Computation of Very Fast Transient Overvoltages in Transformer Windings

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Abstract-- The paper deals with the computation of very fast transient overvoltages (VFTO) in transformer windings. The applied algorithm uses a combination of the Multi-Conductor Transmission Line Model (MTLM) and the Single-Transmission Line Model (STLM). By means of the STLM, the voltages at the end of each coil are calculated. Then, these values are used in the MTLM to determine the distributed overvoltages along the turns. Also, this method significantly reduces the number of linear equations that needs to be solved for each frequency to determine the required voltages in frequency domain.

The algorithm uses a modified continuous Fourier transformation that provides an accurate time domain computation. As an example, the inter-turn voltage distributions for two 500 kV auto-transformers are computed and compared with measurements provided by other publications.

***Key words*: Fast transients, Modified Fourier Transformation, Overvoltages, Switching surges, Transformer**

I. Introduction

SWITCHING operations in a gas insulated substations (GIS) and lightning impulses are known to produce very fast transient overvoltages (VFTO) which are dangerous for the transformer and motor insulation. Also, in medium voltage systems where vacuum circuit breakers [2,3] are used, reignition causes high-frequency oscillation which can be dangerous because of their short rise time. Under special circumstances the terminal overvoltages can arise close to the transformer BIL. Another problem is the external resonance which occurs when the natural frequency of the supplying cable matches the natural frequency of the transformer [7]. Most of the time, the greatest problem is the internal resonance which occurs when the frequency of the input surge is equal to some of the resonance frequencies of the transformer. These overvoltages are characterized by a very short rise time. The experience shows that VFTOs within GIS can be expected to have even a rise time of 0.1 μs and an amplitude of 2.5 p.u. [1]. Most of the time, resonant overvoltages can cause a flashover from the windings to the core or between the turns. The inter-turn insulation is particularly vulnerable to high-frequency oscillation and therefore the study of the distribution of inter-turn overvoltages is of essential interest. The VFTOs produced by switching in GIS depend not only on the connection between the GIS and transformer, but also on the transformer parameters and type of the winding.

This paper deals with the calculation of the inter-turn overvoltage distribution in a shell type auto-transformer. Two transformers with different parameters are studied, and our calculations are verified by some measurements and calculations published by other authors.

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Presented at the International Conference on Power Systems Transients (IPST'05) in Montreal, Canada on June 19-23, 2005
Paper No. IPST05 – 009.

II. Application of the Transmission Line Theory

When every turn in a coil is represented as a transmission line, then the propagation phenomena in transformer coils can be fully described by making a use of the modified telegraphs equations:

$$\frac{\partial \mathbf{V_t}}{\partial x} = -\mathbf{L}\left(\frac{\partial \mathbf{I_t}}{\partial t}\right)$$
$$\frac{\partial \mathbf{I_t}}{\partial x} = -\mathbf{C}\frac{\partial \mathbf{V_t}}{\partial t} + C_0[\mathbf{1}]\frac{\partial E_0}{\partial t} \qquad (1)$$

In (1), $\mathbf{V_t}$ and $\mathbf{I_t}$ are the voltage and current vectors. The order is equal to the number of turns in a coil. $\mathbf{L}$ and $\mathbf{C}$ are square matrices of the inductances and capacitances in the coil, while E_0 and C_0 denote the excitation function and capacitance from one turn to the static plate. The last term in the second equation represents the static induced voltage.

The matrix $\mathbf{C}$ is formed as follows:

$C_{i,i}$ – is a capacitance of turn i to ground and sum of the all other capacitances connected to turn i,

$C_{i,j}$ – is a capacitance between turns i and j taken with the negative sign ($i \neq j$)

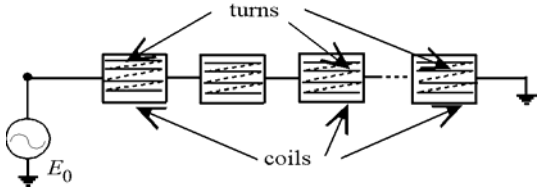

Fig. 1. Turns and coils of the transformer winding

The matrix $\mathbf{L}$ is calculated through the capacitance matrix

$\mathbf{C}$:

$$\mathbf{L} = \frac{\mathbf{C}^{-1}}{v_s^2} \tag{2}$$

where the velocity of wave propagation v_s is calculated as

$$v_s = \frac{c}{\sqrt{\varepsilon_r}} \tag{3}$$

where c is the speed of light and ε_r is the dielectric constant of the transformer insulation. We have to point out that the $\mathbf{L}$ matrix in (2) takes into account the mutual inductances within a specific coil.

