

# Transient Behavior Analysis of Induction Generator at Three-Phase Fault Condition

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**Abstract**— In this paper, we present transient current, active power, and reactive power, etc. analysis of induction generators used in wind power system before and after three-phase fault conditions. The natural approximation to derive analytical formulas for transient conditions is proposed, and the transient behavior of induction generator is analyzed by the developed equations. Furthermore, theoretical discussion also developed to determine the fault conditions and the time at which maximum transient currents flow in the system. These equations confirmed by the numerical simulation that it can be used to study the dynamic behavior of the induction generator under three-phase fault condition.

**Keywords**— induction generator, three-phase fault, theoretical formulas, MATLAB/SIMULINK

## I. NOMENCLATURES

stator voltages	$v_{qs}, v_{ds}$
rotor voltages	$v'_{qr}, v'_{dr}$
stator currents	$i_{qs}, i_{ds}$
rotor currents	$i'_{qr}, i'_{dr}$
stator resistance	$r_s$
rotor resistance	$r'_r$
stator leakage inductance	$L_{ls}$
rotor leakage inductance	$L'_{lr}$
magnetizing inductance	$L_m$
rotor speed(steady state)	$\omega_{r_{ss}}$
rotor speed(fault condition)	$\omega_f$
number of poles	$P$

## II. INTRODUCTION

IN recent year, renewable energy generation is coming up for effective use of natural energy, such as wind energy. Induction generators consisting squirrel-cage rotors are widely used as wind generators because of their salient features like robust rotor design, simple in the construction, maintenance free operation etc. However these induction generators will draw large transient currents, several times as large as the machine rated current, when they are connected to utility grid [1], [2] or occurred various disturbance such as earth fault. Under such situations, there will be a severe voltage

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fluctuations in the power system. Therefore, these systems influence brought to a power system becomes one of main issue for wind power generating system. From such a background, the transient condition connected to utility grid is examined by good many papers[1]-[5], however, there are few examples of analysis about transient condition for various disturbance such as earth fault. Therefore, authors derived the transient current formulas of induction generator at the three-phase fault conditions, and analyzed transient behavior of the induction generator using these formulas [6], [7]. However, since the output equations of the induction generator has nonlinearity nature, and the speed-time characteristic in a fault period is also expressed by the nonlinear equation, above studies assumed that rotor speed during transient condition is constant.

In this paper, we propose at first, the natural approximation to express rotor speed-time characteristic for transient conditions, and then derive the transient current, active power, and reactive power, etc. taking into account of rotor speed change of induction generators at three-phase fault conditions. Furthermore, the transient behavior of induction generator is analyzed by the developed formulas.

The theoretical equations developed are programmed in FORTRAN. For comparison results also obtained using MATLAB/SIMULINK. The simulation block diagram in MATLAB/SIMULINK is constituted using the system equation in consideration of the nonlinearity nature of the induction generator[8], [9]. The simulation results obtained from each theoretical analysis are in close agreement with that of results obtained using MATLAB/SIMULINK simulation. Furthermore, theoretical discussion also developed to determine the fault conditions and the time at which maximum transient currents flow in the system. These equations can be used to study the dynamic behavior of the induction generator under three-phase fault condition.

## III. VELOCITY EQUATION

In this chapter, we derived the velocity equation which express natural approximation the rotor speed of induction generator at three-phase fault. This formula can be obtained from a current equations(rotor speed ; constant)[6], [7] and a equation of motion of rotor.

### A. transient analysis formulas at three-phase fault condition (constant rotor speed)

The block diagram of the system and the specifications under study are shown in Fig. 1 and Table I. Transient phenomenon for a three-phase fault occurred at the some location of generating line when the induction generator running under steady state is analyzed. In the Figure 1, generating line length is 30km, and subscripts of line inductance are distance from fault generating position to induction generator or infinite bus system. Equivalent circuit( $q$ -axis) of the induction generator in  $d$ - $q$  reference frame at three-phase fault condition is shown in Fig. 2, where,  $V_f$  is voltage at the fault generating position. From Fig. 2, on the assumption that the rotor speed of induction generator is constant at fault condition,  $q$ ,  $d$ -axis stator and rotor current can be expressed as following equations[6], [7]. The constant for equations used in this paper are listed in Appendix.

