^2 when $Z_{ex} = \infty$ and $Z_{ex} = 2 \cdot Z_{ch}$.

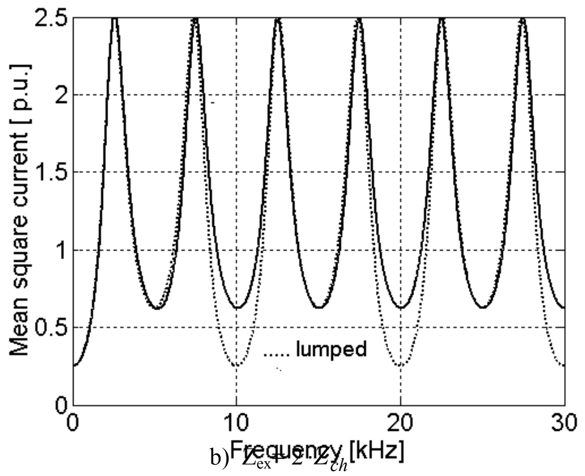
C and D are periodic with 10 kHz periodicity (i.e. adding 10 kHz to the frequency gives the same numerical value). I_L is periodic with 10 kHz periodicity. There is a good agreement between I_D^2 and I_L^2 except for a certain frequency range with center $n \cdot 10$ kHz where n is a positive integer. The values obtained with the lumped resistance model are here significantly lower.

Fig. 3 shows an example where Z_{ex} is less than Z_{ch}. The result is similar to Fig. 2 except that $I_L^2 > I_D^2$ when there is a significant deviation.

The deviation due to the lumping of the resistance can be expressed as an apparent resistance. This resistance is the distributed resistance giving the same losses as the lumped model. The resistance (in p.u.) becomes I_L^2 / I_D^2. Fig. 4 shows the results in Fig. 2 and 3 expressed as the apparent resistance. A fourth result corresponding to Z_{ex} = 0 is included as well.



a) $Z_{ex} = \infty$



b) $Z_{ex} = 2 \cdot Z_{ch}$

Fig. 2 I_D^2 and I_L^2 for two values of Z_{ex} higher than Z_{ch}

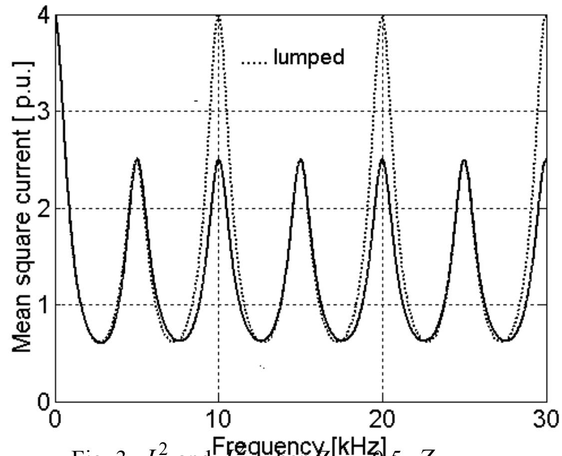
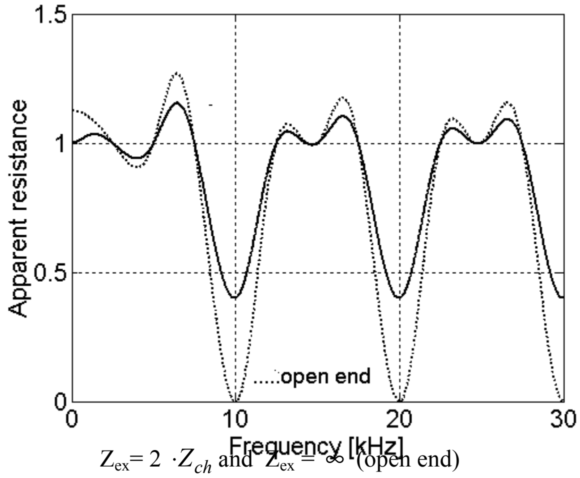
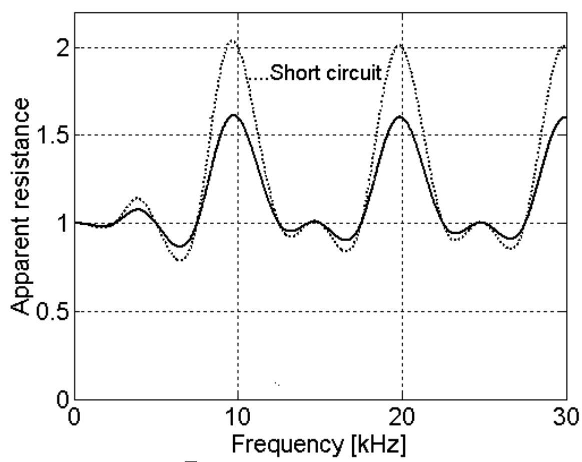