By solving (1), the distribution of voltages and currents can be calculated in a particular coil. This is not very practical because the applied model does not take into account the dielectric and conductor losses due to skin effect. Furthermore, all coils and turns should be considered and this leads to a large number of equations and solving the problem requires very large matrix operations.

Therefore, the problem will be solved in two steps by combination of the STLM and MTLM. Both methods are originally derived from the telegraphs equations and the comparison between the methods is described in [4]. Firstly, each coil is considered as a single transmission line with the following equations for the *i*-th coil:

$$\begin{aligned} V_i(x) &= k_i E_0 + A_i \exp(-\Gamma_i(\omega)x) + B_i \exp(\Gamma_i(\omega)x) \\ I_i(x) &= \frac{1}{z_i}(A_i \exp(-\Gamma_i(\omega)x) - B_i \exp(\Gamma_i(\omega)x)) \end{aligned} \tag{4}$$

where z_i is the characteristic impedance of a turn and Γ_i is the propagation constant that takes into account the dielectric and conductor losses as shown in the Appendix. The description of the coils and turns is shown in Fig. 1. The constants A_i and B_i can be calculated by equating the voltages and currents between two adjacent coils:

$$\begin{aligned} V_i(N_i a) &= V_{i+1}(0) \\ I_i(N_i a) &= I_{i+1}(0) \end{aligned}, \text{ for } 1<i<N_c\text{-}1 \tag{5}$$

where N_i and N_c denote the number of turns in the *i*-th coil and the number of coils respectively, and a is the length of a single turn in a coil.

The number of systems of linear equations that should be solved is equal to the number of coils N_c in the transformer winding.

The last two equations are provided by the first and the last coil (which is earthed), $V_1(0) = E_0, V_{N_c}(N_{N_c}) = 0$. The system of equations (5) can be represented in matrix form:

$$\begin{bmatrix} \mathbf{AU}(j\omega) & \mathbf{BU}(j\omega) \\ \mathbf{AI}(j\omega) & \mathbf{BI}(j\omega) \end{bmatrix} \begin{bmatrix} \mathbf{A}(j\omega) \\ \mathbf{B}(j\omega) \end{bmatrix} = \begin{bmatrix} \mathbf{RU}(j\omega) \\ \mathbf{RI}(j\omega) \end{bmatrix} \tag{6}$$

The description of the matrix elements is given in the Appendix. In this way we can determine the constants A_i and B_i, and the voltages and currents at the beginning of each coil. This represents the STLM. Its advantage is that the whole length of the coil is considered as one line and the parameters mentioned above are computed by solving a minimal number of equations.

The second step, which actually is the MTLM uses the determined voltages in frequency domain from the STLM. From (1) it follows that the relation between the voltages and currents in the *i*-th coil can be expressed as:

$$\begin{aligned} \mathbf{V_t}(x) &= \mathbf{k_i} E_0 + \mathbf{A_t} \exp(-\Gamma_i(\omega)x) + \mathbf{B_t} \exp(\Gamma_i(\omega)x) \\ \mathbf{I_t}(x) &= v_s[\mathbf{C}]\{\mathbf{A_t} \exp(-\Gamma_i(\omega)x) - \mathbf{B_t} \exp(\Gamma_i(\omega)x)\} \end{aligned} \tag{7}$$

