Latency Suitability for the Time-Domain Simulation of Electromagnetic Transients through Network Eigenanalysis

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***Abstract* – This paper complements a companion paper in this same Conference that presents techniques for an accurate and efficient latency exploitation of electric networks using time-domain simulators [1]. However, latency should be exploited with care, since it is generally not true that individual system components may be characterized as being either slow or fast. In this paper, a methodology based on the calculation of the participation matrix in order to verify if the network partitioning is adequate for latency exploitation is presented. Since the participation matrix is a combination of the right and left eigenvectors associated with each mode of the system, it is not dependent on the scaling and units associated with the state variables. Simulation results show networks in which latency exploitation provides very accurate results and also in which it presents some clear limitations.**

Keywords – Latency exploitation, Network eigenanalysis, Integration rules, Electromagnetic transients, Participation matrix.

I. INTRODUCTION

In a companion paper [1] presented at this same Conference, techniques for an efficient and accurate latency exploitation of electric network solutions were presented. Latency is related to the possibility of numerically solving the differential equations governing the behaviour of electric networks with dual or multiple integration steps. The term multirate simulation is also commonly used in the literature to indicate this type of analysis [2,3]. It differs from the traditional EMTP type of solution where a single time step is used for the simulation of the complete network [4]. It should be stressed out that the latency technique proposed in [1] is different from the variable single time step method [5,6]. While the latter simply exploits the latency property by varying the time step through some mechanism that detects the fact that the given signal value is not changing appreciably, the former is a truly multirate simulation in the sense that different integration steps are used for the transient simulation of an electric network at any given time.

Although latency exploitation may lead to very accurate results as reported in [1], its application to the solution of general RLC networks should be analyzed carefully. Results will be far more accurate if each of the subsystems could be characterized as being either slow or fast, i.e., if the dominant eigenvalues in each of the subsystems were fairly independent. However, it is often not true that individual system components can be characterized in an absolute way as being either slow or fast [7].

This paper presents a simple algorithm for the calculation of the dominant poles within the subnetworks that have been separated for latency exploitation. This method is based on the evaluation of the participation matrix $\boldsymbol{P}$ [8] in order to identify the relationship between the states and the modes of a particular network.

Section II of this paper describes the method to evaluate the participation matrix of a network. In section III, the eigenanalysis proposed, which is based on a continuous-time description of the electric network, is shown to be valid for the discrete-time system as well. In section IV, simulations are presented for cases where the network partitioning and corresponding latency exploitation yields situations where results are very much in agreement with the traditional single step EMTP solution and also where the results are not as accurate as one might expect it to be. Finally, in section V the conclusions are stated. Also, in the appendix, a systematic procedure to assign state variables in electrical networks is described.

II. PARTICIPATION MATRIX DETERMINATION

When modelling a system by its state equations, it is straight forward to compute the system's eigenvalues. Latency may then be efficiently exploited if the state variables of the slow subnetwork are virtually independent from the eigenvalues of the fast subnetwork. In a complex system, the correct choice of the state variables is not an easy task. Most of the times, the choice of inductor currents and capacitor voltages do not lead to an independent set of state variables. Appendix A presents a systematic procedure for assigning state variables and writing the dynamical equations of a network.

The simple determination of the eigenvalues of the electrical network, however, is not enough to establish a clear definition on whether latency exploitation should guarantee an accurate transient solution of the network. The problem is that the rate of change of each state variable is a linear combination of all the eigenvalues.

The eigenvectors associated with each mode give an indication of by how much these modes participate in each state variable. The n-column vector $\boldsymbol{\phi}_i$ which satisfies

$$\boldsymbol{A}\boldsymbol{\phi}_i = \lambda_i \boldsymbol{\phi}_i \tag{1}$$

is called the right eigenvector of the state matrix $\boldsymbol{A}$ associated with the eigenvalue λ_i.

Similarly, the n-row vector $\boldsymbol{\psi}_i$ which satisfies

$$\boldsymbol{\psi_i A} = \lambda_i \boldsymbol{\psi_i} \tag{2}$$

is called the left eigenvector of the state matrix $\boldsymbol{A}$ associated with the eigenvalue λ_i.

