Electromagnetic Transients Simulation with Different Time Steps – The Latency Approach

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***Abstract* – This paper presents techniques for an efficient and accurate latency exploitation of electric network solutions based on time-domain transients simulation program, such as the "Electromagnetic Transients Program" (EMTP). Latency or multirate simulation is related to the capability of numerically solving the differential equations governing the behaviour of electric networks with multiple integration steps. With this approach, the limitations of a single and fixed-size integration step as required by EMTP-type programs can be overcome, resulting in a decreased number of numerical operations for a given total simulation time. Using the MATE solution framework of "Multi-Area Thévenin Equivalents", latency exploitation is achieved using non-iterative solutions. A test case is derived and results are presented and compared with those obtained from conventional EMTP simulations.**

Keywords – Latency exploitation, Network decoupling, MATE, OVNI, EMTP, Integration rules, Electromagnetic transients.

I. INTRODUCTION

The computer simulation of electromagnetic transients phenomena in power systems became possible with the development of the EMTP [1]. For just over thirty years now, the EMTP in its several versions has been used with great reliability for power system transients simulation. One of the limitations of conventional EMTP-type simulators, however, is their incapability of performing real-time simulation. There are situations where very sensitive equipment needs to be tested under the same conditions it would face in the actual power system. Thanks to the continuous advance in hardware capability, digital real-time simulation of electromagnetic transients in power systems, once considered impractical due to the small integration step requirements are now becoming common in studies performed by electric utilities and research groups. More details about some of the real-time digital simulators available can be found in [2,3,4,5].

The field of application of EMTP-type real-time digital simulators is still somewhat restricted to small networks, usually with the purpose of testing protective relays and HVDC and FACTS control systems. The next natural evolutionary step in real-time transient simulation is the development of a real-time power system simulator aimed at representing full-size power system networks. Ideally, due to cost and portability reasons, the simulator should be capable of performing real-time and online simulations using very fast and readily-available personal workstations. The real-time research group at UBC is actively involved in the development of very efficient hardware architecture, solution methods, and algorithms with the purpose of achieving this goal.

Regarding the development of very efficient solution methods, our group has primarily been focusing on network partitioning techniques. From our previous experiences with real-time simulation [5], we realized that the exploitation of partitioning techniques in power system networks may result in a much more efficient solution technique for real-time simulation than the traditional approach followed by the EMTP. The EMTP simulates the network as a single entity where any topological change requires the retriangularization of the full admittance matrix. Employing network partitioning across natural boundaries that power networks usually present allows subnetworks changes to be treated individually without having to retriangularize the full system.

One of the natural ways to separate solution regions is to recognize that some subsystems or components have large time constants, and relatively large steps Δt are sufficient for an accurate solution, while some other subsystems have small time constants and require much smaller Δt*'s*. In these cases it is wasteful to solve the slow system at each time step of the fast system solution. The possibility of solving the differential equations governing the behaviour of electric networks with dual or multiple time steps is termed latency exploitation. Although simple in theory, the partitioning technique required for successful latency exploitation is not trivial. The interface between the "fast" and "slow" subsystems has to be adjusted in order to guarantee an accurate and efficient simulation. Incorrect approaches in the coupling of the "slow" and "fast" system solutions can lead to strong numerical oscillations and divergence of long-term simulation.

Section II of this paper describes the MATE ("Multi-Area Thévenin Equivalent") partitioning technique used as a framework for latency exploitation while in section III the network equations for latency exploitation are rigorously formulated. Section IV presents the results obtained when exploiting latency for a particular test case and compares the results in terms of accuracy and efficiency with those obtained from traditional EMTP simulations. Finally, in section V the conclusions are stated.

II. MATE (MULTI-AREA THÉVENIN EQUIVALENT)

The several years of experience of our group in the development of real-time power system transient simulators has most recently culminated in the OVNI ("Object Virtual Network Integrator") simulator [7]. The solution method followed in OVNI is based on network partitioning techniques. Network tearing concepts for the solution of electrical networks were introduced by Gabriel Kron in the 1950's [8]. These techniques, which Kron called diakoptics, take advantage of the very sparse nature of the admittance matrix of power networks and divide the network solution into two parts: the solution of the relatively dense subsystems and the solution of the "links" connecting the subsystems. Following similar ideas as in diakoptics, Ho, Ruehli, and Brennan in 1975, introduced the method of Modified Nodal Analysis (MNA) [10] to combine in the same system of equations nodal equations for "normal" system nodes with branch equations for branches with special characteristics (e.g., current dependent elements)

Even though the original motivation of diakoptics was to take advantage of the sparsity of the system (and was eventually superseded in this arena by Sparsity Techniques pioneered by Tinney and Walker [9]), a secondary benefit, which has become manifest only recently with the advent of real-time simulators, is that it maintains the identity of the component subsystems, something that is very difficult to achieve in solutions based on matrix reductions with Sparsity techniques. With the individuality of the subsystems maintained it is possible to pre-calculate (before the simulation loop) the inverse solution matrices for those subsystems which topology does not change, while only some of the subsystems require recalculation.