The system of equations (7) is used to compute the voltages and currents in the turns of a particular coil. In equation (7) $\mathbf{A_t}$ and $\mathbf{B_t}$ are vectors which can be computed from the boundary conditions in the same way as in the STLM, and $\mathbf{k_i}$ is a vector that consists the capacitive voltage distributions. In a similar way, the matrix equation from where the vectors $\mathbf{A_t}$ and $\mathbf{B_t}$ can be determined is:

$$\begin{bmatrix} \mathbf{MA}(j\omega) & \mathbf{MB}(j\omega) \\ \mathbf{MC}(j\omega) & \mathbf{MD}(j\omega) \end{bmatrix} \begin{bmatrix} \mathbf{A_t}(j\omega) \\ \mathbf{B_t}(j\omega) \end{bmatrix} = \begin{bmatrix} \mathbf{UA}(j\omega) \\ \mathbf{UB}(j\omega) \end{bmatrix} \tag{8}$$

The order of the matrix in (8) that represent the system of equations is equal to twice the number of turns in the observed coil. This significantly helps especially when we deal with large number of linear equations which must be solved for each frequency.

Matrix equation (8) must be solved for each step frequency by providing the voltages at the beginning of each coil which were previously determined by the STLM. The inter-turn voltage is defined as the difference between the voltages of two adjacent turns in a coil:

$$\delta V_{t,i} = V_{t,i}(x) - V_{t,i+1}(x) \tag{9}$$

where $V_{t,i}$ is the voltage in the *i*-th turn of the studied coil.

III. Frequency Analysis

One of the major problems when solving the wave equations directly in time domain is that the parameters of the line, particularly the conductor resistance and inductance are frequency dependent. A detailed approach of the general application of telegraphic equations for multi-conductor transmission lines can be found in [8]. But this method does not take into account the frequency dependency of the transmission line parameters. Therefore, the equations must be solved in the frequency domain. When the transformer is stressed by a sinusoidal voltage, the source function can be expressed as:

$$E_0(t) = E_0 \sin(\omega_p t) \tag{10}$$

The inter-turn voltages in frequency domain are calculated by

multiplying the transfer function with the source function according to Fig. 2,

$$\delta V(j\omega) = H(j\omega)E_0(j\omega) \tag{11}$$

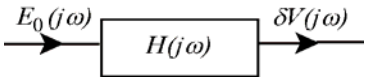

Fig. 2. Frequency domain analysis

From (11) it can be seen that the transfer function $H(j\omega)$ actually is equal to the inter-turn voltage when the input is unity. This approach is used to calculate the transfer function. The time domain results can be calculated either by applying convolution or inverse Fourier transformation.

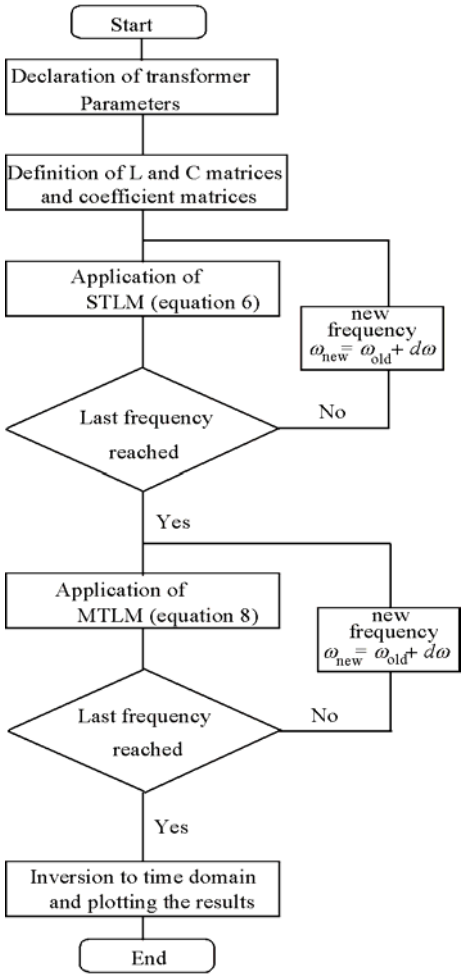

Fig. 3. Algorithm for computation of inter-turn overvoltages

The modified Fourier transformation is defined as:

$$\delta V(t) = \frac{2\exp(bt)}{\pi}\int_{-\Omega}^{\Omega}\frac{\sin(\pi\omega/\Omega)}{\pi\omega/\Omega}H(b+j\omega)\delta V(j\omega)d\omega \tag{12}$$

where the interval $[-\Omega,\Omega]$, the smoothing constant b and the step frequency length $d\omega$ must be chosen properly in order to provide an accurate time domain response [5]. The modified transformation requires the input function $E_0(t)$ to be filtered by an $\exp(-bt)$ window function. The algorithm of the computation is described in Fig. 3.

