Design of a Robust Nonlinear Fuzzy Controller for HVDC Systems

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Abstract -- The design and stability assessment of a new robust nonlinear fuzzy controller that may be applied in the control loops of HVDC systems is presented. A new simplified nonlinear dynamic model is developed for HVDC systems that can be used to design the controller. The proposed model decomposes into several linear systems around important equilibrium points. These local linear models describe the plant dynamic behavior at different operating points. An evaluation is performed by simulation using the Cigré benchmark HVDC model. The simulation results show that, compared to conventional controllers, the new controller gives an improvement in AC/DC/AC system performance during severe faults. Stable behavior is shown to be obtained during a sudden change in effective short ratio (ESCR) when using the proposed controller within a very weak AC/DC system. This is compared with the operation of a conventional controller under similar conditions, during which the HVDC-system becomes unstable.

Keywords: HVDC transmission control, Power System Dynamic modeling, Fuzzy System.

I. INTRODUCTION

HIGH Voltage Direct Current (HVDC) technology finds application in the transmission of power over long unbroken overland distances, significant underwater distances, and in the interconnection of separate, or partitioned, AC systems. HVDC systems are likely to continue to be deployed in the future, especially as technological improvements are making them cost competitive with alternative AC schemes at decreasing power levels; and their unique control characteristics can be exploited to enhance the capacity, level of interconnection, and availability of existing AC systems.

The converter-bridge controller used for HVDC transmission is designed to maintain a specific power transmission characteristic. In an AC/DC power system, the fast-acting converter controller allows hierarchical control of the system following a system disturbance. Conventional HVDC converters currently employ PI type controllers that make use of a fixed gain structure which is optimized for operation at rated conditions. A challenge for future designers is to incorporate sufficient flexibility within the controller to allow the control strategy to be adjusted for different operating conditions.

The tuning of HVDC converter controls involves making a compromise between the speed of response and stability during small disturbances on the one hand, and robustness under large signal disturbances due to faults or switching on the other hand. Furthermore, the highly nonlinear nature of the control loops necessitates the careful selection of control constants to accommodate a range of operating conditions.

The AC/DC interaction becomes more sensitive to disturbances as the effective short-circuit ratio (ESCR) of the AC system interface falls lower and lower [1], and the correct adjustment of control constants for good overall performance becomes much more difficult. To circumvent the above problem, extensive research has been carried out in the area of HVDC control. However, the advanced techniques available in DC adaptive control literature are difficult to apply in practical applications because of the absence of insight into performance with large disturbances, where the adaptive control may be not only ineffective but may degrade the performance rather than enhance it [2],[3].

A gain scheduling adaptive control strategy has been tried in [4] where the effect of large disturbances has been taken into account. In [5], the advantages of automatic continuous fine tuning are combined with predetermined gain scheduling in order to achieve robustness during large disturbances. A robust coordinated control scheme for a parallel AC/DC system is proposed in [6]. The paper describes the derivation and validation of a coordinated controller employing on-line identification of the AC/DC system. Fuzzy-logic-based tuning of the controller parameters for the rectifier side current regulator, and inverter side gamma controller in a HVDC system is introduced in [7-9]. In these, error signals and their derivatives are used as inputs to the fuzzy system, and give optimum system performance under various normal and abnormal conditions. To obtain good performance under various disturbance conditions, the fuzzy system parameters (number of fuzzy IF-THEN rule and their membership functions) need to be adjusted in a trial and error manner.

In this paper, a different approach to designing a suitable controller for HVDC systems is first considered. As shown later, the new controller only uses output variables. Next, HVDC system modeling is presented which is based on the Cigré benchmark model. In section III, a suitable nonlinear dynamic model is developed for the HVDC system that can be decomposed into linear systems around its equilibrium
points. Section IV considers the method of robust nonlinear fuzzy control design using the Takagi-Sugeno fuzzy model. Stability conditions of both fuzzy models and fuzzy control systems are given. The simulation results are shown in section V. Finally, concluding remarks are drawn in section VI.

II. HVDC SYSTEM MODELING

To date, a wide variety of HVDC converter control strategies have been tested and optimized with the help of various digital programs. Great interest in HVDC-system simulation has led to the establishment of a Cigré benchmark model [10], [11] which is used here as a test system [11]. The selected short-circuit ratio (SCR) and the effective short-circuit ratio (ESCR) for the Cigré benchmark model are typical of a weak system. The combination of the weak inverter system, the DC-side resonance approaching fundamental frequency, and the AC-side resonance near the second harmonic make this system particularly onerous for DC control operation.