Fig. 3. I_D^2 and I_L^2 when $Z_{ex} = 0.5 \cdot Z_{ch}$

The apparent resistance is reasonably close to unity (i.e. $\pm$25%) up to about 7.5 kHz. The apparent resistance becomes zero at 10 kHz when $Z_{ex} = \infty$. It is further worth to note that $Z_{ex} = \infty$ does not give an apparent resistance that equals 1.0 when the frequency approaches zero.



$Z_{ex} = 2 \cdot Z_{ch}$ and $Z_{ex} = \infty$ (open end)



$Z_{ex} = 0.5 \cdot Z_{ch}$ and $Z_{ex} = 0$ (short-circuit)

Fig. 4 Apparent resistance

The current at low frequencies is in that case due to the capacitance of the line only and it varies linearly along the line and equals zero at the open end. This current variation gives an apparent resistance equal to 1.125.

The apparent resistance depends on the distribution of the magnitude of the current along the line. Fig. 5 shows as an example this distribution at three different frequencies when $Z_{ex} = 2 \cdot Z_{ch}$. I_L^2 depends on the current at the two ends and at the mid-point only. The current has its minimum value at these three positions at 10 kHz and that results in a very low value for the apparent resistance. The situation is very different at 6.44 kHz where the relatively low value for the current between relative position 0 and 0.5 does not influence I_L^2.

The external impedance Z_{ex} has so far been a resistance. Figs. 6 and 7 show the apparent resistance when Z_{ex} is a capacitance and an inductance respectively. The impedance Z_{ex} is then frequency dependent and the results are roughly in agreement with Fig. 4 except that the apparent resistance is close to 1.5 at 1.2 kHz when Z_{ex} is a 100 mH inductance. A similar peak appears with a lower value at a higher frequency when the inductance is 10 mH.

The current distribution when the inductance is 100 mH is shown in Fig. 8 for three selected frequencies. The current at the end (relative position 1) is inductive and a capacitive component is added when one moves along the line towards the source. The current at the source is at 600 Hz still inductive and a linear variation is observed in Fig. 8. The current at the source becomes capacitive for the two other frequencies. This implies that the current is zero at some location along the line. This location becomes the mid-point at 1200 Hz as shown in Fig. 8. A linear current variation with zero value at the mid-point gives an apparent resistance equal 1.5.

