State-Space Transient Analysis of Single-Phase Transmission Lines with Corona

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Abstract - **State-space techniques are used for the computation of the surge response of a transmission line with corona. The transmission line is treated as the series connection of short line sections, and the effect of corona is represented in each section by a nonlinear (voltage dependent) capacitor-conductance branch. Surge response of the transmission line is obtained for various corona models by taking into account the corona loss and the dynamic variation of the corona parameters. Computed results are compared with measured results available in the literature.**

Keywords - transmission line, electromagnetic transients, corona, state-space technique, nonlinear elements.

I. INTRODUCTION

Corona is an electrostatic discharge mechanism which occurs due to ionisation in an insulation material subjected to electric field intensity over a critical level. Corona is the cause of power loss on transmission lines, audible noise, and electromagnetic interference in communication systems. As the corona phenomenon has significant effects, it has drawn great attention and becomes an important aspect in many engineering areas. Analysis of corona is important for power system protection since lightning surges travelling on transmission lines are significantly infuluenced by corona, and determination of electromagnetic transients for the prediction of insulation level and design of the surge arrestors requires that corona effects be included.

One way to determine the effect of corona is to perform the experiments. However, for every set of new parameters (such as the shape and magnitude of the applied voltage, the dimensions and physical properties of the transmission system, etc.), this requires performing of a new experiment, and in many cases, even the experimental results may not be available because physical system does not exist yet as in the design problems.

The equations describing the corona behaviour cannot be combined with the partial differential equations of transmission lines in a closed-form formulation. Therefore, numerical methods based on time domain solutions such as the finite difference methods [1-3] and the method of characteristics [4,5] are generally used, or models involving distributed nonlinear hysteresis-loop behaviour are evaluated for implementation in the EMTP (Electromagnetic Transients Program) modelling [6-11]. In some of these models, nonlinear (voltage dependent) resistors and capacitors are used, which necessitates the use of special analysis techniques [6-9]. In some others linear constant capacitors and resistors, diodes and dc voltage sources are commonly used [10,11]. However, most of the computational methods available are applicable to some specific models and cannot easily extend to other corona representations.

In this paper, the state-space technique presented in [12] is used for the computation of electromagnetic transients on a single-phase transmission line with corona. The method is based on the formulation of state equations from the lumped-parameter transmission line model and conversion of these equations into a set of linear algebraic equations by the use of trapezoidal rule of integration. To simulate the corona mechanism, a nonlinear shunt capacitance is used to represent *q-v* characteristic of the charge accumulation, and a nonlinear conductance is used to characterize the power loss in corona. Two analytical corona models available in the literature [2,13,14] are examined involving the corona loss and the dynamic variation of the corona parameters. Advantages of the state-space method over the existing methods are that no convergence, initialisation, instability problems, and no restrictions such as the number and configuration of nonlinear elements.

II. STATE-SPACE REPRESENTATION

A. Formulation

Transmission line is a distributed parameter system. Current and voltage relations on a transmission line are expressed by partial differential equations, known as Telegrapher’s Equations. When corona effects are included, solution of these equations is difficult. To overcome this difficulty, lumped parameter transmission line modelling is used, and the transmission line is represented by a large number of identical lumped parameter sections connected in series to simulate its distributed nature [15-17]. Nonlinear (voltage dependent) capacitance-conductance branches of the corona can be combined with the geometric shunt parameters of the transmission line and L-equivalent model shown in Fig. 1 is thus used to simulate one section of the transmission line. In this model, R and L are the resistance and inductance of one section, nonlinear G is the shunt conductance representing corona losses, and nonlinear C represents both the geometric and the corona capacitance. R and L in this model are constants, but G and C are functions of the voltage due to corona. The lumped parameter model for a single-phase transmission line with corona, which is obtained by connecting many lumped parameter sections in series, is shown in Fig. 2. The total number of L-sections in this model is assumed to be n. The state equations for the transmission line with corona can be written in matrix form as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \qquad (1)$$

where $\boldsymbol{x}$ is the state vector with the initial value $\boldsymbol{x_o}$, $\boldsymbol{A}$ and $\boldsymbol{B}$ are the coefficient matrices with proper dimensions, and $\boldsymbol{u}$ represents the vector of inputs. Assuming a voltage source at the sending-end of the line, and using the network theory we obtain the state equations as:

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ v_1 \\ i_2 \\ v_2 \\ \vdots \\ i_n \\ v_n \end{bmatrix} = \begin{bmatrix} -R/L & -1/L & 0 & & & \cdots & 0 \\ 1/C_1 & -G_1/C_1 & -1/C_1 & 0 & & \cdots & 0 \\ 0 & 1/L & -R/L & -1/L & 0 & \cdots & 0 \\ 0 & 0 & 1/C_2 & -G_2/C_2 & -1/C_2 & & \vdots \\ \vdots & & & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & & & 1/L & -R/L & -1/L \\ 0 & \cdots & & & 0 & 1/C_n & -G_n/C_n \end{bmatrix} \begin{bmatrix} i_1 \\ v_1 \\ i_2 \\ v_2 \\ \vdots \\ i_n \\ v_n \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \\ 0 \\ 0 \\ \vdots \\ \\ 0 \end{bmatrix} u(t) \qquad (2)$$

The above set of first order differential equations are obtained by choosing inductor currents and capacitor voltages in the transmission line model as the state variables. In this equation the elements of the state vector are inductor currents (i_j, $j=1,2,...,n$) and capacitor voltages (v_j , $j=1,2,3,..,n$) shown in Fig. 2, and Gj and Cj are conductance and capacitance of section j, respectively.

B. Solution of State Equations

Since the capacitors and conductances of the line with corona are voltage dependent, many elements in matrix $\boldsymbol{A}$ are variable and the state equations describing the behaviour of the system cannot be solved analytically. One way to solve the nonlinear state equations is to transform these equations into a set of linear algebraic equations by employing the numerical integration methods [12]. Time discretization used for numerical integration facilitates representation of nonlinear variations in the system.

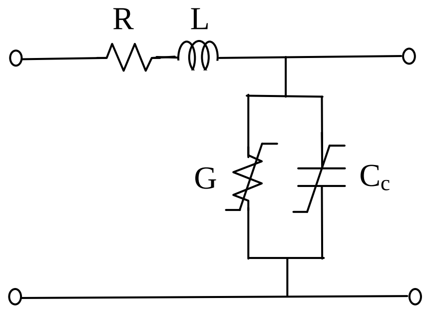


Fig. 1 L-section of a transmission line segment with corona.

By employing the trapezoidal rule of integration, the above set of coupled differential equations can be transformed into a set of linear algebraic equations as

$$(\boldsymbol{I} - \frac{h}{2}\boldsymbol{A_k})\boldsymbol{x_{k+1}} = (\boldsymbol{I} + \frac{h}{2}\boldsymbol{A_k})\boldsymbol{x_k} + \frac{h}{2}\boldsymbol{B}(\boldsymbol{u_k} + \boldsymbol{u_{k+1}}) \qquad (3)$$

This equation can also be written as

$$\hat{\boldsymbol{A}}_{\boldsymbol{k}}\boldsymbol{x_{k+1}} = \boldsymbol{b_k} \qquad (4)$$

where

$$\hat{\boldsymbol{A}}_k = (\boldsymbol{I} - \frac{h}{2}\boldsymbol{A_k}) \qquad (5)$$

and

$$\boldsymbol{b_k} = (\boldsymbol{I} + \frac{h}{2}\boldsymbol{A_k})\boldsymbol{x_k} + \frac{h}{2}\boldsymbol{B}(\boldsymbol{u_k} + \boldsymbol{u_{k+1}}) \qquad (6)$$

In the above set of equations, subscript k and $k+1$ for the state vector x and input u denotes the values of these vectors at $t_k=k\Delta t$ and $t_{k+1}= (k+1)\Delta t$, respectively. Before starting to a new state, the coefficient matrices are renewed depending on terminal voltages of the nonlinear elements in the system. Subsequently, the new coefficient matrix $\boldsymbol{A_{k+1}}$ is determined. Then, the state vector at a discrete time point is determined by solving Eqn. (4) and thus the response of the system is obtained by repetitively solving this set of equations starting from $k=0$. The flowchart for the state-space algorithm is shown in Fig. 3. For the solution of the linear set of equations in (4), LU decomposition given in the next section has been applied.