The ideas of diakoptics are further extended in the MATE solution framework developed at UBC as the base solution for the OVNI real-time simulator [6,7]. The main advance in MATE is the recognition that the component subsystems in the partitioned network can be represented by Thévenin equivalents at the moment of resolving the link equations. This becomes a key point when combining together, in a simultaneous non-iterative solution, subsystems that have been solved with different solution techniques or integration steps. The latter is the latency case which will be further explored in this paper.

Reference [7] explains in detail the MATE formulation and only some of the concepts are repeated here. Figure 1 (from [7]) shows an example network from which the basics of MATE's solution can be indicated.

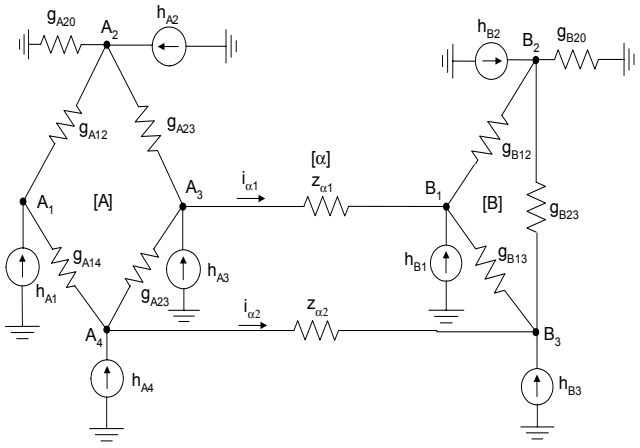

Fig.1: Sample network to explain MATE

The following notation is introduced:

[A] = admittance matrix of subnetwork [A]
[B] = admittance matrix of subnetwork [B]
[α] = links connecting subnetworks [A] and [B]
$i_{\alpha 1}$ = current in link branch α_1
$i_{\alpha 2}$ = current in link branch α_2
$z_{\alpha 1}$ = impedance of link branch α_1
$z_{\alpha 2}$ = impedance of link branch α_2
h_A and h_B are the current sources injected from ground into the nodes of subnetworks [A] and [B] respectively

For the MATE solution the following procedure should be adopted:

1. Advance the solution time by $t = t + \Delta t$;
2. Build the admittance matrices for subnetworks [A] and [B] as if they were completely independent;
3. Update the history sources terms for subnetworks [A] and [B];
4. Solve for the internal node voltages v_{A1}, v_{A2}, v_{A3}, v_{A4} of subnetwork [A] and v_{B1}, v_{B2}, v_{B3} of subnetwork [B], still considering [A] and [B] completely decoupled;
5. Solve for the link currents $i_{\alpha 1}$ and $i_{\alpha 2}$ using the node voltage values calculated in step 4;
6. Update the internal node voltages of subnetworks [A] and [B] by injecting the link currents $i_{\alpha 1}$ and $i_{\alpha 2}$ into the corresponding nodes;
7. Go back to step 1.

It should be realized that the node voltages calculated in step 4 are actually the Thévenin equivalent sources for subsystems [A] and [B] and the interaction between subnetworks occurs when injecting the link currents into the corresponding nodes. The node voltages are then precisely calculated in step 6.

III. THE LATENCY METHODOLOGY

The search for efficient and accurate solution methods exploiting latency is not new. Since the 1980's researchers in the area of "Very Large Scale Integration" (VLSI) have been reporting the limitations of single and fixed-size time

step simulations. The difficulties of the general circuit simulator SPICE, for example, to simulate complex VLSI circuits in a time and cost effective manner are reported in [11]. It was in this context that researchers turned attention to latency exploitation.

