

# Influence of Earth Conductivity and Permittivity Frequency Dependence in Electromagnetic Transient Phenomena

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**Abstract – In this article a more accurate representation of soil behavior is presented. The proposed model takes into account the earth conductivity frequency dependence and the earth permittivity, which normally are not considered. One of the aspects covered in the paper is the importance of properly considering the earth's electromagnetic behavior when calculating transmission line parameters. For an actual 440 kV three-phase transmission line the soil behavior is represented through a unique real value of conductance ( the normal approach ) and through the proposed model.**

**Keywords :** Soil model, Line parameters, Frequency dependence, Electromagnetic transients.

## I. INTRODUCTION

One essential aspect of transmission line modeling is the adequate representation of ground, which has a big influence in line parameters, ahead of being a dominant aspect for analysis and for design of line grounding system. By historical and cultural reasons, most used procedures assume that the ground may be considered as having a constant conductivity, frequency independent, and an electric permittivity that can be neglected ( $\omega \epsilon \ll \sigma$ ). These two assumptions are quite far from reality, and can originate inadequate line modeling.

In the present paper a new soil model is presented. This satisfies the physical coherence conditions concerning the relation between conductivity ( $\sigma$ ) and permittivity ( $\epsilon$ ) in the frequency domain. Some examples of measured ground parameters are presented. The effect of the soil behavior in some transmission line transients and its influence on the overvoltages obtained are discussed.

For an actual 440 kV three-phase transmission line the soil behavior is represented through a unique real value of conductance, the most common assumption, and through a more accurate model of its electromagnetic behavior in relation to the earth conductivity and permittivity frequency dependence.

In some cases, a proper earth model can lead to very different results than the ones obtained with a simple real conductivity value, as shown. The influence of the frequency dependence of the soil parameters, in some line transient phenomena, is analyzed.

The content of the paper is as follows : in Section 2 the soil electromagnetic behavior is described, with the presentation of some measured results; in Section 3 line parameters are calculated; in Section 4 an application to an actual transmission line is presented.

## II. SOIL ELECTROMAGNETIC BEHAVIOR

One essential aspect of grounding systems study and simulation is adequate soil modeling.

Except for very high electric fields, that originate significant soil ionization, soil electromagnetic behavior is essentially linear, but with electric conductivity,  $\sigma$ , and electric permittivity,  $\epsilon$ , strongly frequency dependent. The magnetic permeability,  $\mu$ , is, in general, almost equal to vacuum magnetic permeability,  $\mu_0$ . For slow variation of electromagnetic entities, a hysteresis type behavior may occur. For direct current or very slow variations of electromagnetic entities, humidity migration phenomena, including electroosmosis and effects of temperature heterogeneity may take place, which cannot be dealt with only by means of local soil parameters.

For fast transients, namely those associated to lightning, the soil behavior is important in a reasonably wide frequency range, typically from 0 to 2 MHz.

### A. Field Measurement Procedure

Field measurement procedures have been chosen after measurement tests covering a large number of soil structures and conditions. The basic aspects related to collecting the samples are due to the necessity of [1-5]:

- Assuring maintenance of natural soil consistence and humidity, with sample material "identical" to natural ground.
- Avoiding influence of small depth surface effects, such as sun, wind and vegetation. These effects may originate an important dispersion, in time and space, and special measurement difficulties. To consider such effects correctly, special methods, considering statistical distribution with space and time correlation, may be required. In most applications the error resulting of neglecting such effects is relatively small.
- Avoiding important effects of local soil heteroge-

neity.

- Limit measurement errors related to electrode shape and contact conditions between electrodes and soil material.

Three basic procedures were adopted, namely :

- For rock, cylindrical samples (with 0.1 m diameter with 0.8 m length) are obtained with a boring machine.
- For reasonably consistent soils, a cutting and collecting procedure is applied, obtaining samples with a cuboid shape (1.2 m x 0.2 m x 0.2 m) which are covered with a net, paraffin and a wood box.
- For sand an pulverulent soil, samples are collected with a plastic tube with diameter 0.2 m and 1.2 m long, to which steel pieces are adapted to obtain easy penetration in soil and sample cutting in tube extremity.