$$i_{qs} = \left[ Z_{s1} \cos(\theta_s + \beta_{s1}) - Z_{s2} \cos(\omega_{r_{ss}} t'_s + \theta_s + \beta_{s2}) \right] e^{-\alpha t'_s} \quad (1)$$

$$i'_{qr} = \left[ -Z'_{s1} \cos(\theta_s + \beta'_{s1}) + Z'_{s2} \cos(\omega_{r_{ss}} t'_s + \theta_s + \beta'_{s2}) \right] e^{-\alpha t'_s} \quad (2)$$

$$i_{ds} = \left[ -Z_{s1} \sin(\theta_s + \beta_{s1}) + Z_{s2} \sin(\omega_{r_{ss}} t'_s + \theta_s + \beta_{s2}) \right] e^{-\alpha t'_s} \quad (3)$$

$$i'_{dr} = \left[ Z'_{s1} \sin(\theta_s + \beta'_{s1}) - Z'_{s2} \sin(\omega_{r_{ss}} t'_s + \theta_s + \beta'_{s2}) \right] e^{-\alpha t'_s} \quad (4)$$

Electromechanical torque and active power of induction generator are given by

$$T_{em} = \frac{3}{2} \frac{P}{2} L_m (i'_{dr} i_{qs} - i'_{qr} i_{ds}) \quad (5)$$

$$P_{IG} = \omega_{r_{ss}} T_{em}. \quad (6)$$

Substituting Eqs. (1) to (4) into Eq. (5), and after simplification result the following formula

$$\begin{aligned} T_{em} &= \frac{3}{2} \frac{P}{2} L_m \left[ -Z_{s1} Z'_{s2} \sin(\omega_{r_{ss}} t'_s + \beta'_{s2} - \beta_{s1}) - Z'_{s1} Z_{s2} \sin(-\omega_{r_{ss}} t'_s + \beta'_{s1} - \beta_{s2}) + Z_{s1} Z'_{s1} \sin(\beta'_{s1} - \beta_{s1}) + Z_{s2} Z'_{s2} \sin(\beta'_{s2} - \beta_{s2}) \right] e^{-2\alpha t'_s}. \\ &= \left[ A \sin(\omega_{r_{ss}} t'_s + \theta_A) + B \sin(-\omega_{r_{ss}} t'_s + \theta_B) + C \right] e^{-2\alpha t'_s} \quad (7) \end{aligned}$$

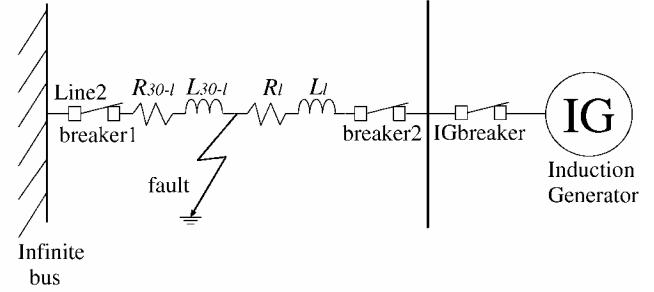


Fig. 1. Infinite bus system.

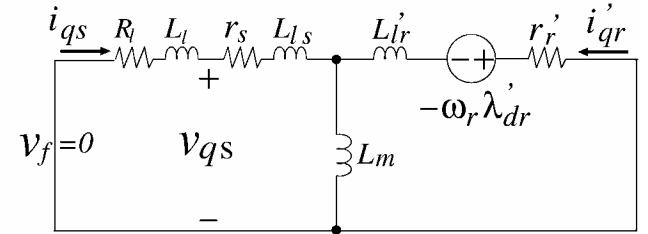


Fig. 2.  $q$ -axis equivalent circuit(three-phase fault).

Table 1. System parameters.