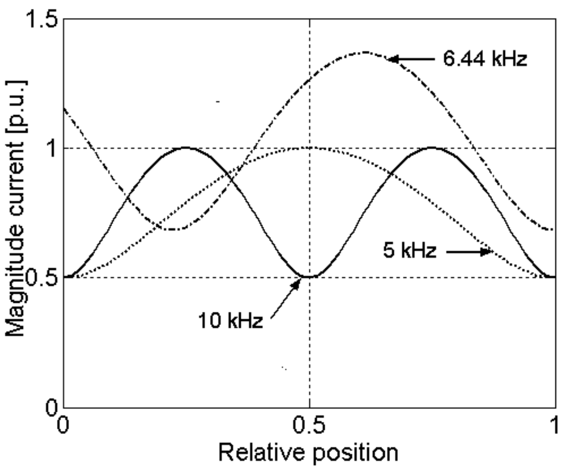


Fig. 5 Current (magnitude) distribution when $Z_{ex} = 2 \cdot Z_{ch}$

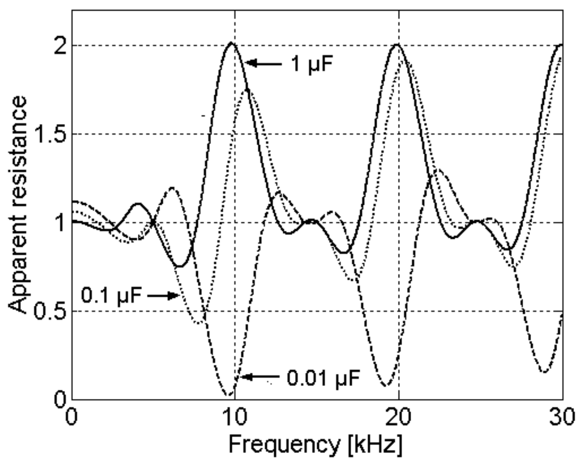


Fig. 6 Apparent resistance when the line is terminated by a capacitance

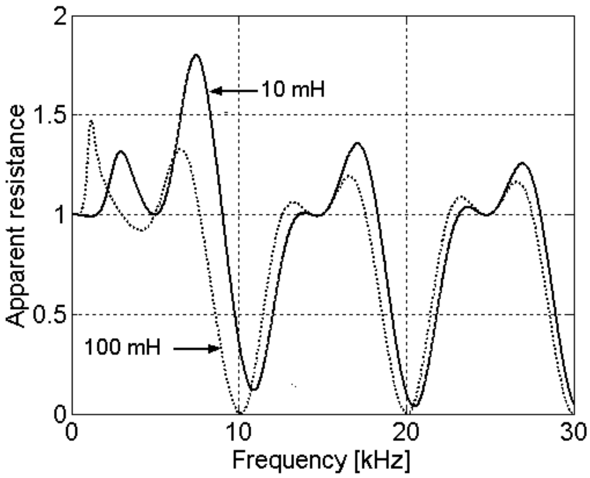


Fig. 7 Apparent resistance when the line is terminated by an inductance

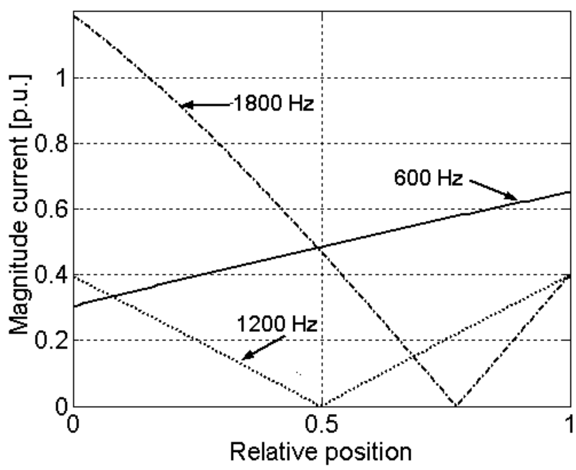


Fig. 8 Current (magnitude) distribution when the line is terminated by a 100 mH inductance

The lumped resistance model uses three lumped resistances and a natural improvement is to use five lumped resistances, i.e. to divide the original line into two equal segments and to consider each segment as a line. This implies roughly that the apparent resistance, as a function of the frequency, will be the same as shown so far if the values for the frequency are multiplied by 2. This approach is strictly not correct since it does not take the junction between the two line segments properly into account. Fig. 9 shows as an example the apparent resistance when using 3 and 5 lumped resistances for a line terminated by a 100 mH inductance. It is seen that the peak value at 1.2 kHz with 3 lumped resistances is substantially reduced without any increase at 2.4 kHz when using 5 lumped resistances.

IV. TRANSIENT RESPONSES

There is no need to lump the resistance in a frequency domain analysis. This approximation was introduced when developing a time domain model. Some time domain examples will therefore be presented here. Fig. 10 shows the step response at the open end when the other end is connected to an ideal voltage source with step voltage 1V. The line parameters are the same as in section III. The computation was performed with ATP and the series resistance was 1 Ω/ km.

Fig. 10 a) shows actually two responses, one corresponding to 3 lumped resistances and one corresponding to 5 lumped resistances. The deviation between the two responses is shown in Fig. 10 b). It is seen that the maximum deviation is about 0,2% of the source voltage. A greater deviation was expected from Figs. 4 and 9. The apparent resistance at 10 kHz is zero in Fig 4 (corresponding to 3 lumped resistances). Fig. 9 gives with 5 lumped resistances an apparent resistance equal to 1 p.u. at 10 kHz when the line is terminated by a 100 mH inductance. This inductance has, however, no significant influence at 10 kHz and above.

