A Comparison of Lightning Induced Voltages on Single and Double Circuit Power Distribution Lines

P. Durai Kannu[1] and M. Joy Thomas [2]
(1)Dept. of Electrical Engineering, National Institute of Technology Karnataka, Mangalore-575 025, India. (2) Dept. of High Voltage Engineering, Indian Institute of Science, Bangalore-560012, India (thoma@hve.iisc.ernet.in).

Abstract— **The induced voltages due to a nearby lightning stroke on the conductors of 3ϕ single and double circuit power distribution lines have been computed. The influence of finite ground conductivity on the induced voltages has been taken into account in the computation. From the results, it is seen that the induced voltage depends on the conductivity of the ground and the number of conductors in the circuit. It is seen that the induced voltage reduces with ground conductivity at line termination where as it increases at line mid point. It is also observed that the induced voltage on a conductor gets reduced if the number of conductors in a power distribution line circuit are more. This reduction is essentially due to the shielding influence of other conductors which are in the circuit.**

Keywords—**Lightning, induced voltage, overhead line, ground conductivity.**

I Introduction

Induced over voltages due to a nearby lightning stroke is one of the main causes of medium voltage power distribution line outages. Hence it is important to estimate the induced voltages on power distribution lines due to a lightning stroke so that suitable protective schemes could be implemented at the time of commissioning the line. For the theoretical estimation of the induced voltages, it is necessary to have an understanding of the influence of various parameters of the line as well as the ground on the induced voltage.

The aim of this paper is to (i) compare the lightning induced voltages on single and double circuit power distribution lines and (ii) to study the effect of ground conductivity on the magnitude and waveshape of the induced voltage on the above overhead power distribution lines. The ground conductivity influences both the radiated electric field and the surge propagation in the line[1-4]. This paper gives the results of the induced voltage due to a nearby lightning for 33 kV, 3ϕ single circuit and double circuit distribution lines. The influence of the presence of other conductors on the induced voltage in a multiconductor system is also studied.

II Theoretical Modeling

The main steps in computing the lightning induced voltage are
1. Modeling the lightning return stroke.
2. Estimating the fields generated by the lightning including the distortions introduced by the finitely conducting ground.
3. Modeling the field to overhead conductor coupling including the influence of ground impedance.

A. Lightning Return Stroke Model

The model adopted for lightning return stroke is the modified transmission line (MTL) model. In this model it is assumed that the current magnitude decreases exponentially with height as it travels up the lightning channel at a constant velocity v. Mathematically this current $i(z',t)$ at height z' and at time t is represented by

$$i(z',t) = e^{(-z'/\lambda)} i(0,t-z'/v) \qquad (1)$$

where λ is the decay constant to account for the effect of the vertical distribution of charge stored in the corona sheath of the leader and subsequent discharge during the return stroke phase.

$i(0,t)$ is the lightning return stroke current at ground level (channel base current). In this paper the induced voltages are computed for a channel base current of 12 kA peak and maximum time derivative of 40 kA/μs. The return stroke velocity v is taken as 1.3×10^8 m/s [5].

B. Electric Fields generated by the Return Stroke

By assuming that the ground is a perfect conductor, the vertical component of electrical field $dE_z(r,z,t)$ and horizontal component of electric field $dE_r(r,z,t)$ due to an infinitesimal lightning channel of length, dz', at height z', carrying current $i(z',t)$ is calculated at an observation point $P(r,\phi,z)$ by the following equations [1,5,6]. These equations representing the MTL model for lightning return stroke in time domain are

