An Algorithm for Calculations of Low Frequency Transformer Transients

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***Abstract* – An algorithm for the calculations of low frequency transformer transients such as inrush current and ferroresonance is developed in this paper. The transformer nonlinearity is represented by nonlinear magnetizing inductance in parallel with nonlinear core loss resistance. Nonlinear curves: magnetizing current – flux linkage and core loss current – supply voltage are piecewise linearized. The stiff differential equation system, which describes transients of electrical circuit, is solved by the *A* and *L*-stable backward differentiation formulas numerical method. It is shown that the *BDF* method completely eliminates numerical oscillation events. Simulation results of the developed algorithm are compared with the results obtained by Matlab/Power System Blockset and also with field measurements during a transformer energization. The proposed algorithm could be successfully applied on numerical calculations of transients with some other nonlinear elements such as surge arresters, power electronic elements, etc.**

Keywords – transformer, stiff differential equations, inrush current, trapezoidal rule, backward differentiation formulas and stability of numerical methods

I. INTRODUCTION

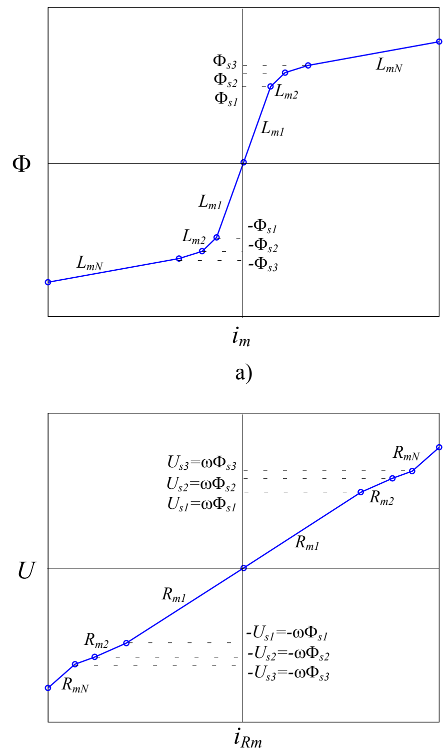
Transformer nonlinearity is represented by nonlinear magnetizing inductance in parallel with nonlinear core loss resistance [1], [2]. This model is reasonably good for low-frequency transformer transients such as inrush current and ferroresonance [1], [3], [4]. It is also used in harmonic loadflow calculations [5]. Nonlinear curves: magnetizing current – flux linkage, fig. 1.a, and core loss current – supply voltage, fig. 1.b, are piecewise linearized. Slopes of some linear regions define inductance and resistance series $L_{m1}, L_{m2}, \ldots, L_{mN}$ and R_{m1}, $R_{m2}, \ldots, R_{mN}$. These curves are obtained by standard no-load transformer tests [1], [2]. During transients, these inductances and resistances are being switched on/off, depending on absolute value of the main magnetic flux linkage, fig. 2.

The magnetizing current i_{mk} and core loss current i_{Rmk} of the *k*-th linear region, are calculated by equation [6]:

$$i_{mk} = \frac{1}{L_{mk}}\Phi + \mathrm{sgn}(\Phi)\sum_{i=1}^{k-1}\Phi_{si}\left(\frac{1}{L_{mi}} - \frac{1}{L_{mi+1}}\right) \qquad (1)$$

$$i_{Rmk} = \frac{1}{R_{mk}}\frac{d\Phi}{dt} + \omega\,\mathrm{sgn}(\Phi)\sum_{i=1}^{k-1}\Phi_{si}\left(\frac{1}{R_{mi}} - \frac{1}{R_{mi+1}}\right) \qquad (2)$$

$k = 1,2,\ldots,N$, *N*-total number of piecewise regions.



b)

Fig. 1. a) Nonlinear curve of core inductance
b) Nonlinear curve of core loss resistance