Two current copper plate electrodes (CE) are adapted at sample extremities (with adjusted pressure) and two copper cylindrical voltage electrodes (VE) are inserted, with exemplificative geometry as in Figure 1. Through an oscillator with variable frequency  $f$ , it is imposed the current through the sample. From voltage at shunt terminal and voltage between voltage electrodes (both measured in amplitude and phase), and geometric factors, it is obtained  $\sigma + i\omega\varepsilon$ .

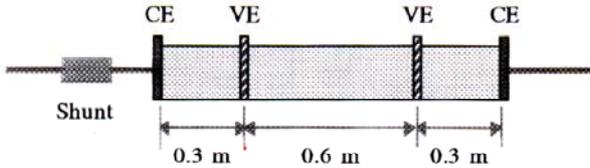


Figure 1- Schematic representation of a soil sample for measurement of  $\sigma + i\omega\varepsilon$  in function of frequency.

The field measurements of real soil have inherent dispersion. A purely mathematical fitting may lead to physically inconsistent models with quite wrong results, e.g. by Fourier methods. It is adequate to have a robust validation criteria of soil models, covering real soil characteristics.

In [1-5] several soil electric models have been presented and justified, which :

- Cover a large number of soil measured parameters, with good accuracy, and within the range of confidence of practical field measurement.
- Satisfy coherence conditions.

In this paper the electrical soil parameters are applied ( $\sigma$ ,  $\omega\varepsilon$ ), in function of frequency, considering a particular set of the models described in [1-5]. The parameters of such models were chosen according to a minimum difference criterion for field measured electrical parameters, in function of frequency, for 68 ground samples at eight sites, in Brazil, covering very different soil types and geological structures. The agreement of obtained models with measured parameters is within or near the confidence range of field measurement values. The measurements were carried out in a frequency range from 100 Hz to 2 MHz. At each site, the maximum dis-

tance between ground points at which samples were collected was less than 500 m.

To show that the influence of small depth surface effects can be neglected in most applications, we indicate, in Table 1, the soil depth  $d$  at which electromagnetic field related to longitudinal line parameters reduces to about 5 % of field at soil surface, in four examples, for three frequencies,  $f$ . The first two (examples 1 and 2) consider, respectively, soil 1 and soil 2 of item II E. of this paper. The last two (examples 3 and 4) consider soils similar to examples 1 and 2, but with a low frequency conductivity of 1 mS/m.

Table 1 – Soil penetration depth,  $d$ , in four examples, for three frequencies

	$d$ [m]			
	Example 1	Example 2	Example 3	Example 4
$f = 60$ Hz	21 742	21 218	6 169	6 155
$f = 10$ kHz	1 694	1 644	510	477
$f = 1$ MHz	48	164	40	48

### B. Soil Models

The models which have been used in the presented results are some of the models described in [1-5].

With the exception indicated below, the models, whose results are presented, are a sum of minimum phase shift parcels,  $W_j$ , which apply to the immittance type magnitude (in complex or tensorial formulation of alternating magnitudes)

$$W = \sigma + i\omega\varepsilon \quad (\omega = 2\pi f, f \text{ being the frequency}) \quad (1)$$

where  $i = +\sqrt{-1}$  and

$$W = \sum_{j=0}^m W_j \quad (2)$$

All submodels used for  $W_j$  are particular conditions of a Type 3 model described in [1], presented below.

Apart from slow phenomena and hysteresis type phenomena, soil behavior is, typically, of minimum phase shift type. For a great number of soils, on frequency range (0, 2 MHz), in a first approach, it is

$$\sigma = a + b \cdot \omega^\alpha \quad \text{and} \quad \omega\varepsilon = c \cdot \omega^\alpha \quad (3)$$

where  $a$ ,  $b$ ,  $c$ ,  $\alpha$  are constant parameters (frequency independent).

For some soils, a similar behavior occurs, but for a smaller frequency range, e.g. (0, 100 kHz), and for higher frequencies, the behavior is different, namely with a lower  $\omega\varepsilon$  increase, or until a  $\omega\varepsilon$  decrease, when frequency increases.

In order to analyze the frequency behavior of  $\sigma$ ,  $\varepsilon$ , it is convenient to consider complex formulations of electromagnetic entities, and to consider  $\sigma + i\omega\varepsilon$  as an immittance. In fact, apart from geometric factors,  $\sigma + i\omega\varepsilon$  may be associated with the admittance of a volume element  $\delta v$ .