IV. Transformer Description

The computation of the inter-turn voltage distribution is a very delicate task and strongly depends on many transformer data such as the type and dimensions of coils and turns, dielectric parameters as well as the influence of the environment. This is the reason why the exact computation of the capacitances, inductance and transformer losses is difficult to do. Furthermore, when studying the VFTOs, a small change in the parameters can cause significant difference in the results.

TABLE I
Transformer Data

	Mean turn length [m]	Turns in a coil	Number of coils	v_s [m/□s]
TR A	7.6	22-50	10	177
TR B	6.83	17-23	12	184

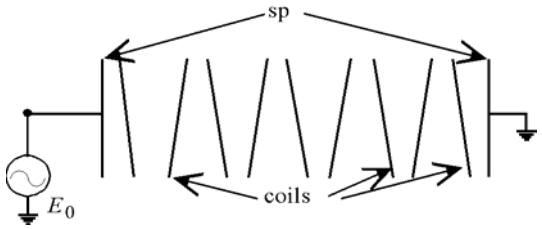

Fig. 4. Description of the transformer coils

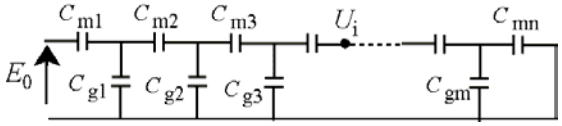

Fig. 5. Static voltage distribution

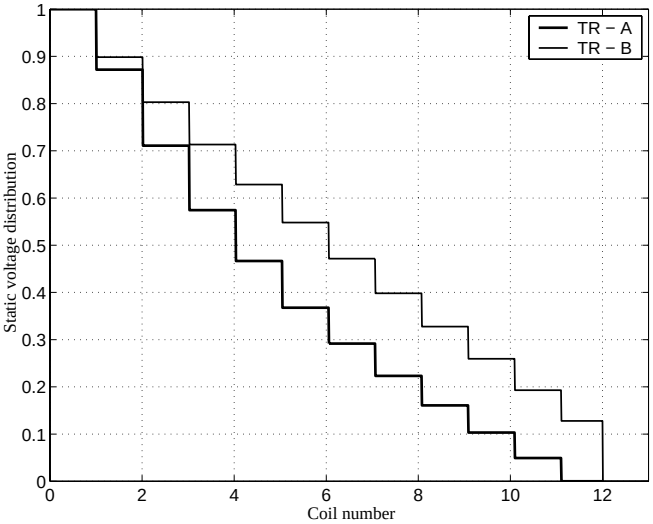

Fig. 6. Computed static voltage distribution

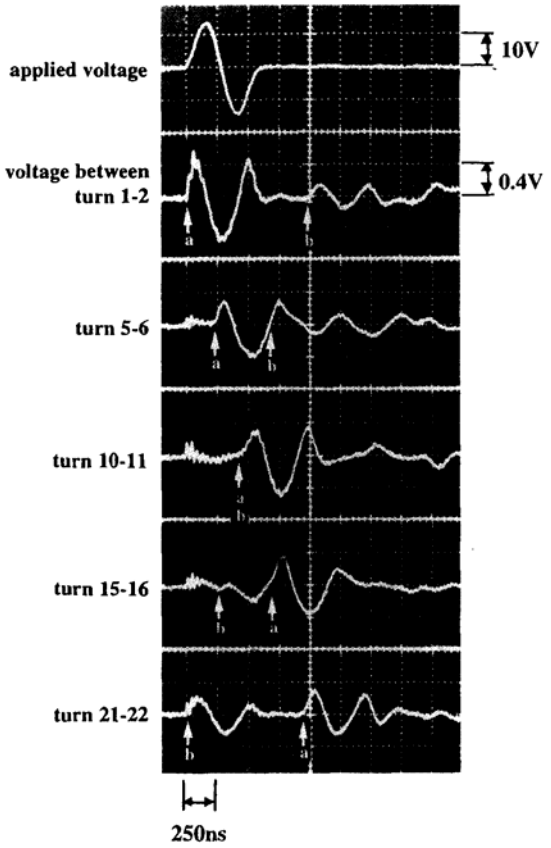

Fig. 7a. Measurements of inter-turn voltage distribution of transformer A in the first coil [6]