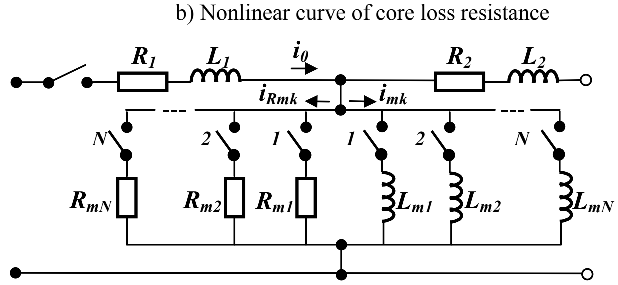


Figure 2. Equivalent transformer model

II. TRANSFORMER ENERGIZATION

Fig. 3. shows the simplified model during transformer's energization. The network is represented by the ideal voltage source $e(t) = E_m\cos\omega t$, with the corresponding network impedance $\bar{z} = R + j\omega L$. Capacitor C represents a power line, cable, shunt filter capacitance, etc. At the moment $t = T_0$, transformer energization occurs.

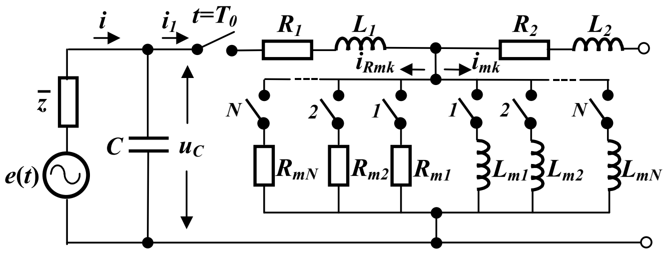


Figure 3. Transformer energization - equivalent model

When the magnetic flux during transient exceeds the first critical value Φ_{s1} (which corresponds to a transformer voltage that exceeded the value of $U_{s1} = \omega\Phi_{s1}$), the inductance L_{m1} and the resistance R_{m1} should be connected and the inductance L_{m2} and the resistance R_{m2} disconnected. If the magnetic flux exceeds the second critical value Φ_{s2}, the inductance L_{m2} and the resistance R_{m2} will be switched off and the inductance L_{m3} and the resistance R_{m3} will be switched on, etc. The operating point will move throughout the regions defined by L_{mk} and R_{mk}, $k = 1,2,...,N$, moving up or down depending on the absolute value of the instantaneous magnetic flux and that magnetic flux actually determines the criteria for the movement within the region. The whole sequence of movements from one region to another occurs at the same moment when the absolute values of the magnetic flux (voltage) take the values defined by the orders Φ_{s1}, Φ_{s2},...,Φ_{sN} and U_{s1}, U_{s2},...,U_{sN}. This gives an idea of organizing the algorithm, into which it introduces the indicator of direction that will continuously determine the position of the operating point. Based on relations (1)-(2), behavior of the circuit in the fig. 3. is described by equation in the state space form on the k-th linear region:

$$dX / dt = A_k X + b_k,\ k = 1,2,...,N \tag{3}$$

State vector, state matrix and free state vector are as follows: $X = [i \quad u_C \quad i_1 \quad \Phi]^T$,

$$A_k = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} & 0 & 0 \\ \frac{1}{C} & 0 & -\frac{1}{C} & 0 \\ 0 & \frac{1}{L_1} & -\left(\frac{R_1}{L_1}+\frac{R_{mk}}{L_1}\right) & \frac{R_{mk}}{L_1 L_{mk}} \\ 0 & 0 & R_{mk} & -\frac{R_{mk}}{L_{mk}} \end{bmatrix},$$