A type 3 model can be described as presented below, for which :

$$W_j(\omega) = k \cdot \left\{ \frac{b^\alpha}{\alpha} {}_2F_1 \left[ 1, \alpha, 1+\alpha, \frac{ib}{\omega} \right] - \frac{a^\alpha}{\alpha} {}_2F_1 \left[ 1, \alpha, 1+\alpha, \frac{ia}{\omega} \right] \right\} \quad (4)$$

representing  ${}_2F_1[\dots, \dots, \dots]$  the hypergeometric function, with four arguments,  ${}_2F_1$ , according to the notation of [6].

This submodel has four independent parameters ( $\mathbf{k}$ ,  $\alpha$ ,  $\mathbf{a}$ ,  $\mathbf{b}$ ).

Considering, in this model (4),  $\mathbf{a} = 0$ , the model becomes :

$$\begin{aligned} W_j(\omega) &= k \cdot \frac{b^\alpha}{\omega} {}_2F_1\left[1, \alpha, 1+\alpha, \frac{ib}{\omega}\right] \\ &= k_1 {}_2F_1\left[1, \alpha, 1+\alpha, \frac{ib}{\omega}\right] \end{aligned} \quad (5)$$

Considering, in the model (4),  $\mathbf{a} = 0$ ,  $\mathbf{b} \rightarrow \infty$  and  $\alpha_j = \alpha$ , the model becomes :

$$W_j(\omega) = K_j \cdot \left[1 + i \tan\left(\frac{\pi}{2}\alpha_j\right)\right] \cdot \omega^{\alpha_j} \quad (6)$$

A parcel  $W_j$  as indicated in (6) is equivalent to parcel  $b \cdot \omega^\alpha$  of  $\sigma$  and to  $\omega \varepsilon = c \cdot \omega^\alpha$ , as indicated in (1) and (3), with  $\mathbf{b} = K_j$ ,  $c = K_j \cdot \tan\left(\frac{\pi}{2}\alpha_j\right)$  and  $\alpha_j = \alpha$ , with the

condition  $\frac{c}{b} = \tan\left(\frac{\pi}{2}\alpha\right)$ . This condition has been verified in soil measurements, within measurement accuracy and soil heterogeneity effects.

Considering, in this model (6),  $\alpha_j = 0$ , the model becomes :

$$\sigma \text{ constant, } \omega \varepsilon \text{ null ("pure" conductor)} \quad (7)$$

Considering, in the model (6),  $\alpha_j \rightarrow 1$ , the model becomes :

$$\sigma \text{ null, } \omega \varepsilon \text{ proportional to } \omega, \varepsilon \text{ constant ("pure" dielectric)} \quad (8)$$

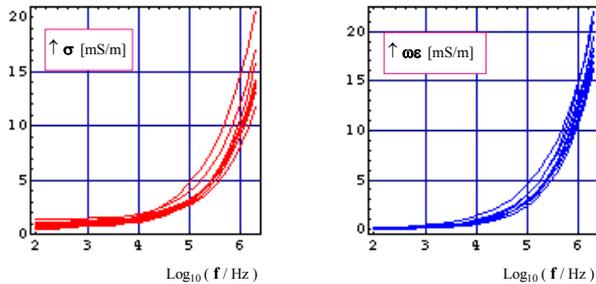


Figure 2 – Electric parameter of soil sample,  $\mathbf{f}$  in logarithmic scale.

Within the range ( 0 , 2 MHz ), for all soil samples modeled in this paper, it is accurate enough to consider two parcels, for  $\sigma + i\omega\varepsilon$ , one constant (in most cases real), and the other of type (4) or of type (5), frequency dependent. In a few cases, there is a net hysteresis effect, that can be modeled with an imaginary part of the constant parcel. For all samples,  $\alpha$  is the dominant parameter of the relative shape of a frequency dependent parcel,  $W_j$ , of  $\sigma + i\omega\varepsilon$ . For  $\alpha = 0$  such a parcel corresponds to a “pure” conductor (  $\sigma$  frequency independent,  $\varepsilon$  null ). For  $\alpha = 1$ , such a parcel corresponds to a “pure” dielec-

tric (  $\sigma$  null ,  $\varepsilon$  constant ). In all samples, for a frequency dependent parcel, it is  $0 < \alpha < 1$ .