The proposed dynamic model is derived from the basic system configuration shown in Fig. 1. In this figure, the rectifier and inverter AC systems are represented by Thevien AC equivalents, i.e. a constant AC voltage source behind a short-circuit impedance. This representation is most commonly assumed for the investigation of large-disturbance and small-disturbance voltage instability in weak AC/DC systems [12-14]. In [15] the time frame for model validity is given as approximately several hundred milliseconds.

A. Power and voltage equations for HVDC system

Based on the per-unit method presented in [16], and by selecting the nominal DC power ($P_{nom}$) and ideal open circuit DC voltage ($V_{dors}$) as base power and base voltage, respectively, the following equations may be derived:

$$V_{dors}^{pu} = \frac{a_{r}}{a_{in}} V_{r}^{pu} \cos \alpha_{r} - \frac{3}{\pi} x_{pu}^{pu} I_{dors}^{pu}$$  \hspace{1cm} (1)

$$V_{dors}^{pu} = \frac{a_{r}}{a_{in}} V_{r}^{pu} \cos \gamma_{r} - \frac{3}{\pi} x_{pu}^{pu} I_{dors}^{pu}$$  \hspace{1cm} (2)

$$P_{dors}^{pu} = \frac{a_{r}}{a_{in}} V_{r}^{pu} I_{dors}^{pu} \cos \alpha_{r} - \frac{3}{\pi} x_{pu}^{pu} (I_{dors}^{pu})^2$$  \hspace{1cm} (3)

$$P_{dors}^{pu} = \frac{a_{r}}{a_{in}} V_{r}^{pu} I_{dors}^{pu} \cos \gamma_{r} - \frac{3}{\pi} x_{pu}^{pu} (I_{dors}^{pu})^2$$  \hspace{1cm} (4)

$$Q_{dors}^{pu} = \frac{3}{\pi} I_{dors}^{pu} \sqrt{V_{r}^{pu} \cos \alpha_{r} - \frac{3}{\pi} x_{pu}^{pu} (I_{dors}^{pu})^2}$$  \hspace{1cm} (5)

$$Q_{dors}^{pu} = \frac{3}{\pi} I_{dors}^{pu} \sqrt{V_{r}^{pu} \cos \gamma_{r} - \frac{3}{\pi} x_{pu}^{pu} (I_{dors}^{pu})^2}$$  \hspace{1cm} (6)

where $a, x, \gamma, V, I_{d}$ and $I_{d}$ are converter transformer turns ratio, commutation reactance, firing angle, extinction angle, AC voltage, DC voltage and DC current, respectively. The subscripts $r, i$ and $N$ denote rectifier, inverter and nominal values, respectively.

Also, $T_{min} = \frac{\pi^2 a_m^2}{3 \sqrt{2} a_{in}}$ and $T_{max} = \frac{2 \pi a_m}{3 \sqrt{2} a_{in}}$, in which $m=r,i$.

B. Power flow equations at AC buses

By assuming that $Z_{rs} = z_{rs} \angle 90^\circ$ and $Z_{is} = z_{is} \angle 90^\circ$, the power flow equations may be given by

$$\frac{V_r}{z_{rs}} E_{rs} \sin \delta_r + P_{dors} = 0$$  \hspace{1cm} (7)

$$\frac{V_r^2}{z_{rs}} - E_{rs} V_r \cos \delta_r - V_r^2 b_{rs} + Q_{dors} = 0$$  \hspace{1cm} (8)

$$\frac{V_i}{z_{is}} E_{is} \sin \delta_i - P_{dors} = 0$$  \hspace{1cm} (9)

$$\frac{V_i^2}{z_{is}} - V_i E_{is} \cos \delta_i + Q_{dors} - V_i^2 b_{is} = 0$$  \hspace{1cm} (10)

C. Converters control system equations

The proposed converters control-system block-diagrams are depicted schematically in Fig. 2. The converter control-system equations (11) and (12) are derived from these diagrams.

$$\alpha_{r} = \frac{K_{\alpha}}{T_{r}} (I_{ord} - I_{dors} + I_{u}) - \frac{1}{T_{r}} \alpha_{r} + \frac{a_{r}}{T_{r}}$$  \hspace{1cm} (11)

$$\beta_{r} = \frac{K_{\beta}}{T_{r}} (\gamma_{ref} - \gamma_{r} + \gamma_{u}) - \frac{1}{T_{r}} \beta_{r} + \frac{\beta_{0}}{T_{r}}$$  \hspace{1cm} (12)

where $I_{ord}, \gamma_{ref}, \beta, I_{u}, \gamma_{r}, K$ and $T$ are current order, reference extinction angle, advance firing angle, current control signal, extinction angle control signal, gain and time constant, respectively.
D. DC transmission line equations