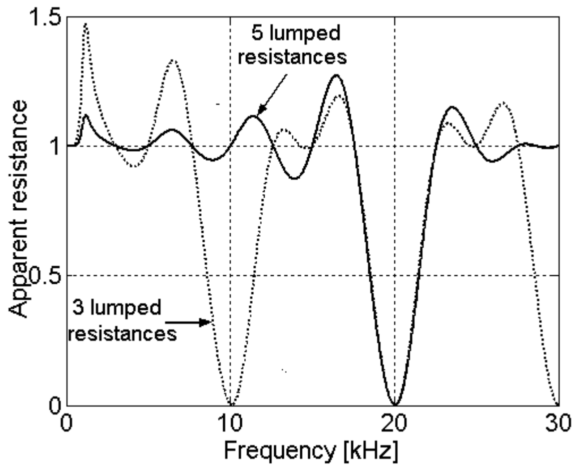
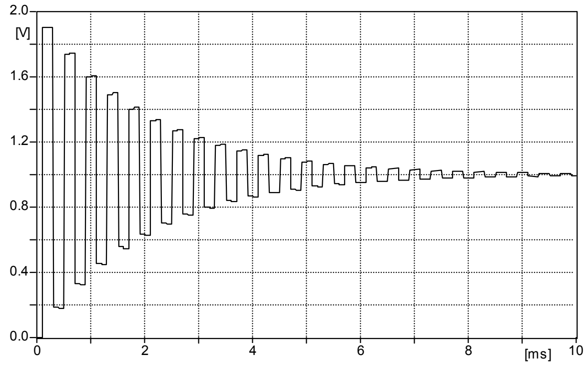
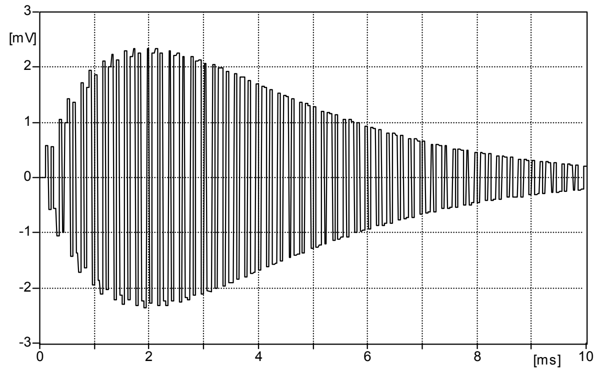


Fig. 9 Apparent resistance when using 3 and 5 lumped resistances. Line terminated by a 100 mH inductance



a) Step response open end



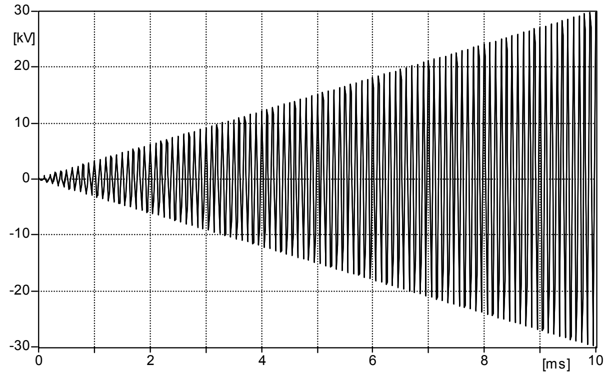
b) Deviation between models with 3 and 5 lumped resistances

Fig. 10 Time domain response, line with open end

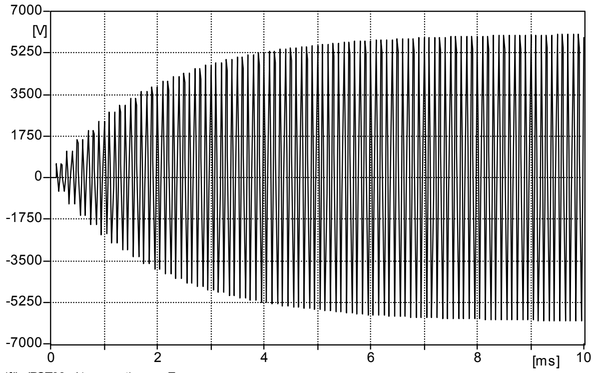
The reason for the minor deviation is probably due to the frequency spectrum of the response in Fig. 10 a). If there is no damping, it can be found that the dominating components correspond to the frequencies 2.5 kHz + $n \cdot 5$ kHz where n is an integer. The apparent resistance in Fig. 4 is 1 p.u for those frequencies. The damping gives additional frequency components (actually a continuous spectrum), but they are probably less important.