Rated Power	674kVA
Rated Line-to-Line Voltage	690V
Rated Frequency	50Hz
Number of Poles $P$	4poles
Stator Resistance $r_s$	0.0118p.u.
Stator Leakage Inductance $L_{ls}$	0.217p.u.
Rotor Resistance $r'_r$	0.0156p.u.
Rotor Leakage Inductance $L'_{lr}$	0.186p.u.
Excitation Inductance $L_m$	7.28p.u.
Line Resistance $R_l$	0.0585p.u.
Line Inductance $L_l$	0.585p.u.
Inertia constant $J$	18.03kg m <sup>2</sup>
Inertia constant $J_1$	$\infty$ kg m <sup>2</sup>

where,

$$A = -\frac{3}{2} \frac{P}{2} L_m Z_{s1} Z'_{s2}, \quad \theta_A = \beta'_{s2} - \beta_{s1}$$

$$B = -\frac{3}{2} \frac{P}{2} L_m Z'_{s1} Z_{s2}, \quad \theta_B = \beta'_{s1} - \beta_{s2}$$

$$\begin{aligned} C &= \frac{3}{2} \frac{P}{2} L_m \left\{ Z_{s1} Z'_{s1} \sin(\beta'_{s1} - \beta_{s1}) + Z_{s2} Z'_{s2} \sin(\beta'_{s2} - \beta_{s2}) \right\} \end{aligned}$$

### B. Equation of Motion

Velocity equation of induction generator at three-phase fault condition are developed using above equations and following equation of motion.

$$J \frac{d\omega_f}{dt} = T_{em} + T_{mech} - T_{damp} \quad (8)$$

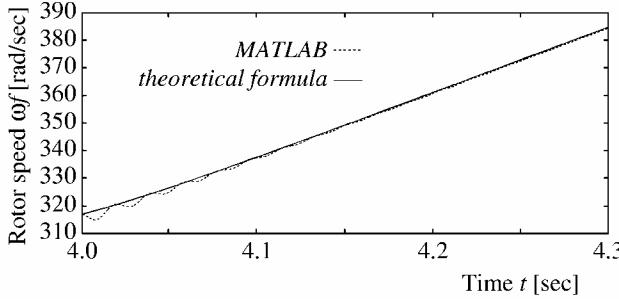


Fig. 3. Rotor speed(three-phase fault).

Where,  $T_{mech}$  is mechanical input torque, and  $T_{damp}$  is damping torque. Substituting Eq. (7) into Eq. (8), and collecting term  $\omega_f$  result in the following equation

$$\begin{aligned} \omega_f = & \frac{-\omega_{r_{ss}} Z_A}{\omega_{r_{ss}}^2 + 4\alpha^2} \left\{ A \cos(\omega_{r_{ss}} t'_s + \theta_A - \theta_{Z_A}) \right. \\ & \left. - B \cos(-\omega_{r_{ss}} t'_s + \theta_B + \theta_{Z_A}) \right\} e^{-2\alpha t'_s} \\ & - \frac{C}{2\alpha J} e^{-2\alpha t'_s} + \frac{1}{J} T_{mech} t'_s + \omega_{r_{ss}} + \frac{C}{2\alpha J} \quad (9) \end{aligned}$$

where,

$$Z_A = \sqrt{1 + \left( \frac{2\alpha}{\omega_{r_{ss}}} \right)^2}, \quad \theta_{Z_A} = \arctan(2\alpha/\omega_{r_{ss}})$$

$$T_{damp} = 0.$$

Furthermore,  $Z_A \approx 1$ ,  $\theta_A \approx \theta_B$  and  $A \approx B$  are realized from  $\left( \frac{2\alpha}{\omega_{r_{ss}}} \right)^2 \ll 1$  and  $L_m \gg L'_{lr}$ . Therefore, first term of the right-hand side of the Eq. (9) becomes smaller than second term of one, and Eq. (9) is rewritten as

$$\omega_f \approx \frac{1}{J} \left\{ \frac{-C}{2\alpha} e^{-2\alpha t'_s} + T_{mech} t'_s \right\} + \omega_{r_{ss}} + \frac{C}{2\alpha J}. \quad (10)$$

The simulation result of velocity equation is shown in Fig. 3. MATLAB/SIMULINK simulation result is also superimposed in the same figure. The simulation results in MATLAB/SIMULINK is take account of non-linearity of induction generator. The simulation result obtained from the theoretical formula is in close agreement with this obtained using MATLAB/SIMULINK simulation.