Latency was first applied in power systems simulation for transient stability studies [12,13]. For electromagnetic transient, latency was proposed for the first time in [14]. Several case studies in [14] demonstrate the validity of latency exploitation as a very efficient simulation method. Resynchronization of the separate solutions for different parts of the network, however, is not made clear in this paper. In a recent paper [15] we have reproduced many of the results shown in [14], and in the present paper we describe in detail the necessary adjustments on the interface between the fast and slow subsystems.

Figure 2 depicts a schematic diagram showing the interconnection of the slow and fast subsystems

As already explained for MATE, the full network is divided into distinct subnetworks, in this case one subnetwork containing fast elements and one subnetwork containing slow elements. These subnetworks are connected through a link. Each subsystem contains its own voltage and current sources, as well as classical passive elements.

The MATE technique guarantees the exact solution of the complete network if both subnetworks are first solved separately and the node voltages are subsequently updated taking into account the current flowing through the link. However, again according to MATE, the solution is only exact if the same integration step is used for both subnetworks.

It is still possible to guarantee very accurate results exploiting latency as shown here if each of the subsystems can be characterized as being either slow or fast, i.e., if the dominant eigenvalues in each of the subnetworks are fairly independent. The generalization of the latency methodology through an eigenanalysis is out of the scope of this paper, but it will be the subject of future publications by the same authors.

In the latency technique proposed in this work, the large time step should always be a multiple of the small time step.

$$\Delta T = n\Delta t \tag{1}$$

Where n is a positive integer, larger than unity and ΔT and Δt are, respectively, the large and small time steps.

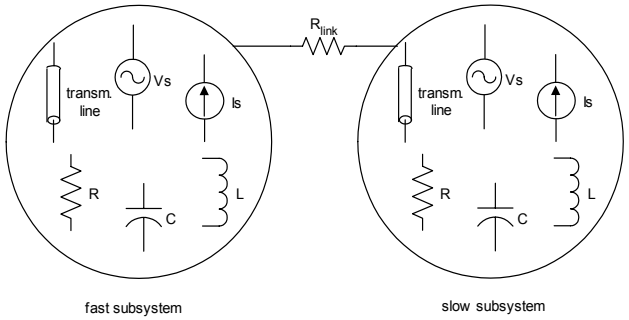

Fig. 2: Diagram showing the interconnection of the fast and slow subsystems

Figure 3 shows the timeline of the simulation at a general time interval.

In order to guarantee a stable solution, both the slow-varying and the fast-varying subsystems are solved at instants multiple of the large time step ΔT. These are the moments when the solutions are resynchronized. At the small time step intervals Δt's within the large step ΔT only the fast varying subsystem is solved. Two important concerns arise here: how to consider the contribution of the slow-varying subsystem when solving only for the fast one and how to take into account the contribution of the fast-varying subsystem through a complete ΔT interval. The approach presented in this paper tries to find the best compromise between accuracy and efficiency of the simulation.

A. History source accumulation for resynchronized solution

Whenever it is time to solve for the entire network (at every instant multiple of the large time step ΔT) the history sources for both the slow-varying and the fast-varying subnetworks should be updated. For the slow-varying subnetwork, the previous solution was calculated at the instant $t-\Delta T$, so the history sources are naturally updated with the values at this time instant. On the other hand, during the large integration step ΔT, the fast-varying subsystem has been solved **n** times, according to (1). It is, therefore, more accurate to take into account all the values calculated for the fast elements during the period ΔT when evaluating their history sources.

We now derive the expanded discretized solution for an inductor and a capacitor using the trapezoidal integration rule, bearing in mind that this formulation is applied only to the network components located within the fast-varying subsystem. Also, it is valid only for the time instants when the full network solution should be obtained. This derivation results in an expanded form of the trapezoidal integration rule for latency exploitation. Starting from the voltage-current relationship in an inductor and a capacitor, respectively:

$$v_L(t) = L\frac{di_L(t)}{dt} \tag{2}$$

$$i_C(t) = C\frac{dv_C(t)}{dt} \tag{3}$$

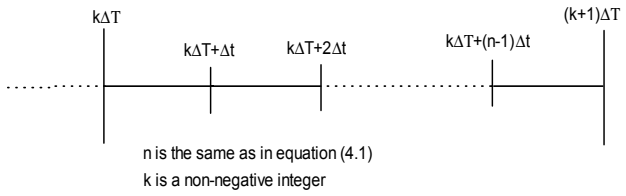

Fig. 3: Timeline of the simulation at a general time interval when latency is exploited

