Finite Difference Method for Lightning Return Stroke Simulation Using the EMTP

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Abstract – **In this work the transmission line equations that represent the lightning return stroke model are resolved via the finite difference time domain (FDTD) method using the leap-frog scheme. The high losses of the lightning channel are considered. The implemented subroutine interfaces with the Electromagnetic Transients Program (EMTP) of which is obtained the boundary conditions of the line. The advantage of the proposed implementation are the line discretization, inherent in the FDTD method can be used to compute electromagnetic fields.**

Keywords – lightning return stroke, EMTP, transmission line, FDTD, leap-frog scheme.

I. Introduction

Lightning [1,2] can be defined as a transient, high-current electric discharge whose path length varying from 2 to 14 km. It takes place when certain regions of the atmosphere attains an electric charge sufficiently large that the electric fields associated with the charge cause electrical breakdown of the air.

A typical discharge between cloud to ground starts in the cloud and eventually neutralizes tens of coulombs of negative cloud charge. The stepped leader initiates the first stroke in a flash by moving from the cloud to ground. As the leader tip comes close to ground, the electric field in the region between the tip and the ground becomes very large and causes one or more upward-moving discharges to be initiated at the ground. When one of the upward-moving discharges contacts the downward-moving leader, some tens meters above the ground, the leader channel is then discharged when a ground potential wave, the return stroke, propagates in the direction of the previously ionized leader path. The upward velocity of the return stroke is typically one-third the speed of the light, and the total transit time from the ground to the top of the channel is typically about 100 μs. The return stroke, at least in its lower portion, produces a peak current of typically 30 kA, with a time from zero to peak of a few microseconds. Current measured at the ground fall to half the peak value in about 50 μs, and current about hundreds of amperes may flow for a few milliseconds or longer.

The return strokes lowers the charge originally deposited on the stepped leader channel to ground and, in so doing, produces an electric-field change with time variations which range from a submicrosecond scale to many milliseconds.

The electromagnetic fields generated by the lightning discharge affect a vast array of electric an electronic systems such as railway, power, communications and airlines industries. These fields sometimes may strongly affect those systems causing among other problems loss of information in the communication system and even destroy electronic components. A very serious damage occurs on the aircraft circuits, in the new carbon composite material structures and may also cause fuel ignition. Elements on the ground are also affected by lightning as electronic systems in buildings, communication towers, rockets on the ground, radar stations and other systems such as electric and telephone networks.

In many aspects the current is the most important single parameter of the lightning discharge. With the knowledge of the wave-shape and amplitude of the current, the electrical problems of protection against lightning can be dealt with. A correct theoretical correlation between lightning current and electromagnetic fields, produced at any range, is a prerequisite for an approximate evaluation of the interaction with affected systems. The model of lightning channel should be capable of providing currents and/or electromagnetic fields incident on the victim structure with typical magnitude and spectral content of a natural lightning.

In this work, the transmission line equations that represent the lightning return stroke model are resolved via the finite difference time domain (FDTD) method and high losses in the line are considered. The EMTP program is used to obtained the boundary conditions of the line and the FDTD method (using CONNEC subroutine) to calculate the current distribution along the line to compute electromagnetic fields. Some time/space interpolation/extrapolation are needed to link the time based equations (EMTP) with space/time numerical discretization of the FDTD method.

The finite differences method is, perhaps, the oldest numerical method used in the solution of partial and ordinary differential equations. The derivatives are substituted by quotients difference involving the values of the solution in discrete points of the domain. The solution of the resultant algebraic equations is obtained from the imposition of the boundary conditions. Using the leap-frog scheme, voltage and current along the line can be obtained and they can be used to calculate the electromagnetic fields and induced voltages in the protection system.

The Electromagnetic Transients Program [3,4] is a comprehensive computer program designed to solve electrical transient problems in lumpy circuits, distributed circuits, or combinations thereof. It is widely used in all the world. Individuals and groups have been adapting, expanding and generally augmenting the techniques, increasing the program's capability.

II. LIGHTNING RETURN STROKE MODEL

A model of the lightning channel [5,6,7] must be adequately built to agree with observed lightning currents and electromagnetic fields wave shapes. Assuming that the lightning channel is a straight vertical uni-dimensional antenna, perpendicular to the conducting ground plane (figure 1), the transmission line approach introduces various simplifications as TEM wave propagation, where the line parameters (*R, G, L* and *C*) are constant.

The current at the channel base i(0,t) can be specified along with other model parameters to allow a determination of the current as a function of height and time along the return stroke channel.