#### IV. THEORETICAL FORMULAS OF INDUCTION GENERATOR AT THREE-PHASE FAULT CONDITIONS

In this chapter, we derived the transient current, active power, and reactive power, etc. taking into account of rotor speed change of induction generators at three-phase fault conditions. These formulas can be obtained by using above velocity equation and Eqs. (1) to (6). Substituting Eq. (10) into Eqs. (1) to (6), i.e.  $\omega_f$  is used instead of  $\omega_{r_{ss}}$ , theoretical formulas of induction generator at three-phase fault condition is rewritten as follows:

$$\begin{aligned} i_{qs} = & \left[ Z_{s1} \cos(\theta_s + \beta_{s1}) \right. \\ & \left. - Z_{s2} \cos(\omega_f t'_s + \theta_s + \beta_{s2}) \right] e^{-\alpha t'_s} \quad (11) \end{aligned}$$

$$\begin{aligned} i'_{qr} = & \left[ -Z'_{s1} \cos(\theta_s + \beta'_{s1}) \right. \\ & \left. + Z'_{s2} \cos(\omega_f t'_s + \theta_s + \beta'_{s2}) \right] e^{-\alpha t'_s} \quad (12) \end{aligned}$$

$$\begin{aligned} i_{ds} = & \left[ -Z_{s1} \sin(\theta_s + \beta_{s1}) \right. \\ & \left. + Z_{s2} \sin(\omega_f t'_s + \theta_s + \beta_{s2}) \right] e^{-\alpha t'_s} \quad (13) \end{aligned}$$

$$\begin{aligned} i'_{dr} = & \left[ Z'_{s1} \sin(\theta_s + \beta'_{s1}) \right. \\ & \left. - Z'_{s2} \sin(\omega_f t'_s + \theta_s + \beta'_{s2}) \right] e^{-\alpha t'_s} \quad (14) \end{aligned}$$

$$\begin{aligned} T_{em} = & \frac{3}{2} \frac{P}{2} L_m \left[ -Z_{s1} Z'_{s2} \sin(\omega_f t'_s + \beta'_{s2} - \beta_{s1}) \right. \\ & - Z'_{s1} Z_{s2} \sin(-\omega_f t'_s + \beta'_{s1} - \beta_{s2}) \\ & + Z_{s1} Z'_{s1} \sin(\beta'_{s1} - \beta_{s1}) \\ & \left. + Z_{s2} Z'_{s2} \sin(\beta'_{s2} - \beta_{s2}) \right] e^{-2\alpha t'_s} \quad (15) \end{aligned}$$

$$P_{IG} = \omega_f T_{em}. \quad (16)$$

On the other hand, terminal voltage of induction generator expressed as following equations(see, Fig. 2)

$$v_{qs} = R_l i_{qs} + p L_l i_{qs} \quad (17)$$

$$v_{ds} = R_l i_{ds} + p L_l i_{ds} \quad (18)$$

where,  $p$  is differential operator. Reactive power of induction generator is given by

$$Q_{IG} = \frac{3}{2} (v_{qs} i_{ds} - v_{ds} i_{qs}). \quad (19)$$

The simulation results using above equations are shown in Fig. 4(a)-(f). MATLAB/SIMULINK simulation are also superimposed in the same figures. Where, fault conditions in figure 4 are three-phase fault occurred( $t_f = 4.0$ sec) at the center of transmission line( $l = 15$ km) when the induction generator running under steady state conditions. The all simulation results obtained from the theoretical formulas are in close agreement with those obtained using MATLAB/SIMULINK simulation. Extensive analysis results with variety of fault generating position on transmission line found to be identical valid as that shown above. Therefore, validity of analysis formulas are confirmed.