$$b_k = \begin{bmatrix} \frac{1}{L}E_m\cos\omega t \\ 0 \\ \frac{R_{mk}}{L_1}\mathrm{sign}(\Phi)\left(\sum_{i=1}^{k-1}\Phi_{si}\left(\frac{1}{L_{mi}}-\frac{1}{L_{mi+1}}\right)+\omega\sum_{i=1}^{k-1}\Phi_{si}\left(\frac{1}{R_{mi}}-\frac{1}{R_{mi+1}}\right)\right) \\ -R_{mk}\mathrm{sign}(\Phi)\left(\sum_{i=1}^{k-1}\Phi_{si}\left(\frac{1}{L_{mi}}-\frac{1}{L_{mi+1}}\right)+\omega\sum_{i=1}^{k-1}\Phi_{si}\left(\frac{1}{R_{mi}}-\frac{1}{R_{mi+1}}\right)\right) \end{bmatrix}$$

For the real parameters of the electric circuits, which include the transformer model, equation (3) represents "*stiff*" differential equations. Eigenvalues of state matrixes A_k, have a ratio of $\left|\lambda_{min}(A_k) / \lambda_{max}(A_k)\right|_{k=1,2,...,N} << 1$. The rigidity of differential equations makes classical explicit numerical rules very hard (Euler, Runge-Kutta, Adams-Moulton etc.) to solve the same equations successfully [7-8]. Explicit rules, applied to "*stiff*" equations, are numerically unstable, what implies an increase of truncation error in each iteration and leads to a method divergence. Numerical rules that successfully solve "*stiff*" differential equations (3) has to be *A*-stable, [7-9]. One of the most commonly used rules is the implicit trapezoidal rule, which is applied in the EMTP software [10]. *A*-stable trapezoidal rule has the drawback of producing slowly damped oscillations ("numerical oscillations") when applied to problems with large negative eigenvalues [10-11]. These problems can occur in transformer energization simulation, [12]. There are different ways for suppression of numerical oscillations that use the special numerical procedure known as CDA [13]. Another solution consists of adding additional damping elements in the circuit [10].
To completely avoid numerical oscillations with the *A*-stability, applied numerical rule has to be *L*-stable [14-15], i.e. following relation has to be fulfilled:

$$\lim_{z\to\infty} R(z) = 0 \tag{4}$$

where $R(z)$ is a stability function of the applied numerical rule. The trapezoidal rule is not *L*-stable, [9]. In this paper is proposed the use of backward differentiation formulas *BDF* that fulfill L-stability. *BDFp* of the *p*-th order is rule applied to (3):

$$\sum_{m=1}^{p}\frac{1}{m}\nabla^m X_{n+1} = h\left(A_k X_{n+1} + b_k\right) \tag{5}$$

$$k = 1,2,...,N,\ n = 1,2,...$$

BDF1 is an implicit Euler method. *BDF2* is both *A*-stable and *L*-stable. Two-step rule is represented by following relation obtained by (5):

$$X_{n+1} = \left[3E - 2hA_k\right]^{-1}\left(4X_n - X_{n-1} + 2hb_k\right) \tag{6}$$

BDFp for $p \geq 3$ are $A(\alpha_p)$ stable with stability angles $\alpha_3 = 86^\circ, \alpha_4 = 73^\circ, \alpha_5 = 51^\circ$, [16].
During numerical solving of stiff differential equations that describe transformer energization, it is shown that eigenvalues of matrixes A_k are commonly in unstable domain of *BDFp* ($p \geq 3$) rules. Because of the above-mentioned reasons, *BDF2* numerical rule is used in this paper. Advantage of the *BDF2* method in comparison with the trapezoidal rule is better stability properties. Order of local truncation error of the *BDF2* method is h^3.
Simplified flowchart with the applied *BDF* numerical method (named "Algorithm") is shown on the fig. 4. Procedure for numerical calculations of state vector is implemented in a special subroutine (*FBDF2*).