The matrices of right and left eigenvectors are respectively given by

$$\boldsymbol{\Phi} = [\boldsymbol{\phi}_1 \boldsymbol{\phi}_2 \ldots \boldsymbol{\phi}_n] \tag{3}$$

$$\boldsymbol{\Psi} = \left[\boldsymbol{\psi}_1^T \boldsymbol{\psi}_2^T \ldots \boldsymbol{\psi}_n^T\right]^T \tag{4}$$

As mentioned in [8], one problem in using right and left eigenvectors individually for identifying the relationship between the states and the modes is that the elements of the eigenvectors depend on units and scaling associated with the state variables. A matrix called the participation matrix $\boldsymbol{P}$ can be evaluated as a measure of the association between the state variables and the modes.

The participation matrix $\boldsymbol{P}$ is given by

$$\boldsymbol{P} = [\boldsymbol{p}_1 \boldsymbol{p}_2 \ldots \boldsymbol{p}_n] \tag{5}$$

with

$$\boldsymbol{P} = \begin{bmatrix} p_{1i} \\ p_{2i} \\ \vdots \\ p_{ni} \end{bmatrix} = \begin{bmatrix} \phi_{1i}\psi_{i1} \\ \phi_{2i}\psi_{i2} \\ \vdots \\ \phi_{ni}\psi_{in} \end{bmatrix} \tag{6}$$

where

ϕ_{ki} = kth entry of the right eigenvector $\boldsymbol{\phi_i}$

ψ_{ik} = kth entry of the left eigenvector $\boldsymbol{\psi_i}$

The element $p_{ki} = \phi_{ki}\psi_{ik}$ is termed the participation factor and measures the relative participation of the kth state variable in the ith mode, and vice versa.

By determining all the participation factors it is possible to verify which modes have most influence in each of the state variables and therefore conclude if the network partitioning employed for latency exploitation will result in accurate results or not.

III. Relationship between the continuous-time and the discrete-time eigenanalyses

If the electrical network is discretized for a time-domain simulation using an EMTP type of program, the eigenvalues that should be calculated in order to apply the eigenanalysis proposed in the previous section are the eigenvalues of the equivalent discrete-time system. However, it is much easier to determine the eigenvalues of the continuous-time system. The question that arises here is if it would be correct to perform the eigenanalysis of the network in its continuous time-domain form and extend the conclusions to the discrete time-domain form. We will see that this is indeed the case.

It is true that the eigenvalues of the discrete-time system are not exactly the same as the eigenvalues of the continuous-time system. They are related, however, by the well known bilinear transformation, when the differential equations are discretized with the trapezoidal rule.

$$\lambda_i = \frac{2}{\Delta t}\frac{z_i - 1}{z_i + 1} \tag{7}$$

where the set of λ_i are the continuous-time and the set of z_i are the discrete-time eigenvalues.

In case the backward Euler (implicit Euler) rule is used in the discretization of the differential equations, the transformation relating continuous-time and discrete-time eigenvalues is given by

$$\lambda_i = \frac{1}{\Delta t}\frac{z_i - 1}{z_i} \tag{8}$$

However, as indicated in [9], the eigenvectors of the continuous and discrete-time systems are the same. This results from the fact that the state space matrix is also mapped from continuous to discrete-time.

In continuous-time, the state equation of a system is written as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{9}$$

For a zero-input system

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) \tag{10}$$

In discrete-time the state equation of the same system also with a zero-input can be written as

$$\boldsymbol{x}(t + \Delta t) = \boldsymbol{A_d}\boldsymbol{x}(t) \tag{11}$$

where $\boldsymbol{A_d}$ is the state space matrix in discrete-time.

The relationship between $\boldsymbol{A_d}$ and $\boldsymbol{A}$ for the trapezoidal and backward Euler rule are respectively given by [9]

$$\boldsymbol{A_d} = \left(\frac{2}{\Delta t}\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\left(\frac{2}{\Delta t}\boldsymbol{I} + \boldsymbol{A}\right) \tag{12}$$

$$\boldsymbol{A_d} = \frac{1}{\Delta t}\left(\frac{1}{\Delta t}\boldsymbol{I} - \boldsymbol{A}\right)^{-1} \tag{13}$$

where $\boldsymbol{I}$ is the identity matrix.