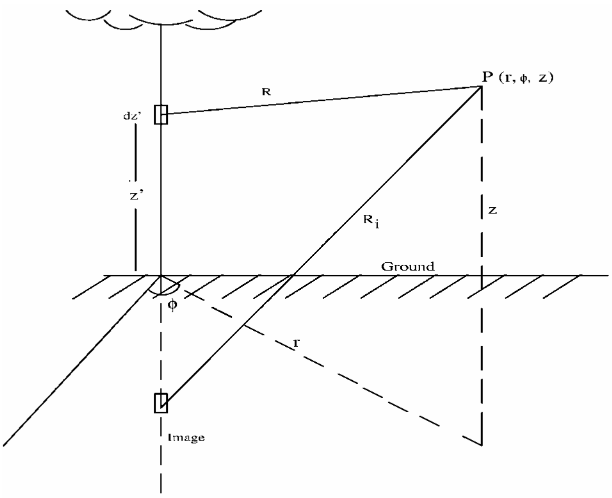

Fig. 1. Sketch showing the lightning channel and its image used in calculating the return stroke fields.

$$dE_z(r,z,t) = \frac{dz'}{4\pi\epsilon_0}$$
$$[\frac{2(z-z')^2 - r^2}{R^5} e^{(-z'/\lambda)}$$
$$\times \int_0^t i(0,\tau - z'/v - R/c)d\tau$$
$$+\frac{2(z-z')^2 - r^2}{cR^4} e^{(-z'/\lambda)} i(0,t - z'/v - R/c)$$
$$-\frac{r^2}{c^2R^3} e^{(-z'/\lambda)} \frac{\partial}{\partial t} i(0,t - z'/v - R/c)] \quad (2)$$

$$dE_r(r,z,t) = \frac{dz'}{4\pi\epsilon_0}$$
$$[\frac{3r(z-z')}{R^5} e^{(-z'/\lambda)} \int_0^t i(0,\tau - z'/v - R/c)d\tau$$
$$+\frac{3r(z-z')}{cR^4} e^{(-z'/\lambda)} i(0,t - z'/v - R/c)$$
$$+\frac{r(z-z')}{c^2R^3} e^{(-z'/\lambda)} \frac{\partial}{\partial t} i(0,t - z'/v - R/c)] \quad (3)$$

where ϵ_0 is the permittivity of free space and c is the velocity of light. $R = \sqrt{r^2 + (z-z')^2}$ is the distance from the current element to the observation point. The effects of the perfectly conducting ground plane on the electromagnetic fields generated by the current element are included by replacing the ground by an image current at a distance z' below the ground. To find the total field, the equations (2) and (3) are integrated along the channel and its image. A sketch showing the lightning channel and its image used in calculating the return stroke fields is shown in figure 1.

C. Effect of Ground on Horizontal Electric Field

The horizontal component of electric field produced by the lightning is significantly influenced by the finite conductivity of the ground. The horizontal electric field including the ground conductivity can be computed using the Cooray-Rubinstein formula [7]. This formula requires the azimuthal component of the magnetic field produced by the lightning which is given as [1,6].

$$dH_\phi(r,z,t) =$$
$$\frac{dz'}{4\pi}[\frac{r}{cR^2} e^{(-z'/\lambda)} \frac{\partial}{\partial t} i(0,t - z'/v - R/c)$$
$$+\frac{r}{R^3} e^{(-z'/\lambda)} i(0,t - z'/v - R/c)] \quad (4)$$

where μ_0 and σ_g are the permeability of free space and the conductivity of the ground respectively.
The horizontal electric field including the ground conductivity $E_{rg}(r,z,t)$ is given as[7]

$$E_{rg}(z=h,r) = E_r(z=h,r) - H_\phi(z=0,r)\frac{\sqrt{\mu_0}}{\sqrt{\epsilon + \sigma_g/j\omega}} \quad (5)$$

where $E_r(z=h,r)$ is the Fourier-transform of the horizontal electric field at height h, $H_\phi(z=0,r)$ is the Fourier-transform of the azimuthal component of the magnetic field at ground level. ϵ is the permittivity of the ground. Both $E_r(z=h,r)$ and $H_\phi(z=0,r)$ are calculated assuming a perfect conducting ground using equations (3) and (4).

