Numerical Analysis of Grounding Resistance of Buried Thin Wires by the FDTD Method

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***Abstract* – For the analysis of grounding resistance with the finite-difference time-domain (FDTD) method for solving Maxwell's equations, an equivalent radius of a naked thin wire in a lossy medium is derived by means of the static-field approximation proposed for derivation of that of an aerial thin wire. It is 0.23 times the side of cells employed, which is the same as that of an aerial thin wire. The validity is tested by comparing the grounding-resistance values obtained through FDTD simulations on simple buried structures with the theoretical values.**

Keywords – FDTD method, grounding electrode, grounding resistance.

I. INTRODUCTION

The role of grounding electrodes is to dissipate fault currents effectively into the soil, and thereby to prevent damage of installations in power systems. Thus, the performance of power systems is influenced by proper functioning of grounding systems.

No formulas of impedance and admittance have been derived even for a simple vertical or horizontal naked conductor buried in a homogeneous ground. Hence, transient characteristics of grounding electrodes have been investigated by experiments and recently by numerical electromagnetic analyses [1],[2],[3] based on the method of moments (MOM), the finite element method (FEM) or the finite-difference time-domain (FDTD) method [4]. Numerical electromagnetic analyses can be performed assuming a well-profiled condition that the values of conductivity and permittivity of a ground are known or set arbitrarily. Such results are useful in understanding the phenomena as well as in confirming measured results.

Numerical electromagnetic analyses based on the FDTD method [4] are effective to analyze the transient response of a large solid conductor or electrode. The accuracy of this method, in the case of being applied to such analysis, has been fully investigated in comparison with an experiment and shown to be satisfactory [5]. As this method requires long computation time and large capacity of memory, an analysis is restricted to a rather small space. A transient analysis of a large system or a system composed of various elements still need to be performed by such tools like the Electro-Magnetic Transients Program (EMTP) [6]. One reasonable process of study, therefore, is to investigate the physical characteristics of a grounding electrode by a numerical electromagnetic analysis, and then to represent the obtained characteristics by an equivalent circuit model or to determine the values of its parameters [2].

So far in most of FDTD analyses of transient and steady-state grounding resistance, large solid electrodes [3],[5], which can be decomposed into many small cubic cells, have been chosen and thin-wire electrodes have not been dealt with. This is because an equivalent radius of a thin wire in a lossy medium has not been made clear. It is, therefore, essential to clarify the equivalent radius of a buried thin wire for more general analyses of grounding systems with this method. In the present paper, an equivalent radius of a thin wire in a lossy medium is derived with the help of the concept proposed for derivation of that of an aerial thin wire [7]. Then its validity is tested by comparing the grounding-resistance values obtained through FDTD simulations on simple buried structures with the theoretical values.

II. METHOD OF ANALYSIS

The FDTD method employs a simple way to discretize a differential form of Maxwell's equations. In the Cartesian coordinate system, it generally requires the entire space of interest to be divided into small rectangular cells and calculates the electric and magnetic fields of the cells using the discretized Maxwell's equations. As the material constants of each cell can be specified arbitrarily, a complex inhomogeneous medium can be easily analyzed. To analyze fields in an open space, an absorbing boundary condition has to be set on each plane which limits the space to be analyzed, so as to avoid reflections there. In the present analysis, the second-order Mur's method [8] is employed to represent absorbing planes.

III. DERIVATION OF EQUIVALENT RADIUS OF BURIED THIN WIRE

In [7], it has been shown that an aerial thin wire has some equivalent radius in the case that the electric-field elements along the thin wire are set to zero in an orthogonal and uniform-spacing Cartesian grid. When the side of cubic cells employed is Δs, the equivalent radius is 0.23 Δs. In the present paper, an equivalent radius of a naked thin wire in a lossy medium is derived. Note that in [9] an equivalent radius of an aerial thin wire has been shown to be 0.135 Δs. In a quasi-steady state, however, 0.23 Δs is more appropriate than 0.135 Δs as an equivalent radius [7].