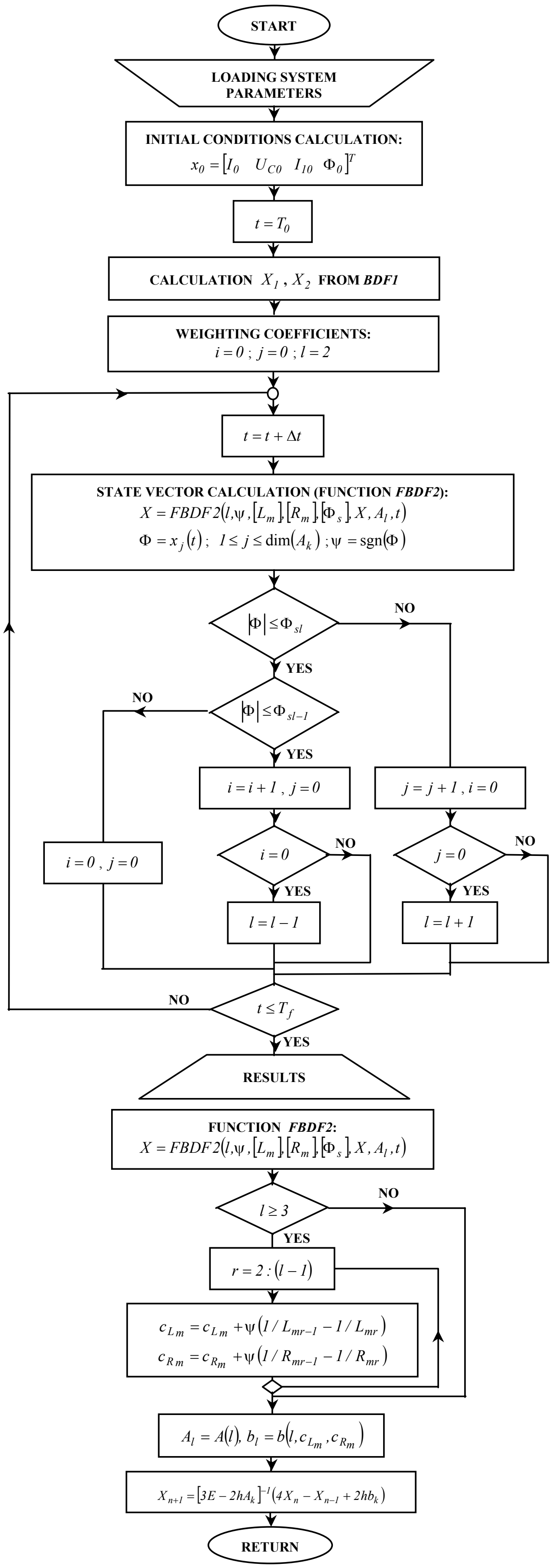


Fig. 4. Simplified flowchart "Algorithm"

III. TEST CASES

A. Results compared to Matlab/Power System Blockset

In order to test the algorithm, a real example from the Power Utility of Bosnia and Herzegovina is used ($220\ kV$ voltage level):

Network parameters:

$E_m = 172\ kV$, $R = 8.82\ \Omega$, $L = 0.281\ \Omega$, $C = 1.218\ \mu F$

Transformer parameters ($220/110\ kV$):

- nominal power $S_{tr} = 200\ MVA$,
- short circuit voltage $u_{k\%} = 15\ \%$,
- resistance per winding phase $R_1 = 0.529\ \Omega$,
- leakage inductance $L_1 = 0.126\ H$,
- iron core losses $R_m = 5.76\ M\Omega$.

Table I: Magnetization curve of 200 MVA transformer

i [p.u.]	0	0.005	0.015	0.03	0.075	1.0
Φ [p.u.]	0	1.05	1.08	1.1	1.12	1.39

It is shown that the "Algorithm" realized by the *BDF2* rule eliminates numerical oscillations in comparison with the traditional trapezoidal rule that can not avoid such oscillations in the case of large eigenvalues range. Fig. 5. shows results of simulations of *A*-stable trapezoidal rule and *A* and *L*-stable *BDF2* rule.