The transmission line parameters that represent the lightning channel are obtained from the simplest channel configuration. The resistance of the channel is inversely proportional to the conductivity and channel radius, assumed to remain constant through out the presence of the return stroke.

The general lossy one-dimensional transmission line equations are

$$-\frac{\partial e(x,t)}{\partial x} = Ri(x,t) + L\frac{\partial i(x,t)}{\partial t},$$
$$-\frac{\partial i(x,t)}{\partial x} = Ge(x,t) + C\frac{\partial e(x,t)}{\partial t}, \qquad (1)$$

where *R, L, C* and *G* are the resistance, inductance, capacitance and conductance per unit length, respectively.

III. FINITE DIFFERENCE METHOD

The solution of (1) can be obtained numerically using the Finite Difference Time Domain method. This will require the transmission line equations to be discretized both in time and space. The discretization can be provided by the FDTD method [8].

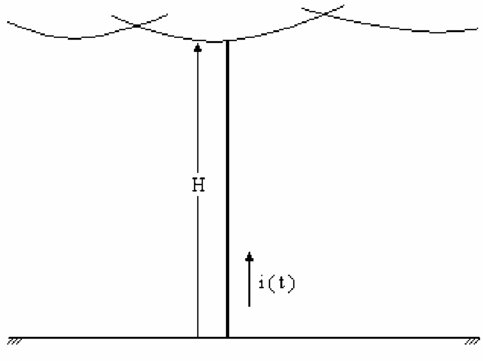

Fig. 1 Lightning return stroke.

The line is divided into *N* sections each of length Δx as shown in figure 2. Similarly, the total solution time is divided into n segments of length Δt. In order to insure stability of the discretization and to insure second-order accuracy the *N+1* voltage and current points are interlaced. Each voltage and adjacent current solution point is separated by $\Delta x/2$. In addition, the times points must also be interlaced and each voltage time point and adjacent current time point are separated by $\Delta t/2$ that imply in the utilization of the leap-frog scheme to calculate the current distribution along the line. The equations (1) approached by first-order central differences with second-order accuracy result in

$$I_j^{n+3/2} = Z_1 I_j^{n+1/2} - Z_2\left(V_{j+1}^{n+1} - V_j^{n+1}\right),$$
$$V_j^{n+1} = Z_3 V_j^{n} - Z_4\left(I_j^{n+1/2} - I_{j-1}^{n+1/2}\right), \qquad (2)$$

where

$$Z_1 = (2L - R\Delta t)/(2L + R\Delta t),$$
$$Z_2 = (2\Delta t)/\Delta x(2L + R\Delta t),$$
$$Z_3 = (2C - G\Delta t)/(2C + G\Delta t),$$
$$Z_4 = (2\Delta t)/\Delta x(2C + G\Delta t).$$

On figure 2, V_1 and I_{N+1} are boundary conditions; V_1 is an ideal voltage source and I_{N+1} is the receiving end current. The solution to the FDTD method represented by the equation (2) is stable [9,10] if

$$v \le \frac{\Delta x}{\Delta t},$$

where v is the wave propagation speed. This implies that the wave must not propagate more than one subdivision in space during one time step. To obtain the exact solution put $\Delta x = v\Delta t$.

IV. COMPUTATIONAL PROCEDURE

Transient analyses can be carried out in circuits with any arbitrary configuration of lumped parameters (*R, L,* and *C*), using the trapezoidal integration rule. Transmission lines, with distributed parameters, transposed or untransposed, using D'Lambert equation solution, can be included in the network. Frequency-dependence of line parameters can be included. Nonlinear resistors (for surge arresters) and nonlinear inductors (for saturable devices) can be represented. It is also possible to open and close switches to simulate breaker operations, flashovers, etc.

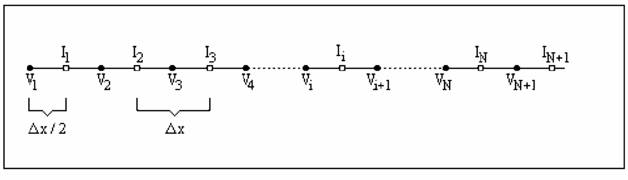

Fig. 2 Finite difference time domain method.