Integrating (2) and (3) over the large time step ΔT:

$$\int_{t-\Delta T}^{t} v_L(t)\,dt = L\left[i_L(t) - i_L(t-\Delta T)\right] \tag{4}$$

$$\int_{t-\Delta T}^{t} i_C(t)\,dt = C\left[v_C(t) - v_C(t-\Delta T)\right] \tag{5}$$

Applying the trapezoidal integration rule to evaluate the left-hand terms of (4) and (5) results in

$$\int_{t-\Delta T}^{t} v_L(t)\,dt = \frac{\Delta t}{2}\left[v_L(t) + v_L(t-\Delta T)\right] + \Delta t\sum_{k=1}^{n-1} v_L(t-\Delta T + k\Delta t) \tag{6}$$

$$\int_{t-\Delta T}^{t} i_C(t)\,dt = \frac{\Delta t}{2}\left[i_C(t) + i_C(t-\Delta T)\right] + \Delta t\sum_{k=1}^{n-1} i_C(t-\Delta T + k\Delta t) \tag{7}$$

The difference equations giving the voltage-current relationships for the inductance and the capacitance can then be written as

$$v_L(t) = \frac{2L}{\Delta t} i_L(t) + eh_L(t) \tag{8}$$

$$v_C(t) = \frac{\Delta t}{2C} i_C(t) + eh_C(t) \tag{9}$$

Where $eh_L(t)$ and $eh_C(t)$ are the history sources of the inductance and the capacitance, respectively, and are given by

$$eh_L(t) = -\frac{2L}{\Delta t} i_L(t-\Delta T) - v_L(t-\Delta T) - 2\sum_{k=1}^{n-1} v_L(t-\Delta T + k\Delta t) \tag{10}$$

$$eh_C(t) = v_C(t-\Delta T) + \frac{\Delta t}{2C} i_C(t-\Delta T) + \frac{\Delta t}{C}\sum_{k=1}^{n-1} i_C(t-\Delta T + k\Delta t) \tag{11}$$

Equations (10) and (11) give the history sources for the inductance and the capacitance according to the trapezoidal integration rule with all the information gathered for these elements within a large integration step ΔT. This consideration prevents the quasi-randomness of taking into account only the single value of the last calculated solution. Although, in principle, it may seem that a significant increase in computational burden is incurred, the fact is that, in the code implementation, the summation part of the history terms (10) and (11) is updated at the end of each small time step Δt. Since this calculation is made over the period that the slow-varying subsystem is latent, there is no significant increase in the number of numerical operations per time step.

B. Thévenin equivalent interpolation for fast-varying subsystem solution

Referring again to the schematic diagram depicted in Fig. 2, it is readily observed that the link current may be calculated if both subsystems are first transformed into their Thévenin equivalents, as suggested by MATE. The Thévenin equivalent voltage source represents the combined effects of the voltage, current, and history sources existent in the original discretized network.

After a particular solution instant when the complete network has been solved, there will be a number of solution steps (depending on the ratio ΔT/Δt) when only the fast subsystem needs to be solved. Thus, the representation of the slow subsystem as a Thévenin equivalent for the interval ΔT seems to be quite reasonable (Fig. 4).

If the Thévenin equivalent were to be maintained constant throughout the simulation interval ΔT, the latent subnetwork would be represented as completely dormant, resulting in a possibly inaccurate solution. If, however, a varying Thévenin equivalent is used at each small time step Δt, accuracy may be significantly increased.

As soon as the global solution is obtained at, for example, **t = ΔT**, it is possible to go one step further and calculate the history sources of the slow-varying elements at **t = 2ΔT**. The Thévenin equivalent voltage source $V_{th\ slow}$ can then be calculated precisely also at **t = 2ΔT**. The contribution of the slow subnetwork when solving only for the fast one may then be obtained by an interpolation process of the Thévenin equivalent voltage source values at **t = ΔT** and **t = 2ΔT**. The Thévenin equivalent impedance $R_{th\ slow}$ is maintained constant throughout the interpolation process since its value depends only on the time step **ΔT** used for the slow subnetwork discretization. With the assumption that the latent subnetwork is varying slowly throughout the interval **ΔT** (otherwise the integration step **ΔT** itself would have to be decreased), a linear interpolation for the Thévenin equivalent voltage source is a very reasonable assumption.