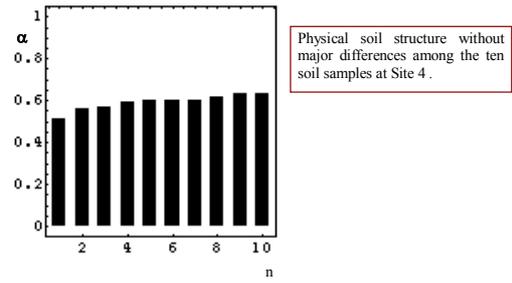


Figure 3 – Electric parameter of soil sample,  $\mathbf{f}$  in logarithmic scale.

### C. Soil Samples

In this section it is presented in graphic form  $\sigma$  and  $\omega\varepsilon$ , in function of frequency,  $\mathbf{f}$ , for models of one of the ground samples obtained in Amazon region. In Figure 2  $\mathbf{f}$  is represented in logarithmic scale. In Figure 3 it is represented, in bar form, the distribution of parameter  $\alpha$  of the function relating  $\sigma + i\omega\varepsilon$  to  $\mathbf{f}$ , either of Type (5) or (6).

### D. Statistical Distribution of Soil Parameters

In order to allow a direct interpretation of statistical distribution of the main electric parameters of ground, in a way which is independent of the model details, the following parameters were chosen, according to the models adopted, independently, for the 68 soil samples, satisfying physical coherence conditions:

$$\sigma_0 = \sigma(100 \text{ Hz}), \sigma \text{ at } 100 \text{ Hz.}$$

$$\Delta_r = \Delta\sigma_1 = \sigma(1 \text{ MHz}) - \sigma(100 \text{ Hz}), \sigma, \text{ increase between } 100 \text{ Hz and } 1 \text{ MHz.}$$

$$\Delta_i = \Delta(\omega\varepsilon)_1 = \omega\varepsilon(1 \text{ MHz}) - \omega\varepsilon(100 \text{ Hz}), \omega\varepsilon \text{ increase between } 100 \text{ Hz and } 1 \text{ MHz.}$$

$$\alpha \text{ parameter of the frequency dependent parcel of } \sigma + i\omega\varepsilon.$$

It was verified that, for these samples, the two parcels of  $\sigma + i\omega\varepsilon$ , one constant, the other frequency dependent, are statistically independent. This fact, and the fact that no significant correlation exists between the pair  $[\Delta_i, \alpha]$ , although it exists between the pair  $[\Delta_r, \alpha]$ , gives rise to the hypothesis that:

- The constant and the frequency dependent parcels of  $\sigma + i\omega\varepsilon$  are related to quite distinct aspects of physical ground behavior.
- The frequency dependent parcel is mainly associated with a dielectric physical process, with related dissipative effects. Such dissipative effects are quite different from conductive behavior associated with the constant parcel.

In Figure 4 we represent the probability density,  $\mathbf{p}$ , of parameters  $\sigma_0, \Delta_r, \Delta_i, \alpha$ , considered separately, and, in Figure 5, the probability density,  $\mathbf{p}$ , of parameters  $[\Delta_i, \alpha]$ , considered together, with Weibull approximations based on the 68 soil samples.

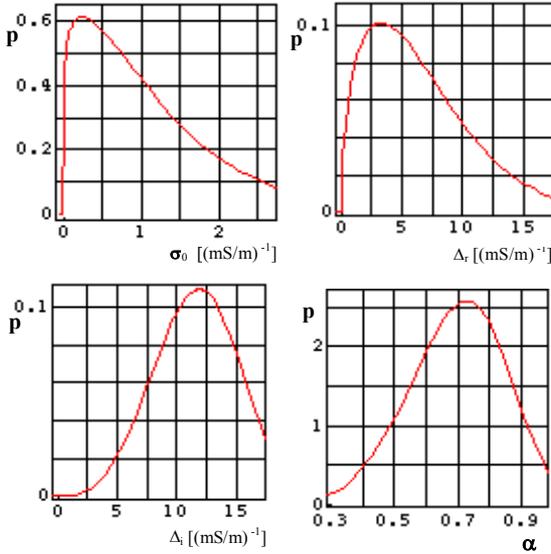


Figure 4- Probability density,  $p$ , of parameters  $\sigma_0$ ,  $\Delta_r$ ,  $\Delta_i$ ,  $\alpha$ , considered separately, with Weibull approximations based in the 68 soil samples. Scales of  $p$  applicable to  $\sigma_0$ ,  $\Delta_r$ ,  $\Delta_i$  are graduated in  $(\text{mS/m})^{-1}$ .