Figure 1 illustrates the cross section of a long thin wire surrounded by a cylindrical sheath conductor. The radii of the thin wire and the sheath are a and b, respectively. The conductivity and the relative permittivity of a medium between the thin wire and the sheath conductor are assumed to be σ and ε_S, respectively. In this condition, the conductance G and the susceptance B between the thin wire and the sheath are given as follows:

$$G = \frac{2\,\pi\,\sigma}{\ln\,(\,b\,/\,a\,)}\,,\qquad B = \frac{2\,\pi\,\varepsilon_0\,\varepsilon_S\,\omega}{\ln\,(\,b\,/\,a\,)}\;. \tag{1}$$

Note that ε_0 is the permittivity of vacuum and ω is the angular frequency (= 2 π f). Therefore, the conductance becomes equal to the susceptance when the frequency f is

$$f_0 = \sigma\,/\,\left(2\,\pi\,\varepsilon_0\,\varepsilon_S\right)\;. \tag{2}$$

For instance, f_0 is 1.5 or 7.5 MHz for a medium of ε_S = 12 and σ = 1 or 5 mS/m, respectively. If the frequency is lower than f_0, the conductance becomes higher than the susceptance and the conduction current in a radial direction is larger than the displacement current. Therefore, electric fields in the medium are mainly determined by σ after several hundreds nanoseconds (> 1 / f_0).

Figure 2 shows the cross section of a thin wire surrounded by a rectangular sheath conductor for an FDTD simulation. Both the thin wire core and the sheath are perfectly conducting. The cross-sectional area of the sheath is 2.5 × 2.5 m^2 and the length is 25 m. This conductor system is represented with cubic cells whose side Δs is 0.25 m. A voltage, which has a risetime of 20 ns and a magnitude of 100 V, is applied between the thin wire and the sheath at its one end. The other end is open. The response is calculated up to 10 μs with a time increment of 0.4 ns.

Figure 3 shows the time-variations of a current at the injection point when σ is 1 or 5 mS/m and ε_S is 12. Figure 4 shows those of E_1, E_2 and E_3 in Fig. 2, which are radial electric fields calculated at 0.5 Δs, 1.5 Δs and 2.5 Δs from the center of the thin wire and at 12.5 m from each end of the conductor. Figure 5 shows the time-variations of the ratios of E_1, E_2 and E_3 to E_2.

From Fig. 5, it is found that the ratios settle down after 100 ns or so although electric fields still change over time. The ratios of E_1, E_2 and E_3 to E_2 are almost equal to those calculated for a thin wire in air [7]: 2.21, 1.00 and 0.59. This is natural because both the conductance and the susceptance of a thin wire follow a similar expression as shown in Eq. (1). Also, the ratios change little even if a different conductivity such as 0.2 or 10 mS/m is employed and a different time increment 0.25 or 0.48 ns is used. Thus, the radial electric field around the thin wire can also be approximated by the following function [7]:

$$E = \frac{3\,\Delta s}{2\,x}\,\cdot \tag{3}$$

Note that x is the distance from the center of the thin wire. In this function, the electric field E is normalized so that E should be unity at x = 1.5 Δs. Figure 6 shows the radial electric fields calculated by this function and those obtained by the FDTD simulation.

If the equivalent radius of the thin wire now in question is assumed to be r_0 and the radial electric field E is assumed to follow Eq. (3), the potential difference between x = r_0 and x = Δs is obtained by integrating Eq. (3) from r_0 to Δs. As a result, it is given as

$$\int_{r_0}^{\Delta s} E\,dx = \frac{3\,\Delta s}{2}\,\ln\frac{\Delta s}{r_0}\;\cdot \tag{4}$$

If the above expression is equated to 2.2 Δs, which is the potential difference obtained by the FDTD simulation, the equivalent radius r_0 is given as

$$r_0 = 0.23\,\Delta s\,\cdot \tag{5}$$

This is an equivalent radius of a naked thin wire in a lossy medium.