Some versions of the Electromagnetic Transients Program EMTP [4] provide an interface that incorporates user-supplied models (nonlinear branch) using the compensation method. In this formulation, the external network is solved independently and reduced to a Thévenin equivalent (if multiphase equivalent, all scalars turn into matrix or vectors, accordingly).

$$e_{km}(t) = e^{0}_{km}(t) - Z_{thev} i_{km}(t),$$

where $e_{km}(t)$ and $i_{km}(t)$ are the voltage across and the current flowing into the nonlinear branch, e^{0}_{km} is the open circuit voltage and Z_{thev} is the Thévenin impedance of the network as seen from the nonlinear branch. The nonlinear branch connected at nodes k and m is expressed by

$$e_{km}(t) = f\left(i_{km}(t)\right),$$

where $f(i_{km}(t))$ is a function of current that depends on the particular user-supplied model.

Using this representation to include the lightning return stroke model in the EMTP, the usual EMTP line equations are replaced by the line equations (2) and they are represented inside CONNEC, where the boundary conditions are now obtained from the Thévenin equivalent from nodes k and m, shown in figure 3. From this figure, we obtain

$$V_1^{n+1} = V_k^{th} - Z_{th1} i_k, \qquad V_1^{n+1} = Z_1 V_1^{n} - Z_2\left(I_1^{n+1/2} - I_0^{n+1/2}\right) \tag{3}$$

and

$$V_{N+1}^{n+1} = V_m^{th} - Z_{th2} i_m, \qquad V_{N+1}^{n+1} = Z_1 V_{N+1}^{n} - Z_2\left(I_{N+1}^{n+1/2} - I_N^{n+1/2}\right) \tag{4}$$

where V_K^{th} and V_m^{th} are the thévenin equivalent source. The space and time shift between voltage and current in FDTD approach led to an approximation to solve equation (3) and equation (4). Since

$$i_k + I_1^{n+1/2} \neq 0 \qquad \text{and} \qquad I_N^{n+1/2} - i_m \neq 0,$$

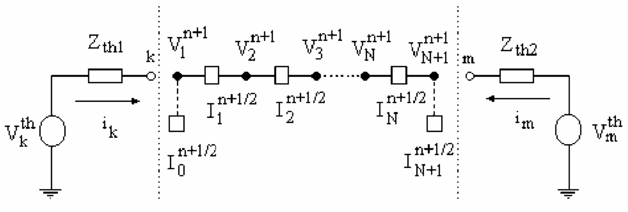

Fig. 3 Boundary conditions using CONNEC.

the nodal currents i_k and i_m can be extrapolated in time and interpolated in space [11] to give the new boundary conditions

$$I_0^{n+1/2} = \frac{-V_1^{n} + Z_{eq1} I_1^{n+1/2} + hist_1 + V_k^{th}}{\frac{3}{4} Z_4 Z_{th1}}, \tag{5}$$

$$I_{N+1}^{n+1/2} = \frac{V_{N+1}^{n} + Z_{eq2} I_N^{n+1/2} + hist_2 - V_m^{t}}{\frac{3}{4} Z_4 Z_{th2}}, \tag{6}$$

where

$$Z_3 = 1, \qquad Z_4 = \frac{\Delta t}{C \Delta x},$$

$$Z_{eq1} = Z_4 - \frac{3}{4} Z_{th1}, \qquad Z_{eq2} = Z_4 - \frac{3}{4} Z_{th2},$$

$$hist_1 = \frac{Z_{th1}}{4}\left(I_0^{n-1/2} + I_1^{n-1/2}\right), \qquad hist_2 = \frac{Z_{th2}}{4}\left(I_N^{n-1/2} + I_{N+1}^{n-1/2}\right).$$

Equations (2) are used to calculate the transmission line voltage and current that represent the lightning return stroke with boundary conditions given by equations (5) and (6). Using the terminal voltages, the equivalent current sources

$$I_1 = \frac{V_k^{th} - V_1^{n+1}}{Z_{th1}} \tag{7}$$

and

$$I_2 = \frac{V_m^{th} - V_{N+1}^{n+1}}{Z_{th2}} \tag{8}$$

are used to compute the correct response of the line which interface with the external linear network represented in the EMTP.