A question that arises here is how the equivalent voltage source can be calculated at an, as yet, unknown solution time **t + ΔT**. The answer is by simply updating the history sources of the slow subnetwork before incrementing the solution step. As soon as the solution of the slow-varying subnetwork is obtained at **t**, the history sources used for the solution at **t + ΔT** can be evaluated. And since the values of independent voltage and current sources are a known function of time, the Thévenin equivalent voltage source for the slow-varying subnetwork at **t + ΔT** can be evaluated even before the next time instant for the slow subnetwork solution is reached. Figure 5 shows how the interpolation procedure works, assuming that **ΔT = 4Δt**.

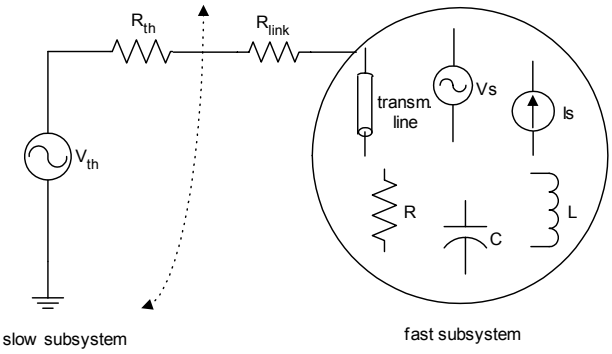

Fig. 4: Network decoupling with the detailed slow subnetwork substituted by its Thévenin equivalent

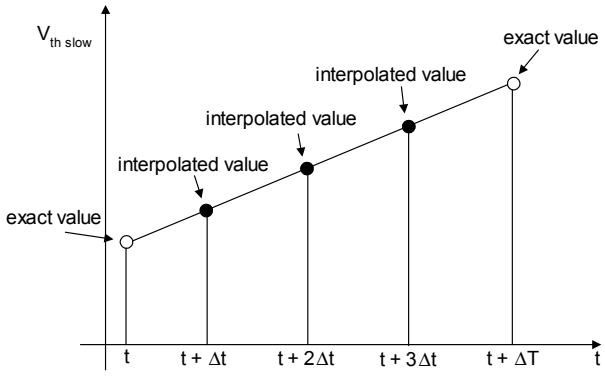

Fig. 5: Thévenin equivalent voltage source linear interpolation for $\Delta T = 4\Delta t$

IV. Simulation Results with Latency Exploitation

An example proposed in [14] was chosen with the purpose of presenting simulation results with latency exploitation. Figure 6 shows the circuit proposed where V_{fast} is the voltage across the fast capacitor (1μF) and V_{slow} is the voltage across the slow capacitor (100μF). This is a simple lumped circuit where the dominant eigenvalues in each of the subnetworks are fairly independent. Therefore it is ideal for latency exploitation. More complex networks, in particular networks containing transmission lines, will be the subject of upcoming papers. The voltage source with a frequency of 60Hz and an amplitude of 1.0V has been connected for a long time, and the switch closes at t = 0. Three different simulation methods are chosen and tested using the trapezoidal integration rule, as follows:

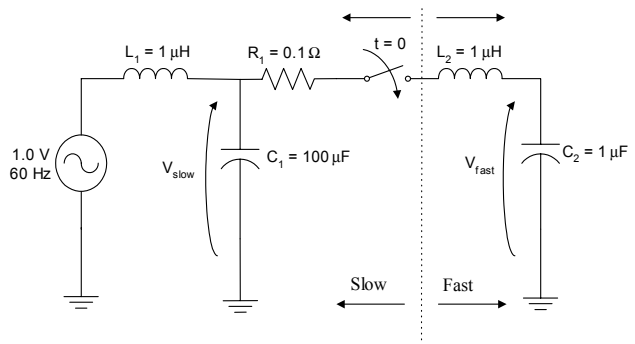

Fig. 6: Lumped circuit for latency exploitation

(1) Standard procedure using a small time step for the complete network solution in order to guarantee maximum accuracy: $\Delta t = 0.2\mu s$ (normal EMTP solution);
(2) Dual step sizes: $\Delta t = 0.2\mu s$ for the fast part of the circuit and $\Delta T = 2.0\mu s$ for the slow part (latency approach with a ratio n = 10);
(3) Large time step for the complete network solution: $\Delta T = 2.0\mu s$ (normal EMTP solution).

Figure 7 shows the voltage across the slow capacitor for the three methods proposed, while Fig. 8 shows the voltage across the fast capacitor for the same three methods.