The complete proof that the eigenvectors of the continuous and discrete-time systems are the same can be found in [9]. Since the eigenvectors are the same, the participation matrix $\boldsymbol{P}$ remains unchanged and it is possible to extend the conclusions from calculating the participation factors of the continuous-time system to the discrete-time equivalent.

IV. Simulation results

The lumped network shown in Fig. 1 will be tested for latency suitability. This network is a good candidate for latency exploitation since inductor L_1 and capacitor C_1 have a resonant frequency ten times lower than inductor L_2 and capacitor C_2. This is the same network presented in the companion paper [1].

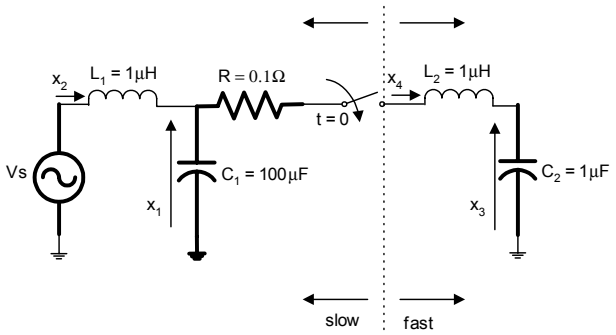

Fig. 1: Lumped circuit for latency exploitation

Following the state variables identification procedure described in the appendix, the normal tree is identified by the heavy lines in Fig. 1. The voltages across the capacitors in the normal tree and the current through the inductors in the links are assigned as state variables, therefore resulting in four state variables, also depicted in Fig. 1.

$$\begin{aligned} x_1 &= v_{C1} \\ x_2 &= i_{L1} \\ x_3 &= v_{C2} \\ x_4 &= i_{L2} \end{aligned} \tag{14}$$

The zero-input state equation is written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C_1} & 0 & -\frac{1}{C_1} \\ -\frac{1}{L_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_2} \\ \frac{1}{L_2} & 0 & -\frac{1}{L_2} & -\frac{R}{L_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{15}$$

Replacing the parameters by their actual values given in Fig. 1, the following eigenvalues are obtained:

$$\begin{aligned} \lambda_{1,2} &= \left(-0.05 \pm j1.0038\right) \times 10^6 \\ \lambda_{3,4} &= \left(-0.000004998 \pm j0.0995\right) \times 10^6 \end{aligned} \tag{16}$$

After obtaining the right and left eigenvectors associated with each eigenvalue using (1) and (2), the participation matrix may be calculated using (5) and (6):

$$\boldsymbol{P} = \begin{array}{cc|c} \lambda_1 & \lambda_2 & \\ \hline 0.00505 + j0.00026 & 0.00505 - j0.00026 & x_1 \\ 0.00005 + j0.00001 & 0.00005 - j0.00001 & x_2 \\ 0.49495 + j0.02464 & 0.49495 - 0.02464 & x_3 \\ 0.49995 - j0.02491 & 0.49995 + j0.02491 & x_4 \end{array} \tag{17}$$

$$\begin{array}{cc|c} \lambda_3 & \lambda_4 & \\ \hline 0.49495 - j0.00008 & 0.49495 + j0.00008 & x_1 \\ 0.49995 - j0.00003 & 0.49995 + j0.00003 & x_2 \\ 0.00505 + j0.00010 & 0.00505 - j0.000010 & x_3 \\ 0.00005 + j0.00000 & 0.00005 - j0.00000 & x_4 \end{array}$$

From the participation matrix, it is easily verified that eigenvalues λ_1 and λ_2 only have a significant contribution on state variables x_3 (voltage across capacitor 2) and x_4 (current through inductor 2). On the other hand, eigenvalues λ_3 and λ_4 only have a significant contribution on state variables x_1 (voltage across capacitor 1) and x_2 (current through inductor 1). Therefore the time response of state variables x_1 and x_2 (on the slow subnetwork) are virtually independent of eigenvalues λ_1 and λ_2 (fast modes). It is then possible to exploit latency for a very accurate time-domain simulation of this circuit according to the network partitioning proposed in Fig. 1.

The simulation results for this circuit have been presented in [1], but are repeated below for completeness. The three methods adopted for the time domain simulation are:

Method 1: Standard procedure using a small time step for the complete network solution: Δt = 0.2μs (normal EMTP solution).