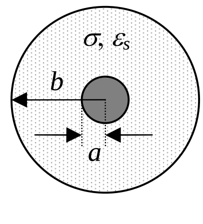


Fig. 1 Cross section of a thin wire surrounded by a cylindrical sheath

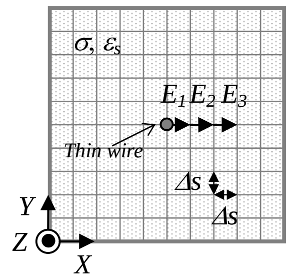
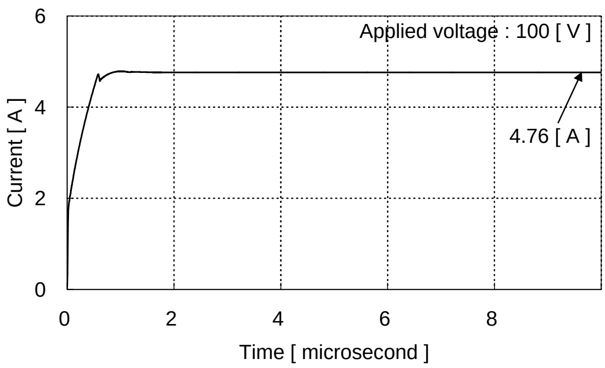
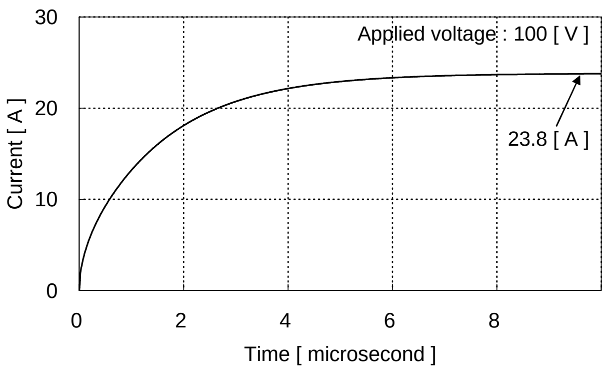


Fig. 2 Electric fields around a thin wire in a rectangular sheath to be used for an FDTD analysis

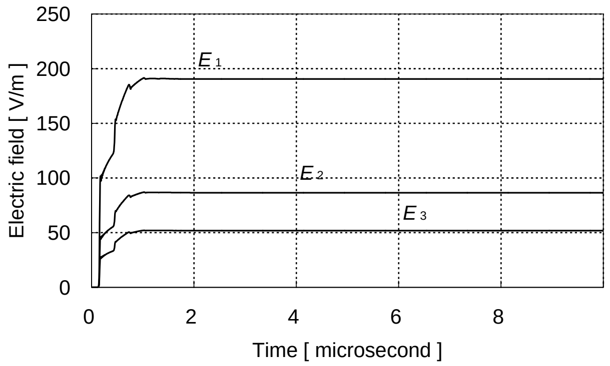


(a) σ = 1 mS/m and ε_S =12

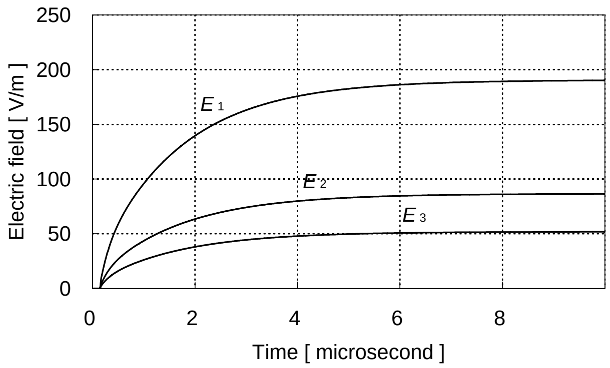


(b) σ = 5 mS/m and ε_S =12

Fig. 3 Time-variations of a current at the injection point calculated by the FDTD method