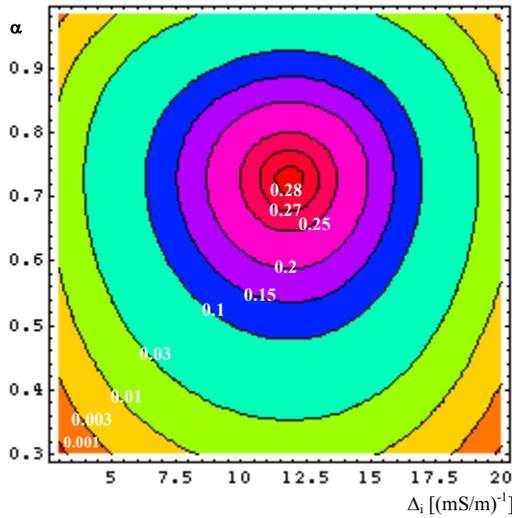


Figure 5- Probability density,  $p$ , of parameters  $[\Delta_i, \alpha]$ , considered together, with Weibull approximations based in the 68 soil samples and without correlation between  $\Delta_i$  and  $\alpha$ . Values of  $p$ , in white, are expressed in  $(\text{mS/m})^{-1}$ .

### E. Soil Parameters Applied

The soil parameters used in these examples were obtained from the experiments described in [3], and are presented below :

- Soil 1 :  $\sigma + i\omega\varepsilon = A + B\omega^\alpha$

with  $A, B, \alpha$  constants, and

$$A = 84.16 \mu\text{S/m}$$

$$B = [0.057849 + 0.12097 i] (\mu\text{S/m}) s^\alpha$$

$$\alpha = 0.71603$$

- Soil 2 :  $\sigma = A$  ;  $\omega\varepsilon = 0$

which results in  $\rho : 11882 \Omega\cdot\text{m}$ , constant.

The conductivity of the studied soils were chosen to be equal at low frequency, in order to compare the obtained results with the results of traditional procedure

that assumes constant conductivity (as measured at low frequency) and  $\omega\varepsilon = 0$ . Soil 1 considers two parcels  $W_j$  as described in II.3, namely one constant parcel,  $A$ , of the type (7) and a frequency dependent parcel,  $B\omega^\alpha$ , of the type (6). In soil 2, only the first parcel,  $A$ , is considered. Although this constant parcel has an apparently low value, it is not uncommon to measure conductivities of this magnitude or lower in several Brazilian regions. By example, for a specific upcoming 1000 km-long transmission line, in the center of Brazil: in 20 % of the line length conductivity was in the range or lower than the adopted value; the minimum measured value was near  $43 \mu\text{S/m}$ .

### III. CALCULATING THE LINE PARAMETERS

In order to implement the soil model, the line parameters were calculated using the approximated formula which includes the earth effect in longitudinal impedance as the equivalent to having an ideal ground surface at a depth  $D'$  (complex) below physical ground surface [7].

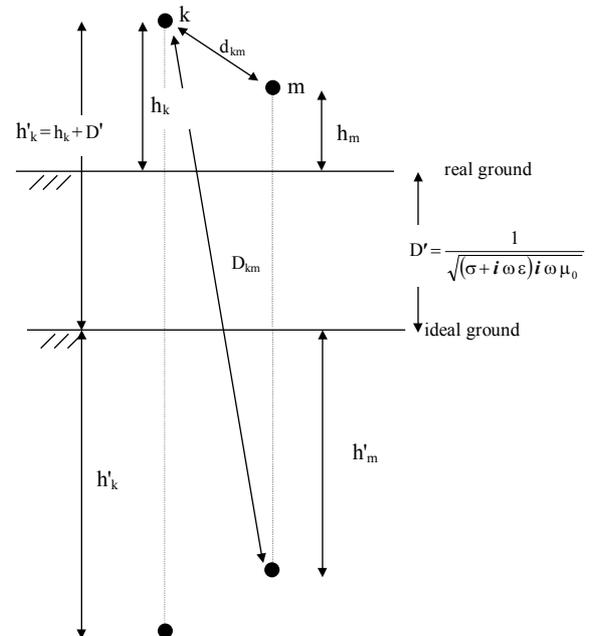


Figure 6 - Conductors  $\underline{k}$  and  $\underline{m}$  position supposing the earth at a complex depth  $D'$ .