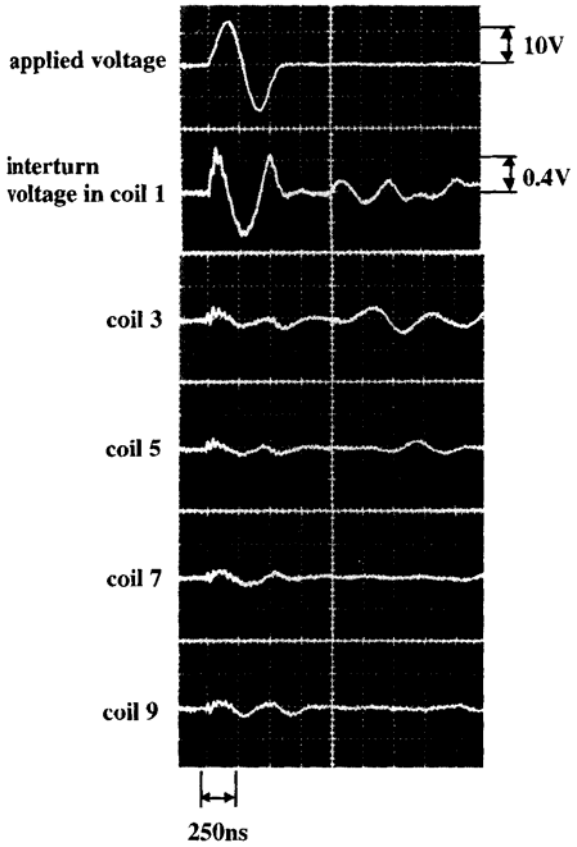

Fig. 7b. Measurements of the inter-turn voltage distribution of 1-2 turn for transformer A [6]

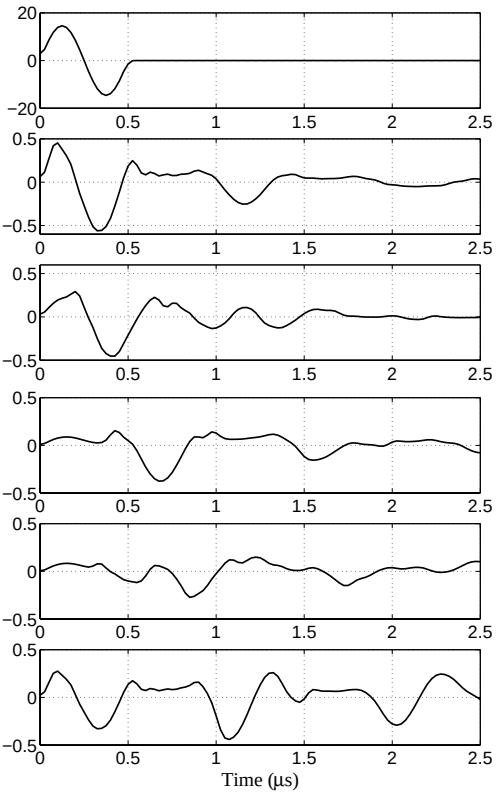

Fig. 8a. Computed inter-turn voltage distribution of transformer A in the first coil