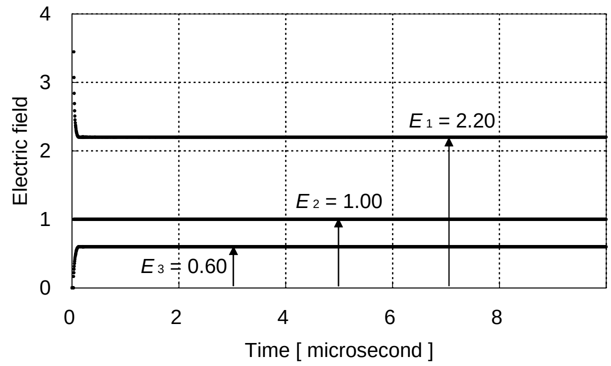


(a) σ= 1 mS/m and ε_S =12

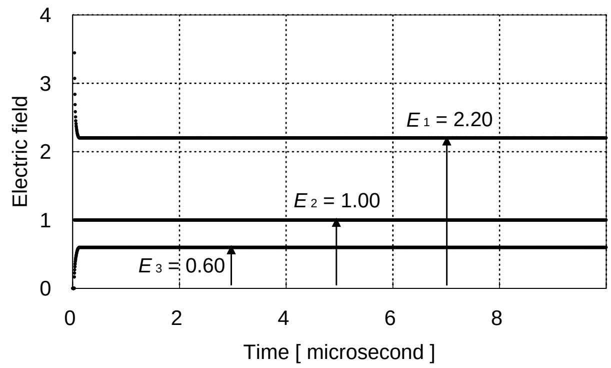


(b) σ= 5 mS/m and ε_S =12

Fig. 4 Time-variations of E_1, E_2 and E_3 calculated by the FDTD method



(a) σ= 1 mS/m and ε_S =12



(b) σ= 5 mS/m and ε_S =12

Fig. 5 Time-variations of the ratios of E_1, E_2 and E_3 to E_2 calculated by the FDTD method

The direct-current (DC) resistance R_{CS} between the core and the cylindrical sheath as shown in Fig. 1 is theoretically given as

$$R_{CS} = \frac{\ln (b / a)}{2 \pi \sigma l}, \tag{6}$$

where l is the length of the conductor. If an equivalent radius of the rectangular sheath shown in Fig. 2 is the radius of a circle that has the same circumference as the rectangular sheath has, b for the rectangular sheath is 1.59 m (= 4 × 2.5 m / 2 π) . The radius of the core a is 57.5 mm (= 0.23 × 0.25 m) from Eq. (5). The length of the conductor l is 25 m. Table I summarizes R_{CS} as a function of σ. The transient resistance obtained from the FDTD simulation is also listed in Table I. In this section, the transient resistance is defined as the instantaneous ratio of an applied voltage to the current at the injection point, and it is evaluated at 10 μs. For instance, 4.2 Ω (= 100 V / 23.8 A) is obtained from Fig. 3. The transient resistance for σ= 10 mS/m is not shown because the injected current does not settle down at 10 μs for the case.

The resistance values obtained from the FDTD simulation agree with the theoretical values. This agreement shows that 0.23 Δs is valid as the equivalent radius of a thin wire in a lossy medium.