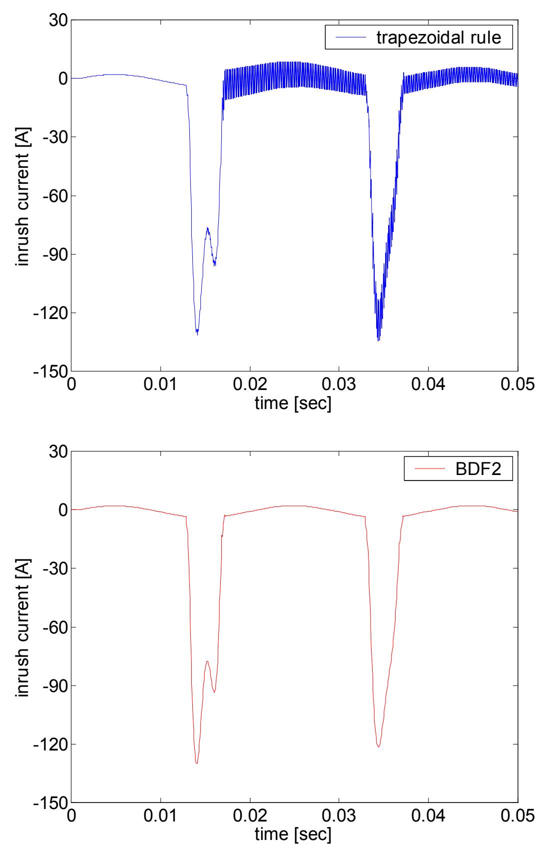


Fig. 5. Inrush current: trapezoidal and BDF method

Further, the simulation results realized by the "Algorithm" with the *BDF* numerical rule are compared to the MATLAB/Simulink/Power System Blockset [17] results. The transformer is energized at $T_0 = 35\ m\,sec$, and the remanent magnetic flux is assumed to be $\Phi_r = 0.5\Phi_{nom}$. All the results are obtained with double-precision arithmetic format. The results are shown in fig. 6.

It can be observed that the core loss resistance R_m, in this case, is considered as a constant value.

From PBS solvers library was chosen the adequate stiff-differential equation system solvers: ode15s (stiff/NDF), numerical differentiation formulas method, [16], [18].