The transmission line longitudinal impedance matrix, per unit length, may be obtained considering:

$$Z_0 = |Z_{0km}| \quad k, m = 1, 2, \dots, n \quad (9)$$

where :

$Z_{0km}$  – longitudinal impedance matrix element, per unit length;

$n$  – total number of conductors

and

$$Z_{0km} = Z_{intkm} + Z_{extkm} \quad (10)$$

where

$Z_{intkm}$  – internal impedance, per unit length, of conductor  $k$ , for  $k = m$ , 0 for  $k \neq m$ ;

$Z_{extkm}$  – external impedance, per unit length, between

conductors  $k, m$ ;  
and

$$Z_{ext} = i \frac{\omega \mu_0}{2\pi} \ln \frac{D_{km}}{d_{km}} \quad k, m = 1, 2, \dots, n \quad (11)$$

where  $D_{km}$  and  $d_{km}$  are defined schematically in Figure 6, and given by :

$$D' = \frac{1}{\sqrt{(\sigma + i\omega\varepsilon)i\omega\mu_0}} \quad (12)$$

For the self terms ( $k = m$ )

$$D_{km} = 2h'_k \quad (13)$$

$$d_{km} = r_k \quad (\text{external radius of conductor } k) \quad (14)$$

and

$$Z_{int} = R_{int} + i X_{int} \quad (15)$$

where

$R_{int}$  – internal conductor resistance

$X_{int}$  – internal conductor reactance

In Figure 7 and Figure 8 the per unit longitudinal parameters for the transposed line using both soil models are presented.

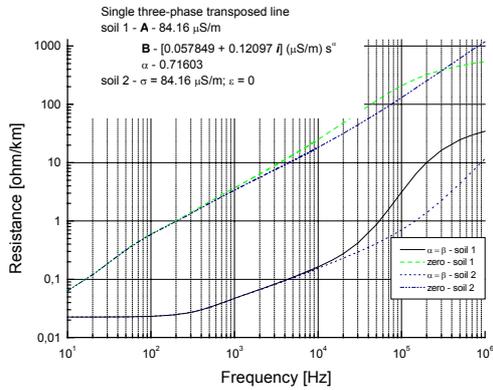


Figure 7 – Resistance per unit length comparing both soil models.

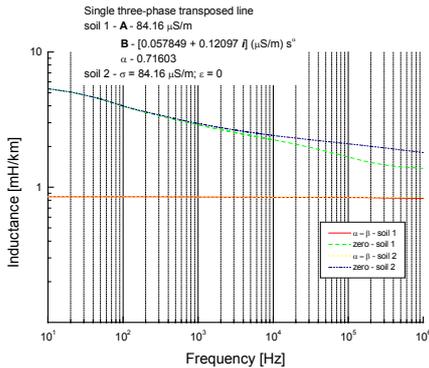


Figure 8 – Inductance per unit length comparing both soil models.

The difference between line parameters for the two soil models is important, namely for the homopolar mode (e.g., 37% in the longitudinal resistance per unit length, at 10 kHz). For fast transients, for which important frequency range may include frequencies above 10 kHz, the difference between the two soil models may also be important for non-homopolar modes. E.g., for 100 kHz, there is an order of magnitude difference in resistance per unit length, between the two soil models, for non-homopolar modes. Typical cases in which fre-

quency range above 10 kHz is important are : transients originating from lightning; front of wave aspects of transients associated with short-circuits. Such cases may be quite important in what concerns insulation coordination.

#### IV. SINGLE THREE-PHASE LINE APPLICATION

In Figure 9 the data of the three-phase line used to illustrate the model are presented.

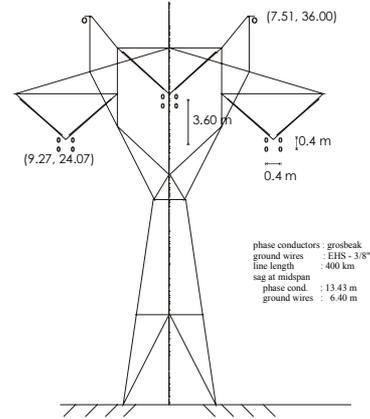


Figure 9 - Schematic representation of the 440 kV three-phase line.