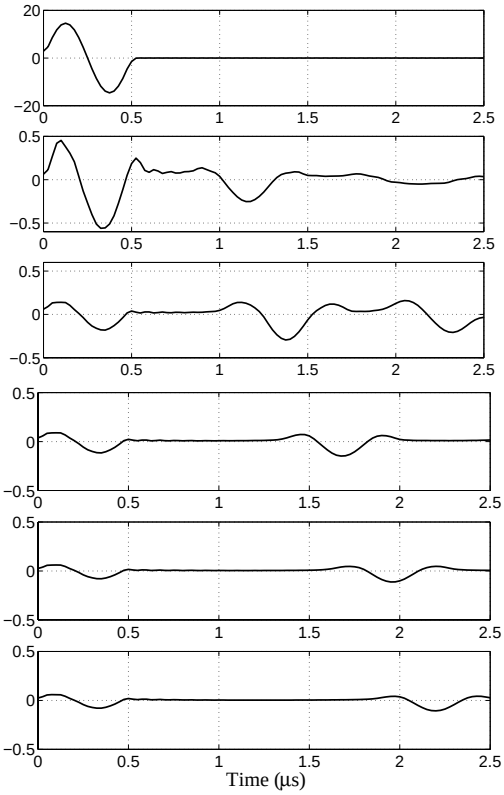

Fig. 8b. Computed inter-turn voltage distribution of turn 1-2 for transformer A

This implies that these kinds of studies require exact data parameters in order to provide more accurate computation. In this work two similar 500 kV auto-transformers are studied. The differences between both transformers are shown in Table I.

The transformer coils are tilted as shown in Fig. 4. The capacitances between the coils are calculated by approximating the coils as plane capacitors and taking into account their mean dimensions. In this way the capacitances between the coils and between the coil and ground are found (see Fig. 5), and the capacitive voltage distribution is determined. Also, the capacitance matrix $\mathbf{C}$ is derived by means of the coil-to-coil and coil-to-ground capacitances.

The capacitive voltage distribution, which is the ratio between any coil voltage U_i and the input voltage E_0 shows how the voltage is distributed among the coils. Thus, the voltage at i-th coil varies around the value $(k_i+k_{i+1})/2$. The computation for both transformers is displayed in Fig. 6. The MTLM takes into account the static voltage distribution.

V. Results and Comparison with Experiment

The calculations are done to two transformers for which the measurements are published in [4,6]. For Transformer A, a single sinusoidal pulse of 2 MHz and amplitude of 15 V is used. Transformer B is excited by a double sinusoidal pulse of 1.6 MHz and amplitude of 1 V. The frequency of the defined pulse excitations are the resonant frequencies and are determined roughly by:

$$f_p = \frac{v_s}{l} \tag{13}$$

where l is the length of a specific coil. Fig. 7 and Fig. 8 show the measured and computed inter-turn voltages in the first coil and in some other coils. All distributed overvoltages for the first coil of transformer A are summarized in Fig. 9.

In Fig. 10 and Fig. 11, the computed overvoltage distributions in the transformer B are presented. The measurements for the first coil of transformer B are published in [4]. Despite the good agreement of the amplitudes and wave forms in the transformer B, the wave forms in the transformer A show slight deviation from the measured wave forms, probably because not enough data are available for this transformer at the time when the calculations were performed. Especially the exact number of turns in some coils and the dimensions of the coils in the transformer were not known. The computations of transformer B show good match with the computations and measurements shown in [4]. We studied the inter-turn overvoltages in each coil and it was shown that the first and the forth coil show most severe overvoltages. Fig. 10 and Fig. 11 show clearly the travelling wave phenomenon in the turns.

The dielectric strength of the inter-turn insulation should withstand the lightning impulse voltage which for 500 kV transformer is approximately 1300 kV. The inter-turn voltage caused by this impulse voltage is about 1.5 % of this value. So, the inter-turn surge voltage that the insulation can withstand is 19 kV. Our study shows that the maximal amplitude of the computed and measured intern-turn overvoltages for the applied resonant frequencies are approximately 5 % of the amplitude of the source voltage.

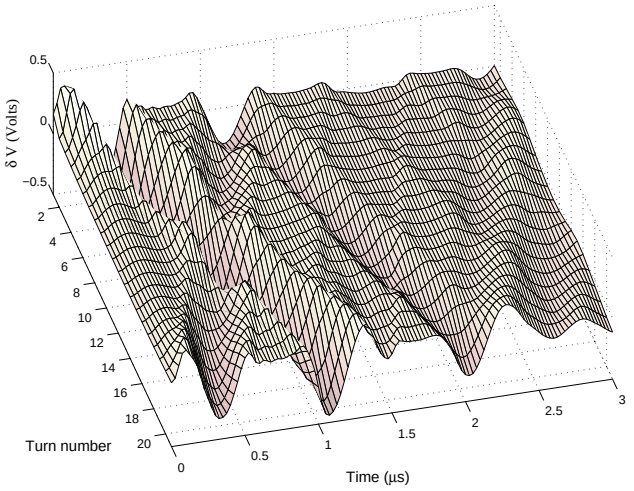

Fig. 9. Computed distribution of inter-turn voltages for the first coil of transformer A