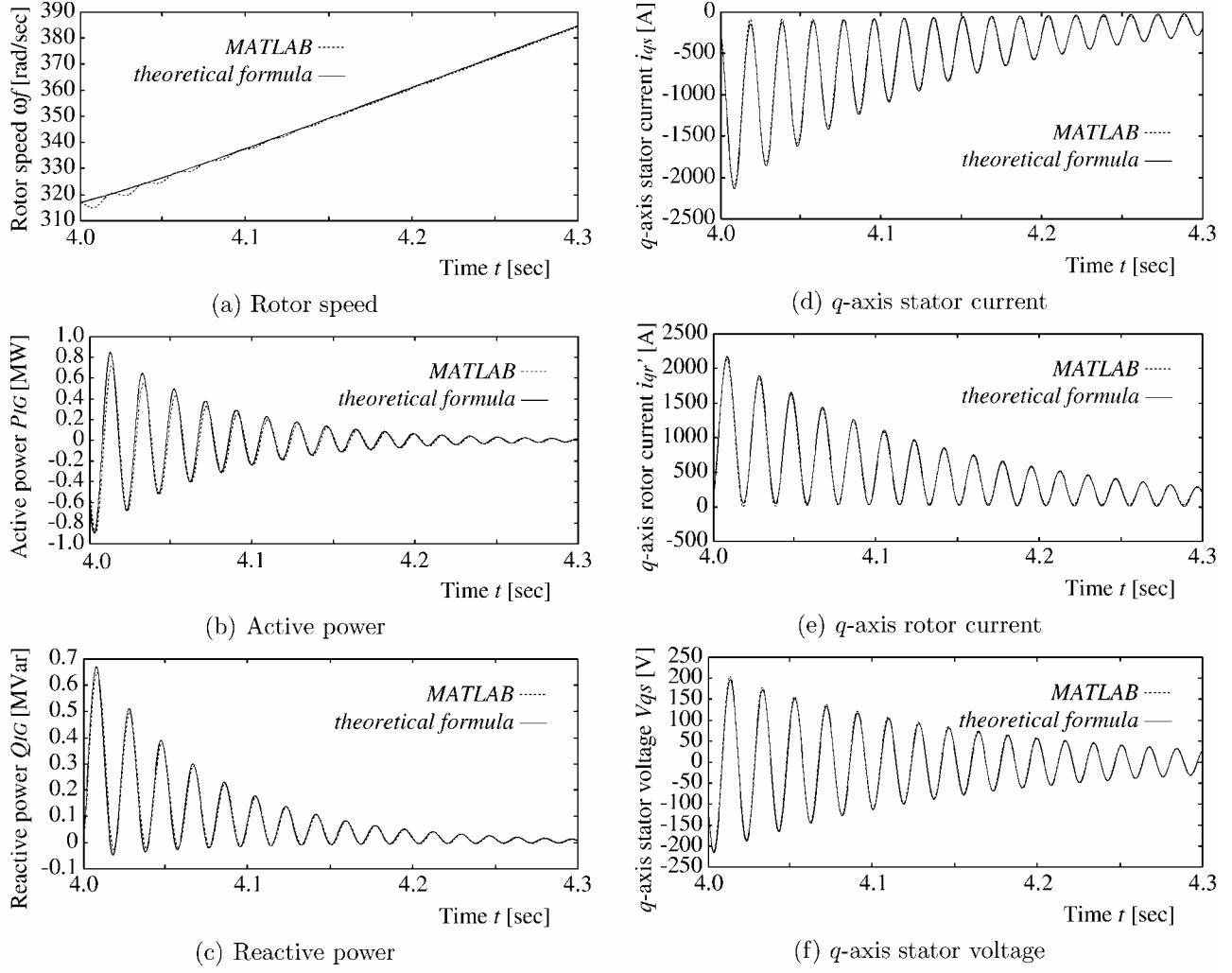


Fig. 4. Simulation results(three-phase fault).

## V. THEORETICAL DISCUSSION

In this chapter, we discussed theoretically about the derived formulas. Theoretical discussion developed to determine the fault conditions and the time at which maximum transient currents flow in the system, and also demonstrated that active power and electromechanical torque of induction generator in three-phase fault are independent of fault conditions.

### A. Theoretical Discussion for transient currents

From Eqs. (11) to (14), transient currents consist of *ac* component with a frequency equals to the rotor speed, a *dc* component, and the components are decreased exponentially. The fault conditions and the time at which maximum transient currents flow in the system are determined by fault phase angle(definition as phase angle of  $q$ -axis stator current at three-phase fault occurred).