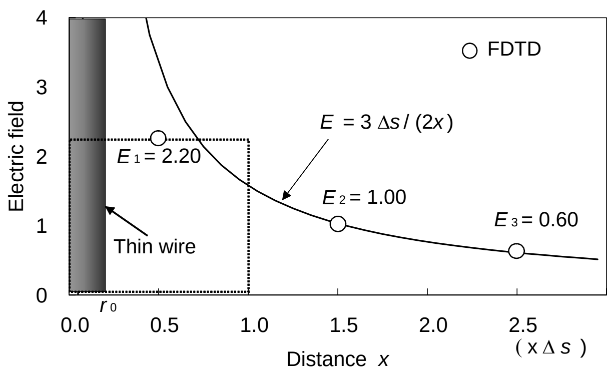


Fig. 6 Radial electric fields as a function of distance from the thin wire [7] (The value of electric field is normalized so that E_2 should be unity)

Table I DC Resistance calculated from Eq. (6) and the transient resistance at 10 μs obtained by the FDTD simulation

	σ= 0.2mS/m	σ= 1 mS/m	σ= 5 mS/m
FDTD	105 Ω	21.0 Ω	4.20 Ω
Eq. (6)	106 Ω	21.1 Ω	4.23 Ω
Difference	0.9%	− 0.5%	− 0.7%

IV. COMPARISON WITH SUNDE'S FORMULA ON GROUNDING RESISTANCE

A. Models for Analysis

Figure 7 shows a side view of an analysis model, which is composed of two naked vertical thin wires and an overhead horizontal thin wire. The buried portion of vertical thin wires is 3 or 5 m. The horizontal thin wire is 30 m long and 1 m high over the surface of a homogeneous ground. This conductor system is excited by a voltage source at a connection point between the horizontal wire and one of the buried vertical wire. The voltage source produces a steep-front wave having a risetime of 10 ns, after which it maintains a magnitude of 100 V.

The conductivity of the homogenous ground σ is set to 1 or 5 mS/m and the relative permittivity ε_S is set to 12. For the FDTD simulation, the conductor system shown in Fig. 7 is accommodated by a large rectangular analysis space of $40 \times 60 \times 30$ m^3. The second-order Mur's absorbing condition [8] is applied to all the planes surrounding the space to be analyzed. To model the system, cubic cells whose side is 0.25 m are used.

The simulations are performed by a personal computer with Pentium IV 1.6 GHz and 1024 MB RAM (about 300 MB of memory is needed in practice). Responses are calculated up to 5 μs with a time increment of 0.4 ns. The computation time for one case is about 20 hours.

B. Analyzed Results

Figure 8 show current waveforms at the voltage source calculated for the model of Fig. 7 in the case that the vertical thin wires are buried up to 3 m. Figure 9 show those for the vertical thin wires being buried up to 5 m. Tables II and III summarizes the values of transient grounding resistance R_{GV} of the 3- and 5-m vertical thin wires evaluated at 5 μs from Figs. 8 and 9, respectively. They are simply calculated from the following relation: $I_S = V_S \,/\, 2\, R_{GV}$. Note that V_S is the magnitude of the applied voltage and I_S is the current of the circuit.

C. Discussion

The wavelength of an electromagnetic field, which corresponds to the evaluation time (5 μs), is several hundred meters. It is ten times longer than the length of the conductor systems shown in Fig. 7. Hence, it is considered that the transient-resistance value at 5 μs is close to the resistance in the steady state. Sunde [10] has derived a theoretical formula of the DC resistance of a vertical conductor buried in a homogeneous ground. It is expressed as

$$R_{GV.SUNDE} = \frac{1}{2\,\pi\,\sigma\,d}\left(\ln\frac{4\,d}{r} - 1\right), \qquad (7)$$

where d is the length and r is the radius of the electrode. The values of grounding resistance calculated by this formula are also included in Tables II and III. The values of transient grounding resistance obtained by the FDTD simulation are only about 5% lower than those calculated by Sunde's formula regardless of the ground conductivity.

When the length of the overhead horizontal thin wire is shortened or enlarged from 30 m to 20 m or 40 m, the transient resistance decreases only by 0.5 Ω (1.7%) or increases by 0.4 Ω (1.4%) for a 5-m buried vertical thin wire in a ground having the conductivity of 5 mS/m. Therefore, it is clear that the influence of 30-m distance between two electrodes is little.