D. Field-to-Transmission Line Coupling Model

For the field-to-transmission line coupling, a set of time domain differential equations describing the voltages induced as proposed by Agrawal et.al.[8] have been used. The surge propagation is affected by the ground impedance. This is represented in the overhead line parameters as i) per-unit series ground impedance and ii) shunt ground admittance. Both the above quantities are frequency dependent and generally the latter term is neglected. The frequency dependent series ground impedance is represented by a convo-

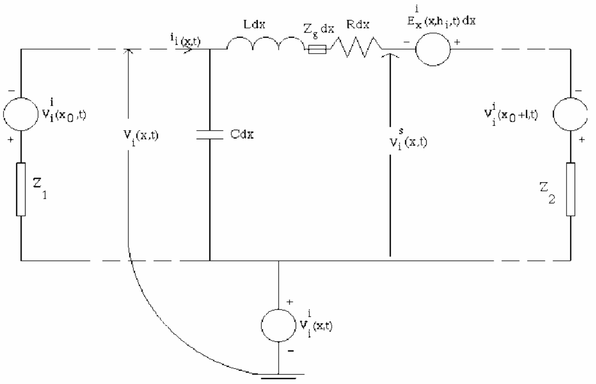

Fig. 2. Equivalent circuit of a single-wire overhead line excited by lightning return-stroke electromagnetic field.

lution integral in the time-domain coupling equations which are given as

$$\frac{\partial}{\partial x}[v_i^s(x,t)] + [L_{ij}]\frac{\partial}{\partial t}[i_i(x,t)] + \int_0^t [\xi_{ij}(t-\tau)]\frac{\partial}{\partial \tau}[i_i(x,\tau)]d\tau = [E_x^i(x,h_i,t)] \tag{6}$$

$$\frac{\partial}{\partial x}[i_i(x,t)] + [G_{ij}][v_i^s(x,t)] + [C_{ij}]\frac{\partial}{\partial t}[v_i^s(x,t)] = 0 \tag{7}$$

where $[E_x^i(x,h_i,t)]$ is the horizontal component of the incident electric field along the conductor at conductor height h_i. The sub index i denotes the particular wire of the multiconductor line. $[L_{ij}], [G_{ij}]$ and $[C_{ij}]$ are the inductance, conductance and capacitance matrices per unit length of the line respectively. $[i_i]$ is the line current vector. $[\xi_{ij}]$ is the transient ground resistance matrix and is equal to the inverse Fourier-transform of $[Z_{g_{ij}}/j\omega]$, ie., $\xi_{ij}(t) = F^{-1}[\frac{Z_{g_{ij}}}{j\omega}]$ and $[Z_{g_{ij}}]$ is the ground impedance matrix.

$[v_i^s]$ is the scattered voltage vector on the i^{th} conductor. This is the voltage due to the field produced by the induced currents on the conductors. The scattered voltage is related to the total line voltage $v_i(x,t)$ by the following relation.

$$[v_i(x,t)] = [v_i^s(x,t)] + [v_i^i(x,t)] \tag{8}$$

where $[v_i^i(x,t)]$ is the incident voltage.

$$[v_i^i(x,t)] = -\int_0^{h_i} E_z^i(x,z,t)dz \tag{9}$$

$E_z^i(x,z,t)$ is the incident(or inducing) vertical electric field at x at a height of z.

The voltage at the end of the line is determined by the boundary condition and the current at the two ends of the line, viz., $i(x_0,t)$ and $i(x_0+l,t)$, where l is the length of the overhead line. The boundary conditions for the scattered voltage are

$$v_i^s(x_0,t) = -[Z_1][i_i(x_0,t)] + \int_0^{h_i} E_z^i(x_0,z,t)dz \tag{10}$$

$$v_i^s(x_0+l,t) = [Z_2][i_i(x_0+l,t)] + \int_0^{h_i} E_z^i(x_0+l,z,t)dz \tag{11}$$

where $[Z_1]$ and $[Z_2]$ are the terminating impedance matrices. Figure 2 shows the equivalent circuit of this model for a single conductor.