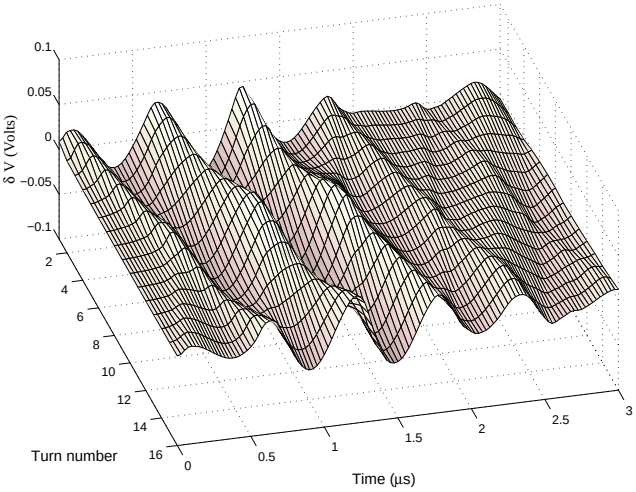

Fig. 10. Computed distribution of inter-turn voltages for the first coil of transformer B

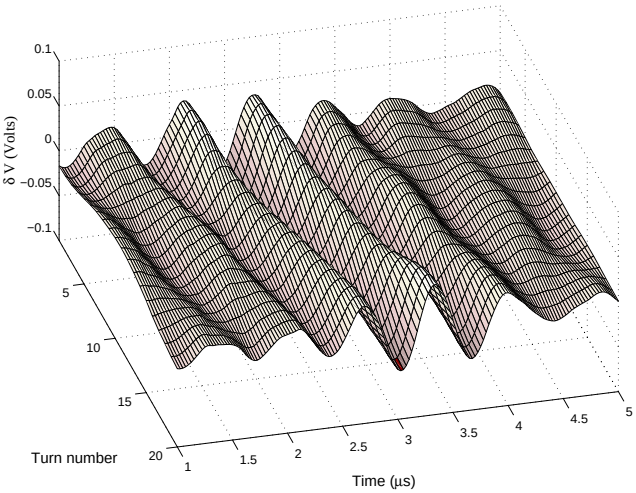

Fig. 11. Computed distribution of inter-turn voltages for the fourth coil of transformer B

This means that for a rated transformer voltage of 500 kV, the inter-turn overvoltages are ($500\sqrt{2}/\sqrt{3}\cdot 0.05$=20.37 kV). When the transformer is stressed by a higher overvoltage for example lightning impulse or GIS switching surges, this value can be even higher. The measured inter-turn voltage distribution versus frequency normally possesses more resonant frequencies. Sometimes they can reach value up to 20 % of the rated voltage which might be rather dangerous and definitely can cause an inter-turn flashover.

VI. Conclusion

The study of VFTO which occur during the switching of the network with GIS, or other phenomena which cause steep-fronted surges is important for insulation coordination. Especially it is important to find the resonance points which depend on the characteristic of the transformer. In this work the resonance of the winding is roughly determined from the speed of surge impulse and the length of a coil. The MTLM and STLM method can be used with full success and there is a good agreement between the measured and calculated inter-turn distributions. However, the computed characteristic of the inter-turn voltage with respect to the frequency most of the time shows a slight deviation from the actual characteristic. Therefore, before applying this method it is recommended to provide the actual (measured) frequency characteristic of the inter-turn voltage distribution from where the resonant frequencies and the amplitude of the inter-turn overvoltage can be observed. Then, for each resonant frequency the voltage distribution along the winding can be calculated. Despite the use of Discrete Fourier Transformation which showed some instabilities in the time domain solution, the modified Fourier Transformation was found to be very stable for these studies. Due to lack of data, our analysis does not take into account the mutual inductances between the coils, but only the mutual inductances between the turns in the coils. It is not known to us if the method is so far applied for other types of windings. Our opinion is that the method can be applied for all types of transformer windings.