Fig. 11 shows an example where the response is strongly modified when using 5 lumped resistances instead of 3. The figure shows the open-end response when injecting a 10 kHz sinusoidal current at the other end. The frequency of the source corresponds to a natural frequency when the line is lossless. Using 3 lumped resistances gives a linear increase in the amplitude in Fig. 11 a). The amplitude is clearly limited by the resistance when using 5 lumped resistances (Fig. 11b). The amplitude after 10 ms is about 6 kV with 5 lumped resistances and about 30 kV with 3 lumped resistances.

The results in Fig. 11 agree well with Figs. 4 and 9.



a) 3 lumped resistances



b) 5 lumped resistances

Fig. 11 Open-end response when injecting a 10 kHz sinusoidal current

V. CONCLUSION

Representing the distributed resistance by 3 lumped resistances causes an error that depends on the variation in the current along the line. This variation is strongly influenced by the network connected to the line. It is possible to find a network where no error is introduced. Other examples give at certain frequencies no losses at all or an apparent resistance that is twice the correct value.

The most extreme deviation was found for frequencies where the line length corresponds to twice the wavelength of the line multiplied by a positive integer.

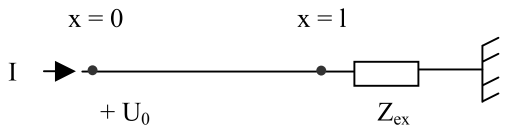
The lumping of the resistance gives in some cases a rather inaccurate result even at rather low frequencies. Dividing the line into two equal segments may then give a significant improvement.

REFERENCE

[1] H.W. Dommel : *Electromagnetic Transients Program (EMTP). Theory Book.* Bonneville Power Administration. Portland, Oregon 1987, section 4.2.2.5

APPENDIX

CURRENT DISTRIBUTION ALONG THE LINE



The general solution for the voltage along the line is:

$$U(x) = A \cdot \cosh(\gamma x) + B \cdot \sinh(\gamma x) \qquad \text{(A1)}$$

where

$$\gamma = j\,\omega / v \qquad \text{(A2)}$$

v is the traveling wave speed
A and B depend on the terminal conditions.

The current equals:

$$I(x) = -[A \cdot \sinh(\gamma x) + B \cosh(\gamma x)] / Z_{ch} \qquad \text{(A3)}$$

where Z_{ch} is the characteristic impedance.

Terminal conditions:

$$U(0) = U_0$$

and (A4)

$$U(l) / I(l) = Z_{ex}$$

gives

$$A = U_0 \qquad \text{(A5)}$$

$$B = -\frac{Z_{ex} \cdot \sinh(\gamma l) + Z_{ch} \cdot \cosh(\gamma l)}{Z_{ch} \cdot \sinh(\gamma l) + Z_{ex} \cdot \cosh(\gamma l)} \cdot U_o \qquad \text{(A6)}$$

An alternative expression for I (x) is:

$$I(x) = C \cdot \exp(-\gamma x) + D \cdot \exp(\gamma x) \qquad \text{(A7)}$$

Comparing A3 and A7 and introducing A5 and A6 give:

$$C = \frac{1}{2} \cdot \frac{(Z_{ex} + Z_{ch}) \exp(\gamma l)}{Z_{ch} \cdot \sinh(\gamma l) + Z_{ex} \cdot \cosh(\gamma l)} \cdot \frac{U_0}{Z_{ch}} \qquad \text{(A8)}$$

$$D = \frac{1}{2} \cdot \frac{(Z_{ch} - Z_{ex}) \exp(-\gamma l)}{Z_{ch} \cdot \sinh(\gamma l) + Z_{ex} \cdot \cosh(\gamma l)} \cdot \frac{U_0}{Z_{ch}} \qquad \text{(A9)}$$