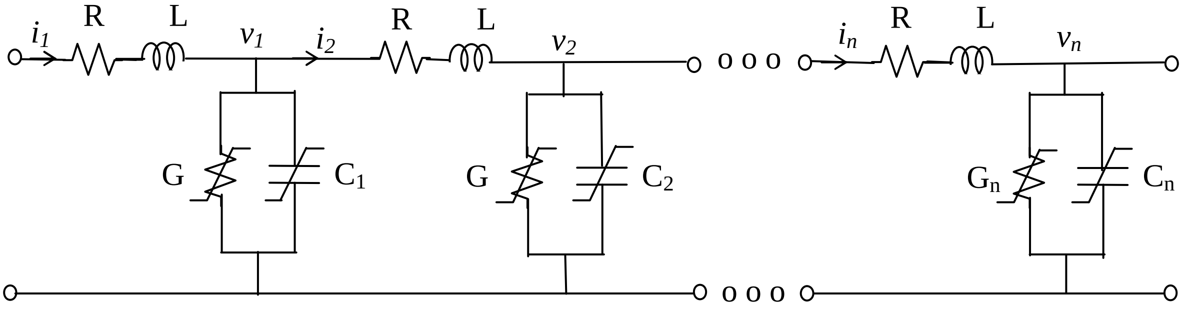


Fig. 2 Equivalent model of a transmission line with corona.

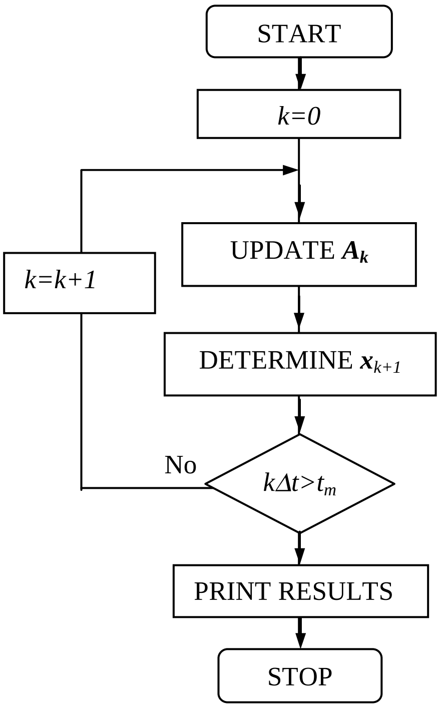


Fig. 3 Flowchart for the state-space algorithm.

C. LU Algorithm

Using matrix operations and inversion for the solution of the set of simultaneous equations in (4) is time consuming and may lead to inaccurate results. To overcome these difficulties, the linear system of equations in the form

$$\boldsymbol{Ax} = \boldsymbol{b} \tag{7}$$

can be solved using LU decomposition. The square matrix $\boldsymbol{A}$ is decomposed as $\boldsymbol{A}=\boldsymbol{LU}$, where $\boldsymbol{L}$ is a lower triangular matrix in which the leading diagonal elements are unity and $\boldsymbol{U}$ is an upper diagonal matrix [20]. Using this decomposition, the set of linear equations can be written as

$$\boldsymbol{LUx} = \boldsymbol{Ly} = \boldsymbol{b} \tag{8}$$

To solve the above equation for $\boldsymbol{x}$, the vector of unknowns, firstly intermediate variable $\boldsymbol{y}$ is determined by solving $\boldsymbol{Ly} = \boldsymbol{b}$ using forward substitution, and then $\boldsymbol{x}$ is determined by solving $\boldsymbol{Ux} = \boldsymbol{y}$. The expression for computing elements of matrix $\boldsymbol{U}$ and $\boldsymbol{L}$ are

$$u_{11} = a_{11}$$

$$\left.\begin{aligned} l_{i,i-1} &= a_{i,i-1} / u_{i-1,i-1} \\ u_{i-1,i} &= a_{i-1,i} \\ u_{i,i} &= a_{i,i} - l_{i,i-1} u_{i-1,i} \end{aligned}\right\} \text{for } i = 2,3,\cdots,n \tag{10}$$

Note that it is possible to overwrite $\boldsymbol{A}$ with the elements of ($\boldsymbol{L}$+$\boldsymbol{U}$-$\boldsymbol{I}$) as they are formed, which reduces computer memory requirements.