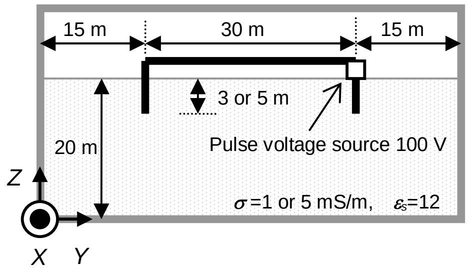
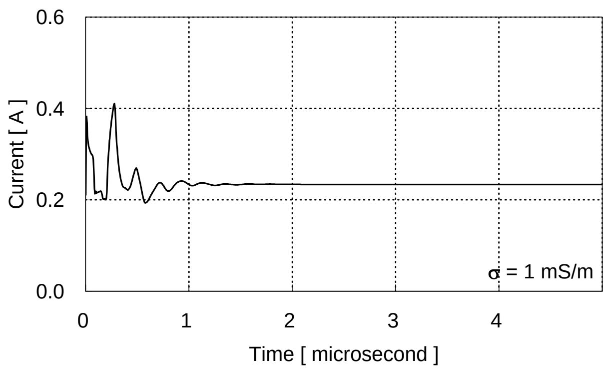
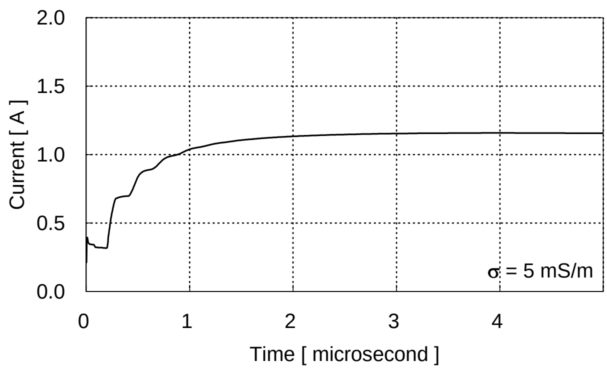


Fig. 7 Two buried vertical thin wires connected by an overhead horizontal wire to be analyzed by the FDTD method

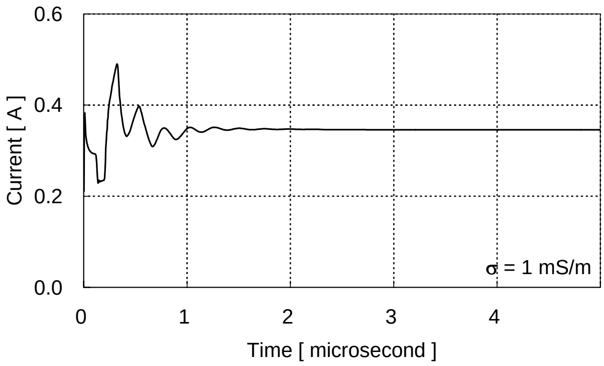


(a) σ = 1 mS/m

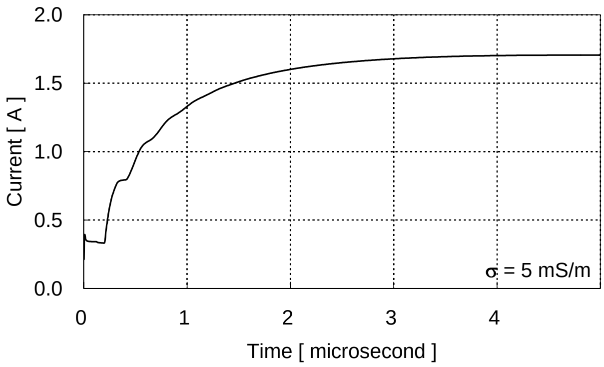


(b) σ = 5 mS/m

Fig. 8 Calculated current waveforms at the voltage source of the model of Fig. 7 in the case that the vertical thin wires are buried up to 3 m



(a) σ = 1 mS/m



(b) σ = 5 mS/m

Fig. 9 Calculated current waveforms at the voltage source of the model of Fig. 7 in the case that the vertical thin wires are buried up to 5 m

As a consequence, it has become clear that 0.23 Δs is valid as the equivalent radius of a thin wire buried in a lossy ground. Note that Sunde has proposed a theoretical formula of resistance also for a horizontal cylindrical electrode [10]. As it is a function of the natural logarithm of the square root of r, the resistance value of a horizontal thin electrode is not so sensitive to the radius of the electrode. This is the reason why a horizontal electrode is not employed for comparison.