E. Transient Ground Resistance

In case of multiconductor transmission lines the ground impedance is a full matrix. The low frequency approximate (ie., assuming $\sigma_g >> \omega\epsilon_0\epsilon_r$) expression for the ground impedance between two conductors i and j is given as [9].

$$Z'_{g_{ij}} = \frac{j\omega\mu_0}{\pi}\int_0^\infty \frac{e^{-(h_i+h_j)x}cos(r_{ij}x)}{\sqrt{x^2+j\omega\mu_0\sigma_g}+x}dx \tag{12}$$

where r_{ij} is the distance between the two conductors in the horizontal plane. The low frequency approximation of the inverse Fourier transform of the ground impedance is given by [9].

$$\xi_{ij}(t) = \frac{\mu_0}{\pi T_{ij}}[\frac{1}{2\sqrt{\pi}}\sqrt{\frac{T_{ij}}{t}}cos(\theta_{ij}/2)$$
$$+\frac{1}{4}e^{T_{ij}cos(\theta_{ij})/t}cos(\frac{T_{ij}}{t}sin(\theta_{ij})-\theta_{ij})$$
$$-\frac{1}{2\sqrt{\pi}}\sum_{n=0}^{\infty}a_n(\frac{T_{ij}}{t})^{\frac{2n+1}{2}}cos(\frac{2n-1}{2}\theta_{ij})$$
$$-\frac{cos(\theta_{ij})}{4}] \tag{13}$$

where $a_n = \frac{2^n}{1.3....(2n+1)}$
with $[\frac{h_i+h_j}{2} + j\frac{r_{ij}}{2}]^2\mu_0\sigma_g = T_{ij}e^{j\theta_{ij}}$

In (13), if $h_i = h_j$ then r_{ij}=0 and θ_{ij}=0 we get the expression for diagonal terms of $[\xi_{ij}(t)]$. In the above expression, $\xi_{ij}(t)$ tends to infinity as $t \to 0$. Loyka[10] has shown that $\xi_{ij}(t)$ will tend to be a constant if it is calculated without low frequency approximation. In the present work, the incremental time Δt is selected such that $\xi_{ij}(t)$ is calculated using the equation (13). In solving the convolution integral the ground transient resistance is taken as constant in the range $t - \Delta t < \tau < t$. Hence for the calculation of line current at n^{th} incremental time, $t = n\Delta t$, the numerical expression for the convolution integral [11] at any node is obtained as follows,

$$\begin{aligned} & \int_0^{n\Delta t} \xi(t-\tau)\frac{\partial i}{\partial \tau} d\tau \\ = & \sum_{m=0}^{m=n-2} \xi^{m+1}(t)(i^{n-m} - i^{n-m-1}) \\ & +\xi^1(t)\frac{(i^n - i^{n-1})}{2} \\ & +\xi^n(t)\frac{(i^1 - i^0)}{2} \end{aligned} \tag{14}$$

with $\xi^k(t)$ and i^k representing the value of the transient ground resistance for the elements in $[\xi_{ij}(t)]$ and the line current respectively at time $t = k\Delta t$.

III Results and Discussions

The lightning induced voltages are calculated for 33 kV, 3ϕ, single circuit and double circuit overhead power distribution lines. The configuration of the lines are shown in figure 3. The length of the transmission line is taken as 1 km. The line conductors are terminated at both ends by their characteristic impedances and the earth wire is terminated by the surge impedance of the tower[12]. The induced voltages are calculated at the line ends as well as at the mid point of the line for a lightning stroke located at a distance of 50 m from the line center and perpendicular to the line for (1) a perfectly conducting ground and (2) with ground conductivity σ_g of 0.001 S/m and relative permittivity ϵ_r of 10.

Figure 4 shows the induced voltages on conductor 1 of single circuit vertical configuration at line termination as well as at line mid point. From the figure it is seen that the finite ground conductivity reduces the peak of the induced voltage at line termination whereas it increases its peak at line mid point. The induced voltage goes negative initially at the line termination.