III. CORONA MODELS

A number of models have been used for the simulation of corona in power systems. Many of them are presented to investigate overvoltages in high-voltage power transmission lines due to lightning, energization, and other sources of transients. These models can be classified as static and dynamic models depending on the functions describing the variation of the corona capacitance. In static models, straight line or parabolic approximations of *q-v* curves of corona are used. On the other hand, in dynamic models, the change of corona capacitance is dependent on both the voltage and the rate of change of the voltage. In this study, two available analytical corona models are adopted; i) Gary's model [2], ii) Skilling [13]-Umoto [14] model. These models are described in the next sections briefly. Note that, the corona capacitance and conductance are extra elements connected to the circuit when the voltage across these elements v is greater than the corona inception voltage v_c and $(dv/dt) > 0$, otherwise they are zero. The second condition characterises the dynamic variation in the corona parameters, and the derivative of the voltages can be determined from their algebraic equivalents in right hand side of Eq. (2).

A. Gary's model

The corona capacitance presented by Gary et al. [2] involving the geometric line capacitance is defined as

$$C_c = C_0 \eta (v/v_c)^{\eta\text{-}1} \tag{11}$$

where C_c is the corona capacitance, C_0 is the geometric line capacitance, v_c is the corona inception voltage and η is a coefficient, which for a single conductor, is given by the following experimental formula [21]

$$\eta = 0.22r + 1.2 \tag{12}$$

where r is the conductor radius in centimetres.

B. Skilling and Umoto Model

As a second, the nonlinear corona capacitance presented by Skilling [13] and Umoto [14] is adapted for the state-space simulation. The corona capacitance is defined as

$$C_C = 2k_C(1 - v_C / v) \quad \text{F/m} \tag{13}$$

where v_C is the corona inception voltage,

$$k_C = \sigma_C \sqrt{r/2h} x 10^{-11} \tag{14}$$

and σ_C is corona loss constant, r and h are radius and height above ground of conductor, respectively.

C. Modelling of Corona Loss

The occurrence of corona discharge produces changes not only in the instantaneous values of the line capacitance, but also the line conductance. The corona attenuation loss is modelled by a resistive current loss through the resistive branch to ground which is defined as [21]

$$G_C = k_R(1 - v_C / v)^2 \quad \text{mho/m} \tag{15}$$

where

$$k_R = \sigma_G \sqrt{r/2h} x 10^{-11} \quad \text{mho/m} \tag{16}$$

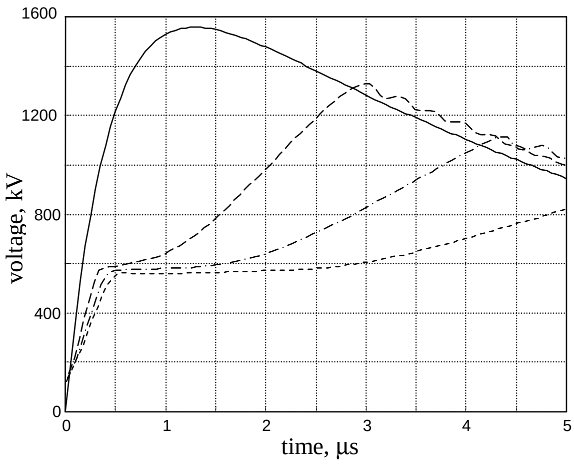
and σ_G is the corona loss constant.