VII. References

[1] Cornick K., Filliat B., Kieny C., Müller W.: Distribution of Very Fast Transient Overvoltages in Transformer Winding, *CIGRE* paper 12-204, Paris, 1992.

[2] Craenenbroek T., De Ceuster J., Marly J.P., De Herdt H., Brouwers B., Van Dommelen D.: Experimental and Numerical Analysis of Fast Transient Phenomena in Distribution Transformers, *IEEE Winter Meeting*, Singapore, January 2000.

[3] Popov M., van der Sluis L.: Improved Calculations for No-load Transformer Switching Surges, *IEEE Transactions on Power Delivery*, Vol. 16, No. 3, July 2001, pp. 401-408.

[4] Shibua Y., Fujita S., Hosokawa N.: Analysis of Very Fast Transient Overvoltages in Transformer Winding, *IEE Proceedings on Generation Transmission and Distribution*, Vol.144, No. 5, September 1997, pp.461-468.

[5] Bickford J.P., Mullineux N., Reed J.R.: *Computation of Power System Transients*, IEE, Peter Peregrinus Ltd., 1976, ISBN 0901223859.

[6] Fujita S., Hosokawa N., Shibua Y.: Experimental Investigation of High Frequency Voltage Oscillation in Transformer Windings, *IEEE Transactions on Power Delivery*, Vol. 13, No. 4, October 1998, pp. 1201-1207.

[7] Paap G., Alkema A., van der Sluis L.: Overvoltages in Power Transformers Caused by No-load Switching, *IEEE Transactions on Power Delivery*, Vol. 10, No. 1, January 1995, pp. 301-307.

[8] Djordjevic A.R., Sarkar T.K. Harrington R.F.: Time-Domain Response of Multiconductor Transmission Lines, *Proceedings of the IEEE*, Vol.75, No. 6, June 1987, pp. 743-764.

VIII. Biographies

Marjan Popov (M'95, SM'03) received his Dipl-Ing. and M.S. in electrical engineering from the St. Cyril and Methodius University in Macedonia in 1993 and 1998 respectively, and the Ph.D. degree from Delft University of Technology, the Netherlands in 2002. In 1997 he was an academic visitor at the University of Liverpool, UK. Currently he is an assistant professor in the group of Electrical Power systems with the Power System Laboratory at TU Delft. His major fields of interest are arc modeling, transients in power systems, parameter estimation and power system protection.

Lou van der Sluis was born in Geervliet, the Netherlands on July 10, 1950. He obtained his M.Sc. in electrical engineering from the Delft University of Technology in 1974. He joined the KEMA High Power Laboratory in 1977 as a test engineer and was involved in the development of a data acquisition system for the High Power Laboratory, computer calculations of test circuits and the analysis of test data by digital computer. In 1990 he became a part-time professor and since 1992 he has been employed as a full-time professor at the Delft University of Technology in the Power Systems Department. Prof. van der Sluis is a senior member of IEEE and past chairman of CC-03 of Cigre and Cired to study the transient recovery voltages in medium and high voltage networks.

Gerardus C. Paap was born in Rotterdam, the Netherlands on February 2, 1946. He received his M.Sc. Degree from Delft University of Technology in 1972, and his Ph.D. degree from the Technical University of Lodz in 1988. Since 1973 he has been with the Department of Electrical Engineering at Delft University of Technology. From 1973 to 1985, he was with the Division of Electrical Machines and Drives. Since 1985 he has been with the Power Systems Laboratory where he is currently Associate Professor. Dr. Paap's main research interests include power-system transients, stability and control, dynamics of electrical machines and the large-scale implementation of renewable energy. He is a member of the Dutch National CIGRE Committee no. 11: Rotating Machines and senior member of IEEE.

Hans De Herdt was born in Rumst, Belgium on October 7, 1964. He graduated in 1987 as electrical-mechanical engineer from the Katholieke Universiteit Leuven. In 1989 he joined Pauwels Trafo, where he is currently working as R&D project engineer. His main interests are electromagnetic field calculations, transformer models and transient overvoltage.