The magnitude and waveshape of the induced voltages for such a long line which is few meters above the ground mainly depends on the horizontal electric field along the conductor. Since the horizontal electric field has a negative peak initially due to the finite conductivity of the

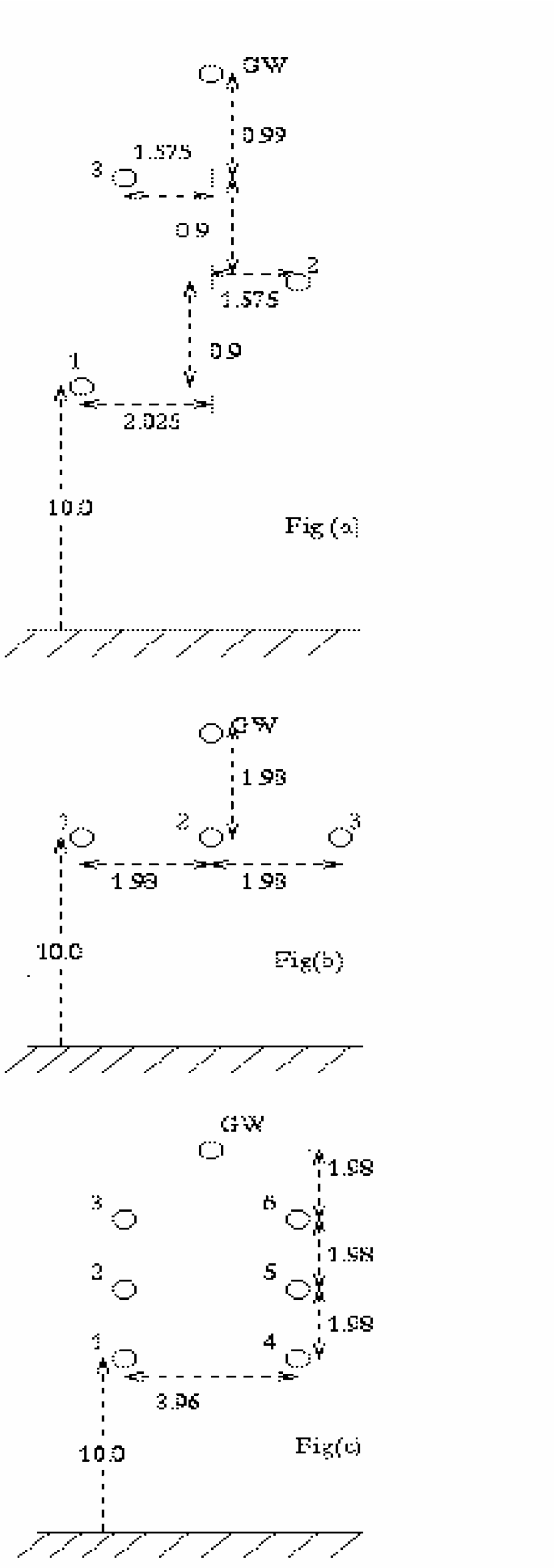

Fig. 3. Conductor configurations used.(a) Single circuit vertical,(b) Single circuit horizontal and (c) Double circuit. GW-ground wire. All dimensions are in meters.