V. CONCLUSIONS

In the present paper, for the analysis of grounding resistance with the FDTD method, an equivalent radius of a naked thin wire in a lossy medium has been derived with the help of the static-field concept proposed for an aerial thin wire. It is 0.23 times the side of cells employed, which is the same as that of an aerial thin wire. The validity has been investigated by comparing the grounding-resistance values obtained through FDTD simulations on simple buried structures with the theoretical values, and shown to be satisfactory.

Table II Transient grounding resistance of a 3-m vertical electrode obtained by the FDTD analysis and the DC resistance calculated by Sunde's formula

	σ = 1 mS/m	σ = 5 mS/m
FDTD	214 Ω	43.2 Ω
Eq. (7)	230 Ω	46.1 Ω
Difference	− 7 %	− 6 %

Table III Transient grounding resistance of a 5-m vertical electrode obtained by the FDTD analysis and the DC resistance calculated by Sunde's formula

	σ = 1 mS/m	σ = 5 mS/m
FDTD	145 Ω	29.3 Ω
Eq. (7)	154 Ω	30.9 Ω
Difference	− 6 %	− 5 %

Table IV Dependency of transient grounding resistance of a 5-m vertical electrode, calculated by the FDTD analysis, on the distance between two electrodes

Distance	20 m	30 m	40 m
Resistance	28.8 Ω	29.3 Ω	29.7 Ω

REFERENCES

[1] L. Grcev and F. Dawalibi, "An electromagnetic model for transients in grounding systems," IEEE Trans. Power Delivery, Vol.PWRD-5 (4), pp.1773-1781, 1990.
[2] Y. Liu, M. Zitnik and R. Thottappillil, "A time domain transmission line model of grounding system," Proc. Int. Symp. High Voltage Engineering 2001, pp.154-157, 2001.
[3] K. Tanabe, "Novel method for analyzing dynamic behavior of grounding systems based on the finite-difference time-domain method," IEEE Power Engineering Review, Vol.21 (9), pp.55-57, 2001.
[4] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equation in isotropic media," IEEE Trans. Antennas and Propagation, Vol.AP-14 (4), pp.302-307, 1966.
[5] K. Tanabe, A. Asakawa, T. Noda, M. Sakae, M. Wada and H. Sugimoto, "Verifying the novel method for analyzing transient grounding resistance based on the FD-TD method through comparison with experimental results," CRIEPI Report, no.99043, 2000. (in Japanese)
[6] W. Scott-Meyer, *EMTP Rule Book*, B.P.A., 1977.
[7] T. Noda and S. Yokoyama, "Thin wire representation in finite difference time domain surge simulation," IEEE Trans. Power Delivery, Vol.PWRD-17 (3), pp.840-847, 2002.
[8] G. Mur, "Absorbing boundary conditions for the finite-difference approximation of the time-domain electromagnetic-field equation," IEEE Trans. Electromagnetic Compatibility, Vol.EMC-23 (4), pp.377-382, 1981.
[9] K. R. Umashankar, A. Taflov and B. Beker, "Calculation and experimental validation of induced currents on coupled wires in an arbitrary shaped cavity," IEEE Trans. Antennas and Propagation, Vol.AP-35 (11), pp.1248-1257, 1987.
[10] E. D. Sunde, *Earth Conduction Effects in Transmission Systems*, Dover, New York, 1968.