The line parameters were calculated in the range of 10 Hz to 10 kHz. As it is a single line, to represent its modes (exact ones for a transposed line and quasi-modes for a non-transposed line) Clarke's transformation matrix was applied as explained in [8-10]. With the longitudinal impedance and transversal admittance in mode domain, the synthetic circuits were calculated, composed of one cascade of  $\pi$ -circuits for each mode, each representing 10 km length. The 10 km length  $\pi$ -circuit represents properly the line response up to 7 kHz. The line was supposed ideally transposed.

##### A. Frequency Analysis

A frequency scan analysis was performed for both models where the sending terminal had a 1 V source and the receiving end was open. The relations between the line ends were analyzed in the range of 10 Hz to 7 kHz. In Figure 10 the zero sequence response is presented for the transposed line.

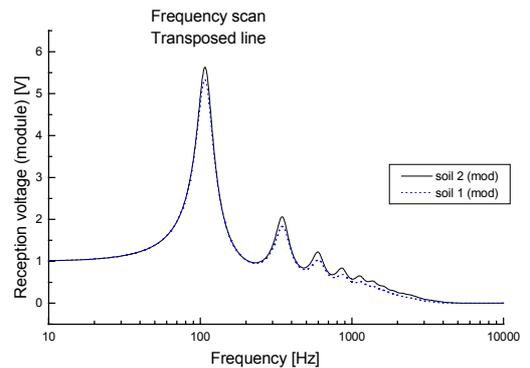


Figure 10 - Zero sequence - Transposed line.

The results for both soil models are discussed below :

- The positive sequence response was similar for both models.
- The zero sequence response for the frequency dependant soil model is more damped than the one which uses only constant conductance.
- The difference between the zero sequence response for the two soil models is important, e.g. 14 % at 1000 Hz.

## V. CONCLUSIONS

In some existing systems there are unexplained differences, sometimes of the order of magnitude, between calculated and measured values, such as between induced voltages, electric field in ground and transferred voltages. These differences can be related to the soil models used.

So, it is essential for most applications concerning grounding systems, or involving electromagnetic phenomena affected by ground, to adequately model the ground behavior, including several aspects not considered in common practice.

For transmission lines, according to specific conditions, and the phenomena being studied, it may be quite important to correctly model the soil, considering frequency dependence of  $\sigma + i\omega\varepsilon$ .

We have presented some illustrative results for a 440 kV three-phase transmission line. The soil behavior is represented through two alternative soil models. In the first soil model we have considered an accurate soil representation, satisfying coherence conditions and with  $\sigma + i\omega\varepsilon$  frequency dependent. In the second soil model, we have considered a constant, frequency independent, conductance and  $\omega\varepsilon$  much lower than  $\sigma$ . The two models have similar behavior at low frequency, but quite a distinct one at higher frequency.

In some cases, an adequate earth model can lead to results quite different from those obtained with the usual procedure of considering the parameter  $\sigma$  of soil frequency independent and parameter  $\varepsilon$  frequency independent with a relatively small value. The conditions in which such a difference can be important include the following examples :

- Switching conditions in which an important homopolar component may occur, either due to the spread of switching of the three poles, or to fault conditions, and in which the important frequency spectrum is not restricted to extremely low frequencies ( $< 1$  kHz), and includes frequencies up to about 10 kHz.
- Network sustained operation, faults and maneuvers in which conditions occur near resonance, for the homopolar component, for frequencies not restricted to extremely low frequencies ( $< 1$  kHz), and, e. g. , for frequency between 1 and 10 kHz.
- Fast transients, for which important frequency range may include frequencies above 10 kHz. In this case, the difference between an accurate soil model and usual assumptions may be important also for non-homopolar modes. Typical cases in which frequency

range much above 10 kHz (for some conditions above 1 MHz) is important are: transients originated by lightning; front of wave aspects of transients associated with short-circuits, transients in gas-insulated substations. From the presented results it is expected that, for transient phenomena with dominant frequency spectrum till above 10 kHz, the distinct homopolar mode response may have quite important effects, not restricted to higher attenuation. By example, overvoltage shape, namely in front of wave, may be quite different, with important consequences in insulation coordination.

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