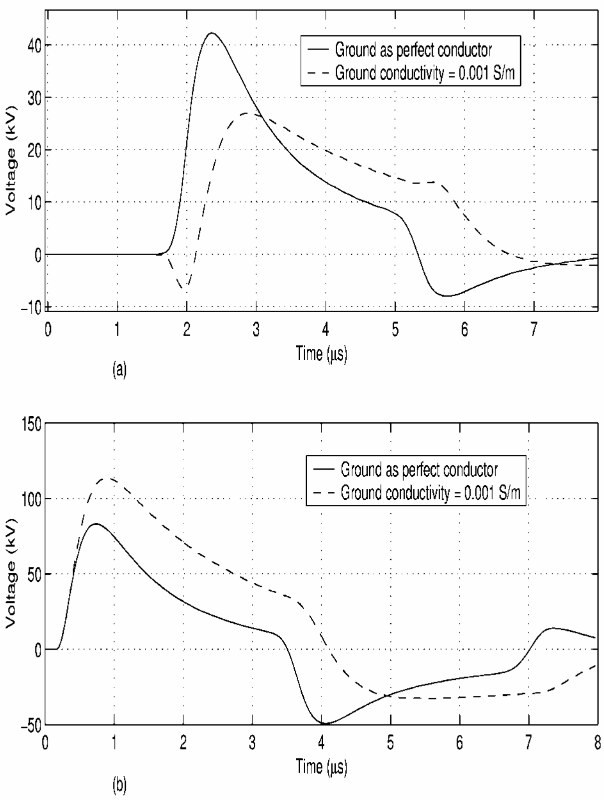

Fig. 4. Induced voltage on conductor 1 of single circuit vertical configuration (a) at line termination and (b) at line mid point.

ground (see figure 5), the voltage induced also posesses initial negative polarity at the line termination whereas at the mid point of the line, the horizontal electric field along the direction of the line is zero for the above mentioned stroke location and hence there is no initial negative peak in the induced voltage at the line mid point.

For a lightning return stroke current of 12 kA at ground level, the plots(figures 6, 7 and 8) show that the positive peak of the induced voltage at line terminations for conductor 1 is about 1) 27 kV for single circuit vertical configuration 2) 28 kV for single circuit horizontal configuration and 3) 20 kV for double circuit configuration. At the line mid point, the induced voltages for the above three configurations for conductor 1 are approximately the same($\simeq$ 113 kV).

The induced voltage at line termination is also computed for a single conductor of length 1 km at a height of 10 m above the ground. The figure 9 shows the induced voltage on conductor 1, which is at the same height and same distance from strike point for a single conductor, single circuit vertical and double circuit configurations. It is seen that the induced voltage at line termination is less for the double circuit configuration case (which is about 20 kV as against 27 kV and 40 kV for the single circuit vertical configuration and single conductor respectively). This is because when the number of conductors are more, there is a mutual shielding effect coming into effect there by reducing the induced voltage. The waveshape of the induced voltage on a conductor in multiconductor system is also affected by the presence of the other conductors.

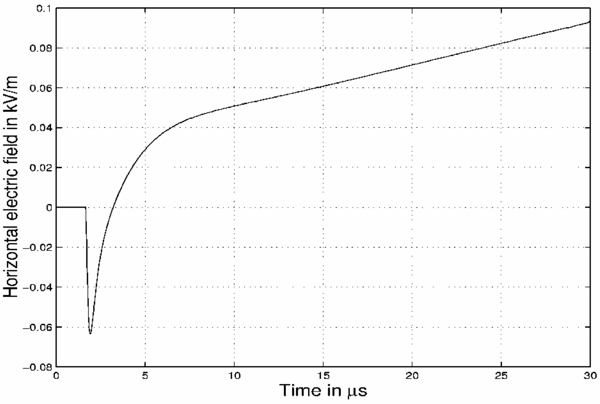

Fig. 5. Horizontal electric field at a height 10 m and distance of 500 m from the lightning stroke location for the ground parameters σ_g=0.001 S/m and ϵ_r=10.

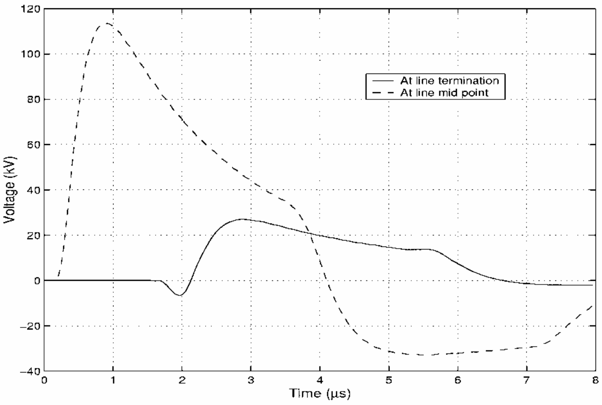

Fig. 6. Induced voltage on conductor 1 of the single circuit vertical configuration.