Method 2: Dual step sizes: Δt = 0.2μs for the fast part of the circuit and ΔT = 2.0μs for the slow part (latency exploitation).

Method 3: Large time step for the complete network solution ΔT = 2.0μs (normal EMTP solution).

Figure 2 shows the voltage across the "slow" capacitor for the three methods proposed, while Fig. 3 shows the voltage across the "fast" capacitor for the same three methods.

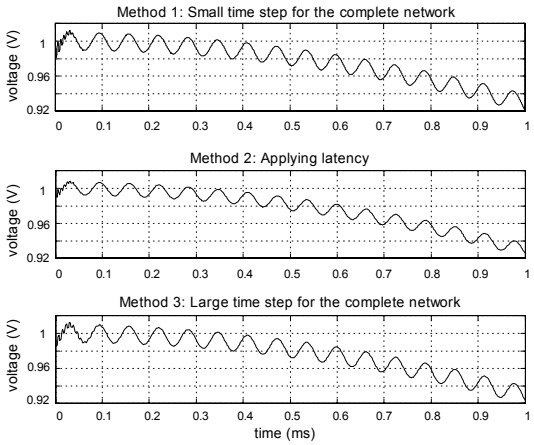

Fig. 2: Voltage across the "slow" capacitor

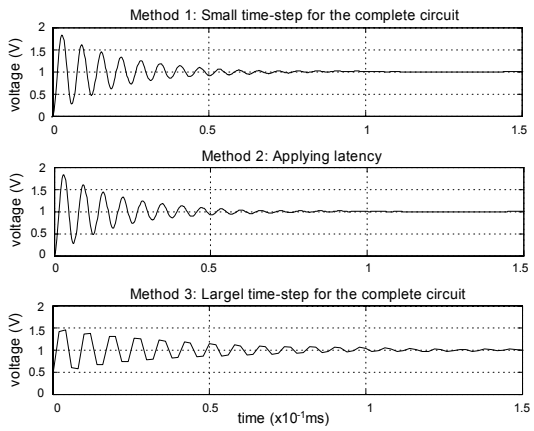

Fig. 3: Voltage across the "fast" capacitor

The voltages across the "slow" and "fast" capacitors are accurately predicted by method 2, even though the integration step of the slow subcircuit is ten times larger than the integration step of the fast subcircuit since the participation factors of the "fast" modes on the "slow" state variables are very small. Latency exploitation therefore, produces very accurate results.

By slightly modifying the circuit in Fig. 1 by adding an additional LC cell, as shown in Fig. 4, a case where latency exploitation presents restrictions can be observed.

The resonant frequency of the additional LC cell at the right-side of the figure is equal to that of the "slow" LC cell of the previous circuit. It is assumed that the new cell has a "slow" behaviour, so that the partitioning adopted in Fig. 4 is valid. We will verify, however, that this assumption is not correct. As before, assigning the voltages across the capacitors in the normal tree and the currents through the inductors in the links as state-variable:

$$\begin{aligned} x_1 &= v_{C1} \\ x_2 &= i_{L1} \\ x_3 &= v_{C2} \\ x_4 &= i_{L2} \\ x_5 &= v_{C3} \\ x_6 &= i_{L3} \end{aligned} \tag{18}$$

The zero input state equation is written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C_1} & 0 & -\frac{1}{C_1} & 0 & 0 \\ -\frac{1}{L_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_2} & 0 & -\frac{1}{C_2} \\ \frac{1}{L_2} & 0 & -\frac{1}{L_2} & -\frac{R_1}{L_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{C_3} \\ 0 & 0 & \frac{1}{L_3} & 0 & -\frac{1}{L_3} & -\frac{R_2}{L_3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \tag{19}$$

Replacing the parameters by their actual values given in Fig. 4, the following eigenvalues are obtained:

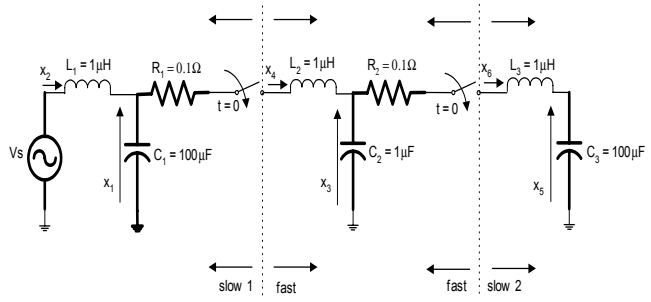

Fig. 4: Modified lumped circuit for latency exploitation

$$\begin{aligned} \lambda_{1,2} &= (-0.049999 \pm j1.416872)\times 10^6 \\ \lambda_{3,4} &= (-0.019417 \pm j0.116982)\times 10^6 \\ \lambda_{5,6} &= (-0.030584 \pm j0.051016)\times 10^6 \end{aligned} \tag{20}$$

As before, the participation matrix may be obtained:

$$\boldsymbol{P} = \begin{array}{|cc|c|} \hline \lambda_1 & \lambda_2 & \\ \hline 0.001256 + j0.000045 & 0.001256 - j0.000045 & x_1 \\ 0.000006 + j0.000001 & 0.000006 - j0.000001 & x_2 \\ 0.497500 + j0.017555 & 0.497500 - j0.017555 & x_3 \\ 0.250003 - j0.008822 & 0.250003 + j0.008822 & x_4 \\ 0.001244 + j0.000044 & 0.001244 - j0.000044 & x_5 \\ 0.249991 - j0.008823 & 0.249991 + j0.008823 & x_6 \\ \hline \end{array}$$

$$\begin{array}{|cc|c|} \hline \lambda_3 & \lambda_4 & \\ \hline 0.549100 + j0.117230 & 0.549100 - j0.117230 & x_1 \\ 0.342619 + j0.205052 & 0.342619 - j0.205052 & x_2 \\ 0.000457 + j0.000548 & 0.000457 - j0.000548 & x_3 \\ 0.028401 - j0.122851 & 0.028401 + j0.122851 & x_4 \\ 0.048940 - j0.076261 & 0.048940 + j0.076261 & x_5 \\ 0.030483 - j0.123719 & 0.030483 + j0.123719 & x_6 \\ \hline \end{array}$$

$$\begin{array}{|cc|c|} \hline \lambda_5 & \lambda_6 & \\ \hline -0.050356 - j0.090034 & -0.050356 + j0.090034 & x_1 \\ 0.157375 - j0.245457 & 0.157375 + j0.245457 & x_2 \\ 0.002043 + j0.000167 & 0.002043 - j0.000167 & x_3 \\ 0.221596 - j0.064686 & 0.221596 + j0.064686 & x_4 \\ 0.449817 + j0.463155 & 0.449817 - j0.463155 & x_5 \\ 0.219526 - j0.063144 & 0.219526 + j0.063144 & x_6 \\ \hline \end{array} \tag{21}$$

From a careful analysis of the participation matrix, it can be observed that no clear independence between fast and slow behaviour can be verified for some state variables. This is the case, for example, of state variable x_6 which is equally dependent on the pair of eigenvalues λ_1 and λ_2 (very fast) and the pair λ_5 and λ_6 (slow). Latency exploitation in the way proposed in Fig. 4 would then not guarantee results as accurate as in the previous circuit.

The same three methods employed for the time-domain simulation of the previous network are repeated here. Since the analysis of the participation matrix has indicated that inductor 3, which is supposedly located in a "slow" part of the network, is actually very dependent upon the eigenvalues with the highest oscillating frequency, it is very illustrative to present the voltage across inductor 3, as shown in Fig. 5.

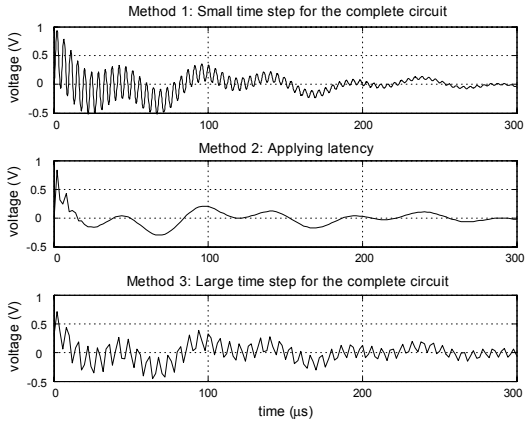

Fig. 5: Voltage across inductor 3 (supposedly "slow")