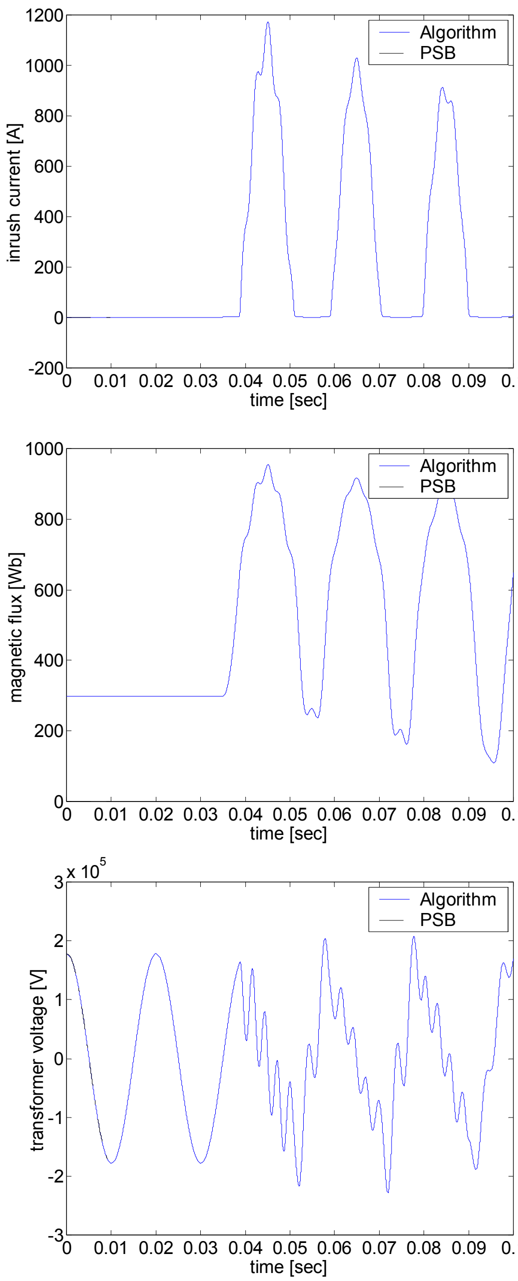


Fig. 6. Inrush current, magnetic flux and transformer voltage: "Algorithm" and Power System Blockset results

B. Results compared to laboratory measurements during transformer energization

It must be pointed out that the realized program for power transformers is hard to test for two reasons:

- Software tool Power System Blockset does not have the option for nonlinear representation of the resistance caused by the iron core losses due to the fact that those losses were strictly considered as constant values, and
- Power transformers usually have, in manufacturer's documentation, only three, maximum four measurement points of magnetizing curve. Those points are usually measured at 95%, 100% and 105% of nominal transformer voltage on the low-voltage side, which makes impossible to construct a magnetizing curve.

Therefore, it is necessary to determine all the relevant data from the measurements with non-load transformer in order to determine how much does the non-linearity of iron core affects the results of simulation. Because of the above-mentioned reasons, the realized algorithm is tested with the data obtained from the laboratory measurement of energization of the small transformer.

Input data: vector of core inductances $L_m = [L_{m1}\ L_{m2}\ \ldots\ L_{mN}]$ and vector of core loss resistances $R_m = [R_{m1}\ R_{m2}\ \ldots\ R_{mN}]$, are given from curves $i_m - \Phi$ and $i_{Rm} - u$. These curves are obtained by standard non-load transformer test, [1]. Based on these nonlinear curves it is possible to recalculate input vectors from the relations:

$$L_{mk} = \frac{\Delta\Phi(k)}{\Delta i_m(k)} = \frac{\Phi(k+1) - \Phi(k)}{i_m(k+1) - i_m(k)} \tag{7}$$

$$R_{mk} = \frac{\Delta u(k)}{\Delta i_{Rm}(k)} = \frac{u(k+1) - u(k)}{i_{Rm}(k+1) - i_{Rm}(k)} \tag{8}$$

for $k = 1, 2, \ldots, N-1$

Laboratory measurement of inrush current and transformer voltage during transformer energization is done according to fig. 3.

Parameters of network model:

$E_m = 210\sqrt{2}\ V$, $R = 14\ \Omega$, $L = 0.25\ H$, $C = 4.22\ \mu F$,

Parameters of transformer model ($220 / 24\ V$):

- nominal power $S_{tr} = 300\ VA$,
- resistance per winding phase $R_1 = 1.99\ \Omega$,
- leakage inductance $L_1 = 2.54\ mH$,
- input vectors of iron core inductances $[L_m]$ and iron core resistances $[R_m]$:

K	1	2	3	4	5	5	7	8
L_{mk}	7.96	12.15	9.70	6.01	3.76	2.28	1.79	1.40
R_{mk}	2897	3909	4370	4726	4855	5071	4318	4545
K	9	10	11	12	13	14	15	16
L_{mk}	1.33	1.13	0.95	0.77	0.56	0.26	0.18	0.11
R_{mk}	4208	3630	4038	3301	3295	2133	3035	2273

Results of measurements and results realized by "Algorithm" are shown in fig. 7.