The simulation results of three-phase fault with two types of rotor inertia( $J$  and  $J_1$ ) are shown in Fig. 5,

where the simulation conditions are changed only inertia constant of rotor. As shown in Fig. 5, since the change of rotor speed is very small(about  $\pm 1.5\%$  of steady state rotor speed), both magnitude and phase angle of two simulation results( $J$  and  $J_1$ ) are in close agreement. Since, the transient current decrease exponentially, its peak value will be achieved within one cycle from fault occurred. Therefore, the time  $t_{s_{max}}$  at which maximum transient currents flow in the system is given[6] by

$$t_{s_{max}} = \frac{1}{\omega_r} \arccos \left\{ \frac{\alpha Z_{s2}}{Z_s Z_{s1}} \cos(\theta_s + \beta_{s1}) \right\} - \frac{\gamma_s}{\omega_r}. \quad (20)$$

The above formula is the time which transient current becomes maximum with some fault phase angle. Substituting Eq. (20) into Eq. (1) and differentiating  $\theta_s$ , the fault phase angle  $\theta_{s_{max}}$  at which the transient current becomes maximum can be obtained. Simulation result( $-\pi \sim \pi$ ) is shown in Fig. 6. The maximum value of  $\theta_s$  can be obtained from this characteristic at

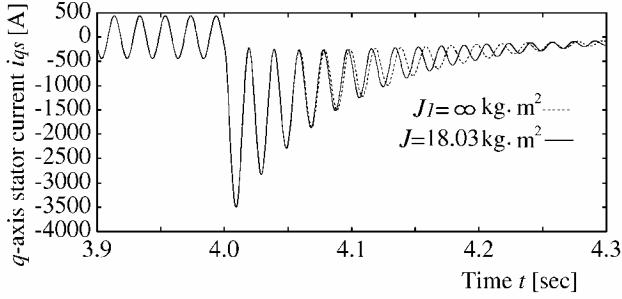
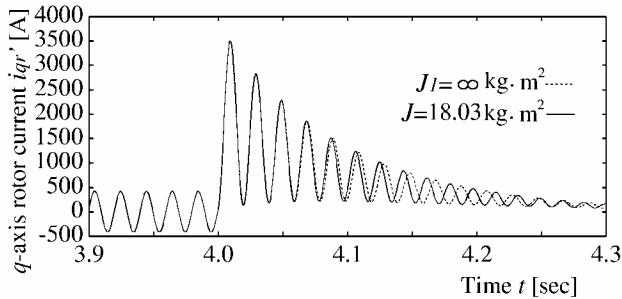

 (a)  $q$ -axis stator current

 (b)  $q$ -axis rotor current

Fig. 5. Simulation results(MATLAB/SIMULINK).

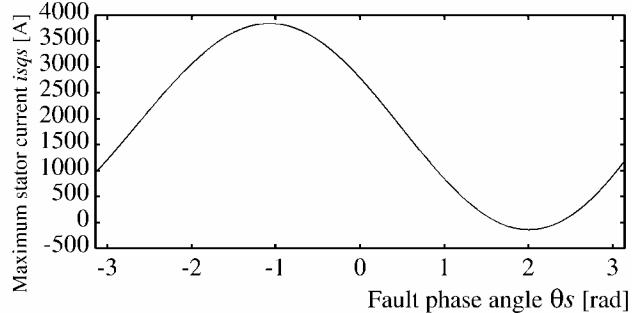
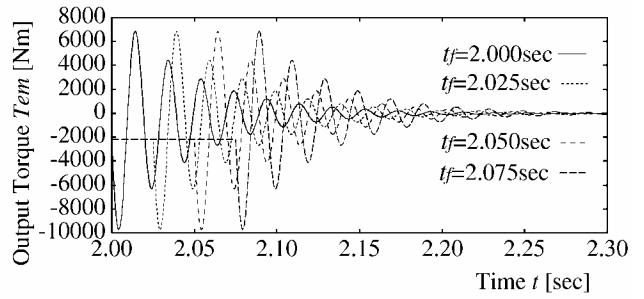
which the gradient is zero, and its value found to be  $\theta_{s_{max}} = -1.206\text{rad}$ . for become positive stator maximum transient current, whereas  $\theta_{s_{max}} = 2.000\text{rad}$ . become for negative stator maximum transient current. Furthermore, the peak value of transient current is changing sinusoidally, the fault phase angle at which transient current is minimum is expressed by the following equation