The lightning electric field induces a current in all the conductors including ground wire and phase conductors of the overhead line which intern produces a magnetic field which couples with all the other conductors. This mutual coupling between the conductors decreases the induced voltage which is very clearly seen at the line termination. However, at line mid point, and for the lightning location chosen in this study, the influence of mutual coupling on the induced voltages gets nullified as the contributions from either

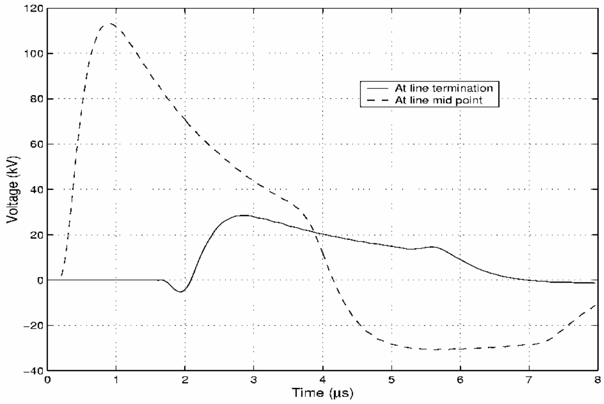

Fig. 7. Induced voltage on conductor 1 of the single circuit horizontal configuration.

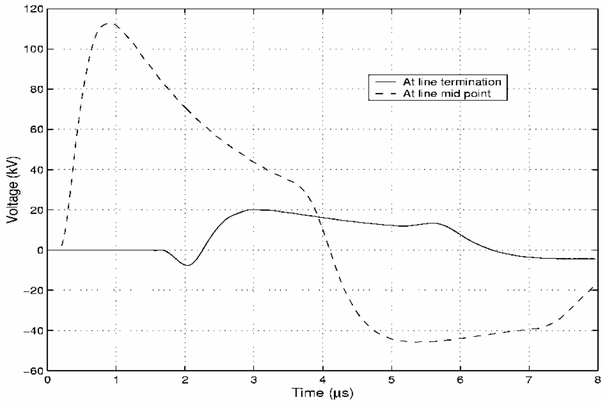

Fig. 8. Induced voltage on conductor 1 of the double circuit configuration.

side of the line mid point cancel each other. Hence at line mid point, the induced voltage remains almost constant for different configurations of the power distribution line studied.

IV Conclusions

Induced voltage due to a nearby lightning on 33 kV, 3ϕ single circuit and double circuit power distribution lines have been calculated. The effect of ground conductivity is included in the computation of field as well as in the coupling of field to the overhead conductors. When finite ground conductivity is taken into account, the first peak of the induced voltage is reduced at the line termination where as it increases at conductor mid point. The induced voltage at the line termination is less for the double circuit line than that of single circuit line. The presence of large number of conductors reduces the first peak at the line termination appreciably.

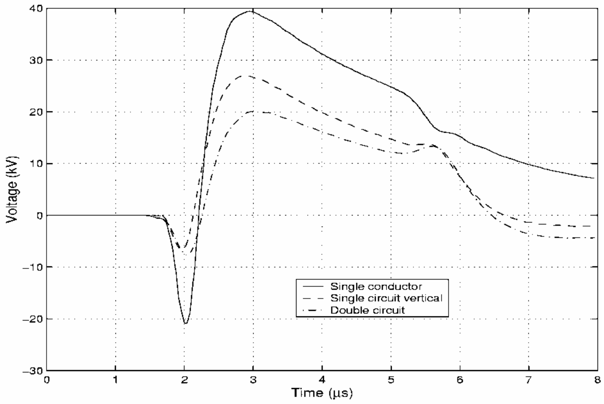

Fig. 9. Induced voltage on conductor 1 for single and multi conductor lines.

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