IV. APPLICATION AND RESULTS

As an application, the surge response of a 2185.4 m transmission line subjected to a 1.3/6.2μs double exponential voltage surge with an amplitude V=1560 kV is computed by the state-space method. The simulation attempts to reproduce a field test reported in [22]. The corona voltage is v_c=550 kV, conductor radius of the transmission line is 2.54 cm, and the average height above ground is 18.9 m. High frequency parameters of the line are r=11.35 Ω/km, l=1.73 mH/km, and c=7.8 nF/km. Variation of the voltages obtained by the state-space method for x=655.6 m, x=1291.4 m and x=2185.4 m together with the applied impulse wave (voltage at x=0 m) using Gary's model and Skilling-Umoto model are given in Fig. 4a and b, respectively. The corona constants for the Skilling-Umoto model are taken as σ_C=30 and σ_G=10x10^6 [6]. The voltage curves are plotted by shifting in the order of their travel delay on time axis and another transmission line with the same characteristic is used for the termination at sending-end to avoid the reflections. Considering fast rising time of the applied surge, step length for numerical integration is chosen as Δt=0.05 μs and 110 lumped parameter L-sections are used for the simulations. The measured surge voltages [22] on the line are shown in Fig. 5. When the computed results are compared with the measured ones, a good agreement can be observed between the curves obtained by the state-space technique using Skilling-Umoto model and experimental ones. However, there are some differences between the measured results and the curves obtained by using Gary's model, which may be due to corona capacitance being more effective. The ripples after the wave crest in the computed results are also more definite in the case when Gary's model is used.

V. CONCLUSIONS

State-space techniques are used for the computation of surge response of a transmission line with corona. Available analytical corona models in the literature are adapted considering the corona losses and the dynamic change of the corona capacitance. The obtained results are compared with the experimental results available in the literature. Comparisons show that the model suggested by Skilling and Umoto with proper corona constants is more satisfactory than Gary's model in reproducing the experimental results.



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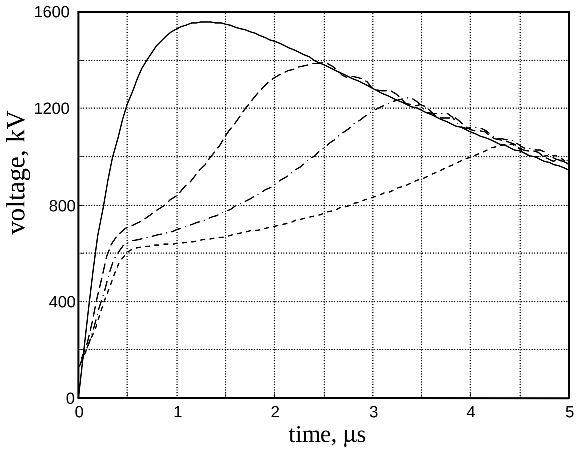

b

Fig. 4 Surge response of the line with corona computed using *a*) Gary's model, *b*) Skilling-Umoto model.
—— x=0 m, – – – x=655.6 m,
–·– x=1291.4 m - - - - x=2185.4 m

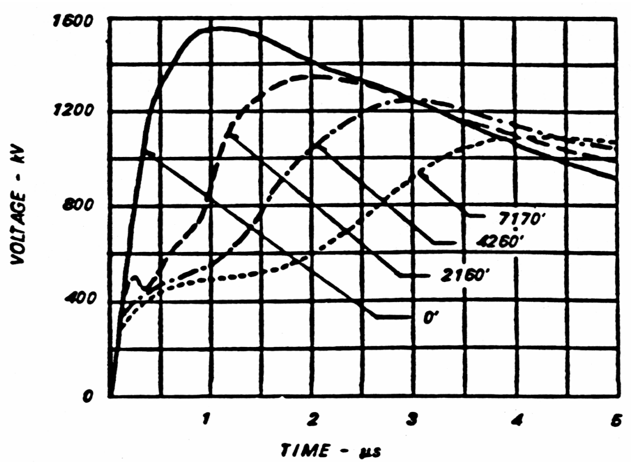

Fig. 5 Measured surge response of transmission line with corona [22]. Voltages at;
—— x=0 m, – – – x=658.4 m,
–·– x=1295.5 m - - - - x=2185.4 m

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