The voltage across the slow capacitor, as shown in Fig. 7, is accurately predicted by the three distinct methods. However, the voltage across the fast capacitor, as shown in Fig. 8, is predicted very poorly by method 3. Since V_{fast} presents a very high oscillating frequency, a time step of 2.0μs imposes a Nyquist rate too close to this frequency and, therefore, the results are not accurate anymore. Method 2, on the other hand, guarantees very accurate results for the voltage across capacitor C_2, to the point where the simulation results obtained with methods 1 and 2 become virtually indistinguishable. Table I shows the number of floating point numerical operations (FLOPS) performed for a total simulation time of 1.0ms for each of the three methods proposed.

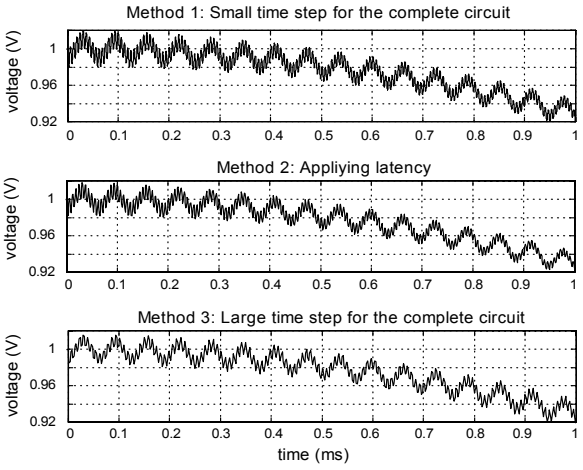

Fig. 7: Voltage across the slow capacitor

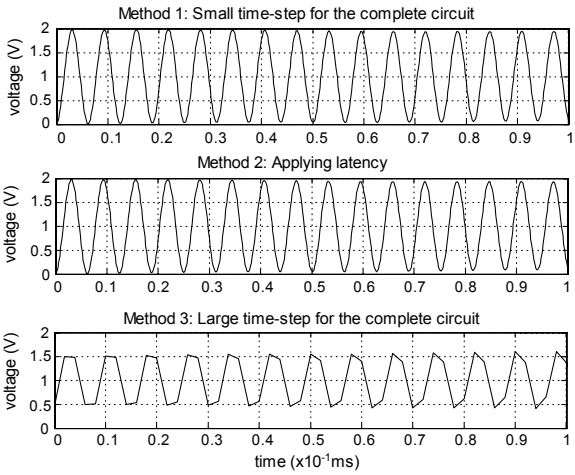

Fig. 8: Voltage across the fast capacitor

Table I: Number of FLOPS for the simulated case considering a total simulation time of 1.0ms

Type of simulation performed	FLOPS
Method 1	405,085
Method 2	257,123
Method 3	40,585

In this case a significant decrease in the number of numerical operations was obtained. In this particular example, since each subnetwork is composed of two nodes, it is virtually impossible to obtain a decrease greater than 50% of the number of FLOPS from method 1. Given that the reduction obtained is around 40% for a ratio of step sizes equivalent to 10, the latency approach is very efficient. Note that method 3, as expected, provides the fewest number of FLOPS. However, the results are very inaccurate.

V. CONCLUSIONS

This paper presented a new methodology for latency exploitation in time-domain EMTP-type simulators. The latency concept in this paper is related to using different integration steps in different parts of the electric network.

The electric network is partitioned according to the MATE technique, and the resulting subnetworks are discretized with time steps adequate to the accurate solution of each particular subnetwork. The purpose of the latency solution is to allow a decrease in the computational burden, eventually facilitating the achievement of real-time simulation for large power system networks.

The proposed latency simulation method has been described in detail, with emphasis on the two proposed techniques for increased efficiency and accuracy of the simulation: the history source accumulation of the fast subnetwork elements for resynchronized solution, and the Thévenin equivalent voltage source interpolation for the slow subnetwork representation. These two methods allow a compromise between accuracy and efficiency of the simulation, and guarantee a smooth interface between the slow and fast variables.

The proposed latency technique was developed within the MATE framework of the OVNI real-time simulator. By exploiting latency using non-iterative solutions, the robust characteristics of EMTP-type simulators are maintained.

An example network simulated in time-domain making use of the developed latency technique shows the validity of the method. Accuracy is maintained while achieving a significant decrease in numerical operations when compared to the traditional EMTP method of single constant integration step.

A promising application of latency exploitation is in power electronics based circuits, such as HVDC transmission and FACTS controllers. In these cases, very high switching frequencies (and consequently small integration steps) are needed for the converters while the power network may be simulated with larger integration steps. This and other applications are currently under development and contributions are expected in the near future.

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