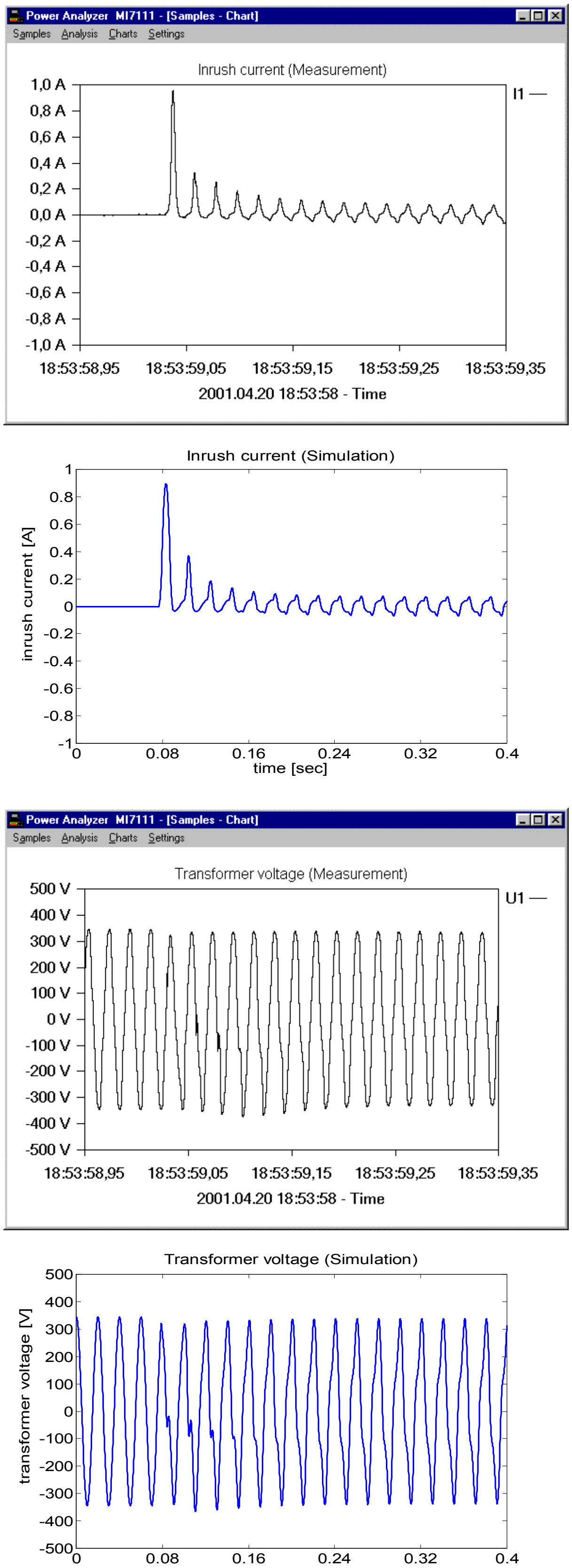


Figure 7. Inrush current and transformer voltage: measured and simulated by the "Algorithm"

IV. CONCLUSIONS

The proposed algorithm can be used for analyzing any low frequency transient phenomenon such as inrush current, ferroresonance, load rejection, etc., for a non-linear transformer. The stiff differential equation system is solved by the *A* and *L*-stable backward differentiation formulas numerical method. *L*-stability puts in advantage *BDF* numerical rule in comparison to the traditionally used trapezoidal rule. This property of the *BDF* method completely eliminates numerical oscillations. The results presented in this paper are obtained with double-precision arithmetic in *FORMAT LONG G* – the best of fixed or floating point format with 15 accurated digits.

All the numerical results obtained by this algorithm are checked with the Power System Blockset - electromagnetic transient software, integrated in the MATLAB/Simulink 6.0. It is also shown that the calculated results with the applied algorithm are in good agreement with the measured results of the small transformer energization. The computing time (CPU time) of the developed program is close to CPU time of the commercial software such as MATLAB/Power System Blockset.

BDF2 rule proposed in this paper eliminates numerical oscillations also in the cases when this can not be achieved with the trapezoidal rule.

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