V. Case Studies

To evaluate the performance of the proposed routine, the lightning return stroke was simulated using the transmission line return stroke model. The used line parameters in the simulations (L=20 μH/m, C=2,96 pF/m and G=0 S/m [12]) resulting in a constant propagation velocity of 130 m/μs and a surge impedance of 2600 Ω. They were considered constant, although of the capacitance and inductance vary with the height. The height of the channel was considered 4 km and the current wave form was monitored at P1, P2 and P3, respectively, 0, 1 and 2 km height. For the lightning channel discretization in time and space were considered, respectively, Δt=0,1 μs and Δx=13 m. For the channel losses were used R=1 and 10 Ω/m. The typical double exponential (1,2x50 μs) function was used as current source at the ground level:

$$i(t) = I_0 (e^{-\alpha t} - e^{-\beta t}),$$

where $I_0 = 1{,}0167\,pu$, $\alpha = 0{,}01423 s^{-1}$ and $\beta = 6{,}0691 s^{-1}$. In this work , the boundary condition "open line'' was chose to represent the cloud, but other boundary conditions can be used as well as G different of zero.

Figure 4 present the current waveforms for a lossless lightning channel. For R=0 Ω/m neither distortion nor attenuation are observed. For this reason some reflection at the end of the channel occurs. For R=1 Ω/m (figure 5) some attenuation and distortion are present. For R=10 Ω/m (figure 6) bigger attenuation is present. This results in a complete attenuation of the current waveform at the top of the channel. As the boundary condition of the channel top doesn't influence in the current waveform the same results can be obtained using the MTL model.

VI. Conclusions

This paper presented a new procedure to simulate the lightning return stroke using the FDTD method and the EMTP program. A new routine was written to interface with the EMTP and validation tests were made with good results.

The lightning return stroke model represented by a transmission line whose equations are resolved via FDTD method are discretized in the space what permits to obtain the current and the voltage along the channel. These currents and voltages can be used to calculate electromagnetic fields and induced voltages.

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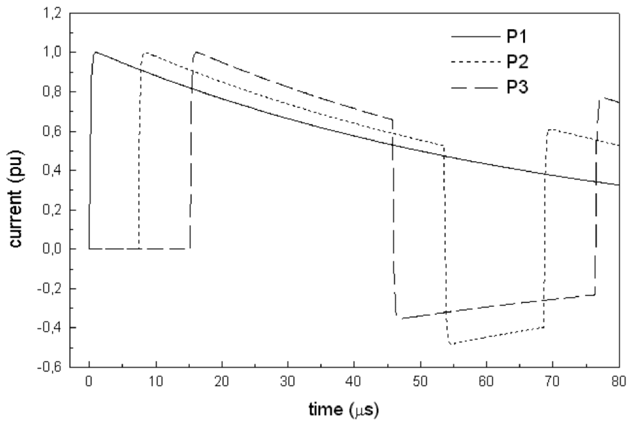

Fig. 4 Lossless lightning channel.

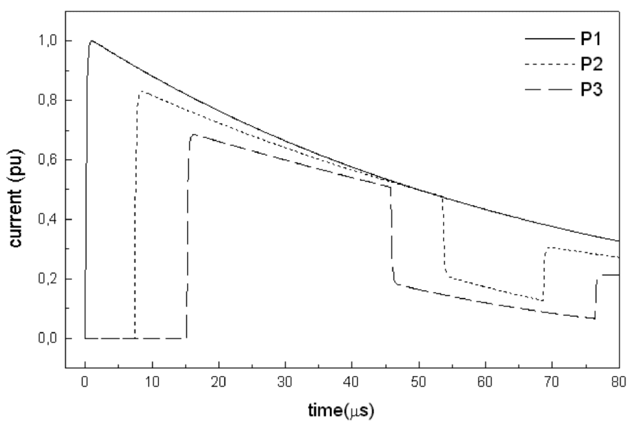

Fig. 5 Lightning channel with 1 Ω/m of the losses.

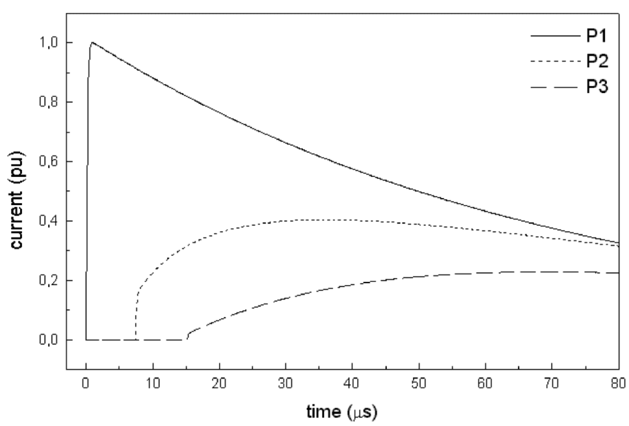

Fig. 6 Lightning channel with 10 Ω/m of the losses.