Figure 5 clearly shows the strong dependence of the voltage across inductor 3 on the fast modes. The assumption that inductor 3 is located in a "slow" part of the network is not very accurate. However, latency might still be used in this case to track the average value of the signal. Even if the high frequency components were very slowly damped, the proposed latency technique would still be capable of following the slow modes associated with this variable accurately, since latency acts as a low-pass filter. Latency exploitation would of course fail if in a particular simulation, obtaining the average values of a state variable is not sufficient

V. CONCLUSIONS

This paper presented a methodology based on the calculation of the participation matrix in order to verify if the matrix partitioning is adequate for latency exploitation. The participation matrix only depends on the right and left eigenvectors associated with each mode of the system. Simulation results show that in general RLC lumped networks it may not be easy to separate slow and fast components simply by visual inspection even for very simple networks. The decision on whether to exploit latency when there is no clear separation between time constants depends on the simulation needs of the user, and a proper judgment has to be made.

In power systems however, the separation of time constants is usually more explicit than in general lumped RLC networks. This is the case, for example, of an HVDC converter station, where, for an accurate simulation, the switching frequency of the valves requires an integration step much smaller than the power network connected to the converter. Synchronous generators electromechanical transients are much slower than electromagnetic transients and the decoupling of time constants is again well defined in this case. Future papers will present simulation results exploiting latency in these situations.

Through the concept of the participation matrix, a generalized latency methodology that allows an automatic and efficient network partitioning may be developed. Different network partitionings may be performed and the most suitable for latency exploitation would be the one which participation matrix indicates that state-variables predominantly fast or slow have been decoupled.

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APPENDIX – ASSIGNING STATE VARIABLES TO A GENERAL RLC LUMPED NETWORK

The state variable description of a system is an alternative to the input-output description, which is applicable only when the system is initially relaxed.

If a system contains "p" inputs and "q" outputs, an n-dimensional linear and time invariant state variable description is of the form

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t) \end{aligned} \qquad \text{(A.1)}$$

where $\boldsymbol{x}(t)$ is the state vector of the system and $\boldsymbol{y}(t)$ is the output.

Matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, and $\boldsymbol{D}$ are respectively of "$n_x n$", "$n_x p$", "$q_x n$", and "$q_x p$" dimensions.

From the analysis of (A.1), general properties such as controllability, observability, and stability of the system; as well as the exact responses of the system due to some excitation, may be determined.

In the case where the dynamic systems are electric networks, a systematic procedure for assigning state variables and writing the dynamical equations may be described. This procedure is extracted from [10].

It is well known that if all the state variables of an RLC network are known, then the behaviour of the network is uniquely determinable for any input.

The procedure described here requires the definition of *tree*, *link*, and *cutset* of a network. A *tree* of a network is

defined as any connected graph (connection of branches) containing all the nodes but not any loops.

Every branch in a given *tree* is called a *tree branch*. Every branch not in the *tree* is called a *link*. A *cutset* of a connected network is any minimum set of branches such that the removal of all the branches in this set causes the remaining network to be unconnected. With respect to any fixed *tree*, every *link* and some *tree branches* form a unique *cutset* called a *fundamental cutset*. With these definitions in mind, the procedure for assigning state variables is as follows:

1) Choose a tree called a *normal tree*. The branches of the normal tree are chosen in the following priority order: voltage sources, capacitors, resistors, inductors, and current sources. Hence, a normal tree consists of all the voltage sources, the maximum number of permissible capacitors, the resistors, and finally the minimum number of inductors. Usually, it does not contain any current source.

2) Assign the charges or voltages of the capacitors in the normal tree and the flux or current of the inductors in the links as state variables. The voltages of the capacitors in the links and the currents of the inductors in the normal tree do not need to be chosen as state variables.

3) Express the branch variables (branch voltage and current of all the resistors, the capacitors in the links, and the inductors in the normal tree) in terms of the state variables and the inputs by applying the 1^{st} or 2^{nd} Kirchhoff laws to the fundamental loops or cutsets of these branches.

4) Apply the 1^{st} or 2^{nd} Kirchhoff laws to the fundamental loop or cutset of every branch that is assigned as a state variable.