$$\theta_{s_{min}} = \theta_{s_{max}} \pm \frac{\pi}{2}. \quad (21)$$

The validity of these time and fault conditions are demonstrated by the trial-and-error simulation. Also, it is confirmed by theoretical formulas and MATLAB/SIMULINK simulations that these transient currents are 2 to 4 times the machine rated current.

#### B. Theoretical Discussion for Electromechanical Torque and Active Power of Induction Generator

From derived formulas, electromechanical torque and active power of induction generator in the three-phase fault are independent of fault condition(fault phase angle  $\theta_s$ ). Characteristic of electromechanical torque with fault occurred time  $t_f$  are shown in Fig. 7. Fig. 7 shows simulation results for each fault occurred time  $t_f$  obtained from MATLAB/SIMULINK simulation. As mentioned above, it has confirmed that electromechanical torque and active power are independent of fault condition. Therefore, in the three-phase fault, these peak value are appeared at the same time.


 Fig. 6.  $i_{sas} - \theta_s$  characteristic curve( $n=0$ ).

 Fig. 7.  $T_{em} - t_f$  characteristic curve.

## VI. CONCLUSIONS

In this paper, we present transient current, active power, and reactive power, etc. analysis formulas of induction generators used in wind power system before and after three-phase fault conditions. These formulas are taking into account of rotor speed change of induction generators at three-phase fault conditions. Furthermore, theoretical discussion also developed to determine the fault conditions and the time at which maximum transient currents flow in the system. The simulation results obtained from each theoretical analysis are in close agreement with that of results obtained using MATLAB/SIMULINK simulation, and the validity of derived fault condition are demonstrated by the trial-and-error simulation. These equations can be used to study the dynamic behavior of the induction generator under three-phase fault condition.

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## Appendix

$$\theta_s = \omega t_s + \theta_{is}, \quad \phi_s = \theta_{ir} - \theta_{is}$$

$$A_s = L(L_1 L_2 I_{ms} + L_2 L_m I_{mr} \cos \phi_s)$$

$$A'_s = L(L_1 L_m I_{ms} + L_m^2 I_{mr} \cos \phi_s)$$

$$B_s = L(L_2 L_m I_{mr} \sin \phi_s - (r_s L_2 + r'_r L_1) I_{ms} / 2\omega_{r_{ss}})$$

$$B'_s = L(L_m^2 I_{mr} \sin \phi_s + (r_s L_2 + r'_r L_1) I_{ms} / 2\omega_{r_{ss}})$$

$$C_s = L(L_m^2 I_{ms} + L_2 L_m I_{mr} \cos \phi_s)$$

$$C'_s = L(L_1 L_m I_{ms} + L_1 L_2 I_{mr} \cos \phi_s)$$

$$D'_s = L(L_1 L_2 I_{mr} \sin \phi_s + (r_s L_2 + r'_r L_1) I_{ms} / 2\omega_{r_{ss}})$$

$$Z_{s1} = \sqrt{A_s^2 + B_s^2}, \quad Z'_{s1} = \sqrt{A'_s{}^2 + B'_s{}^2}$$

$$Z_{s2} = \sqrt{C_s^2 + D_s^2}, \quad Z'_{s2} = \sqrt{C'_s{}^2 + D'_s{}^2}$$

$$\beta_{s1} = \arctan(B_s/A_s), \quad \beta'_{s1} = \arctan(B'_s/A'_s)$$

$$\beta_{s2} = \arctan(B_s/C_s), \quad \beta'_{s2} = \arctan(D'_s/C'_s)$$

$$Z_s = \sqrt{\alpha^2 + \omega_r^2}$$

$$\gamma_s = \theta_s + \beta_{s2} - \delta_s, \quad \delta_s = \arctan \frac{\omega_r}{\alpha}$$