The DC transmission line is represented by an equivalent T network with the lumped charging capacitance at the midpoint of the DC link, thereby dividing the series impedance into two parts. From this model equation (13–15) may be derived:

\[ \dot{i}_{dr} = \frac{\omega_0}{X_i} V_x \cos \alpha_r - \frac{\omega_0}{X_i} V_y - \left( \frac{3\omega_0 X_i}{\pi} + R \omega_0 \right) i_{dr} \]  
\[ \dot{i}_{di} = \frac{\omega_0}{X_i} V_x \cos \gamma_i + \left( \frac{3\omega_0 X_i}{\pi} + R \omega_0 \right) i_{di} \]
\[ V_c = \omega_0 XL - \omega_0 XI_{di} \]

where \( X = \frac{1}{\omega_0 C}, \ X_i = \omega_0 L_a \) and \( X_i = \omega_0 L_i \) are appropriate reactivities.

![Fig. 2 Converters control system block diagrams, a) Rectifier, b) Inverter.](image)

III. DYNAMIC SYSTEM MODEL

In this section a mathematical model of the dynamic system in Fig.1 is derived. The DC transmission line differential equations and load flow equations results in the set of differential-algebraic (DA) equations (16).

\[ \dot{x} = f(x, y, u) + h_f(x, y, u)u \]
\[ 0 = g(x, y, u) + h_g(x, y, u)u \]

where \( x = \begin{bmatrix} I_{dr} & I_{di} & V_c & \alpha_r & \beta_i \end{bmatrix}^T \), \( y = \begin{bmatrix} V_r & V_i & \delta_r & \delta_i \end{bmatrix}^T \), \( \mu \) and \( u \) are vectors of state variables, vector of algebraic variables, vectors of AC/DC system parameters and input vector, respectively. \( f \) and \( g \) are vectors of functions of these DA variables. The set of algebraic equations may be viewed as a manifold over which the dynamics of the differential system are constrained to occur [17]. Under equilibrium conditions, the system of equations (16) is described by:

\[ 0 = f(x_0, y_0, \mu_0, u_0) \]
\[ 0 = g(x_0, y_0, \mu_0) \]

where \((x_0, y_0, \mu_0)\) is an equilibrium or fixed point. An operating point is an equilibrium point. Hence at each operating point, the system may be described by a linearized form of equation (16). The general linear system model is given by:

\[ \dot{x} = f(x, y, \mu_0)\Delta t + f_y(x, y, \mu_0)\Delta y + f_r(x, y, \mu_0)\Delta u \]
\[ 0 = g(x, y, \mu_0)\Delta t + g_y(x, y, \mu_0)\Delta y + g_r(x, y, \mu_0)\Delta u \]

where \( f, f_y, g_y, \) and \( g_r, g_y, \) and \( g_r, g_y, \) are the Jacobian sub-matrices comprising partial derivatives of \( f \) and \( g \) with respect to \( x \) or \( y \), as indicated by the subscript label. Substituting \( \Delta y \) from the second equation of (18) into the first one gives equation (19).

\[ \Delta \dot{x} = f_x - f_y g_y^{-1} g_r \Delta t + h_r \Delta u = A(x, y, \mu_0) \Delta t + B \Delta u \]

where \( A \) is the dynamic state matrix describing local dynamic behavior of the nonlinear system, as given by Eq.20, assuming that \( g_r \) remains nonsingular along system trajectories as the system parameters vary.

\[ A(x, y, \mu_0) = f_x - f_y g_y^{-1} g_r \]

The per-unit values of the proposed dynamic model are calculated based on the method presented in [16]. Then, the nonlinear system equation (17) is solved for different operating conditions and several operating points are obtained.

IV. ROBUST NONLINEAR FUZZY CONTROL DESIGN

In the proposed controller design, the nonlinear system model decomposes into linear system models in accordance with the cases for which linear models are suitable, and then individual linear models are aggregated into a single nonlinear model in terms of membership functions. This is a nonlocal approach which is conceptually simple and straightforward. The nominal model of a nonlinear system is:

\[ \dot{x}(t) = f(x, t) + g(x, t)u(t) \]
\[ y(t) = Cx(t) \]

where \( x(t), u(t), y \) and \( C \) are vector of state variables, vector of control inputs, vector of outputs and constant matrix, respectively. Adding model uncertainties \( \Delta f \) to system (21) gives:

\[ \dot{x}(t) = f(x, t) + \Delta f(x, t, \xi) + g(x, t)u(t) \]

where \( \xi \) is the vector of uncertain parameters which is restricted to a prescribed bounding set [16]. By considering the Takagi-Sugeno fuzzy model [18, 19], the uncertain nonlinear system (23) can be modeled as the following uncertain fuzzy system:

**Plant Rule i:** IF \( \Theta_i \) is \( \mu_{ij} \) and...and \( \Theta_p \) is \( \mu_{ip} \)

THEN \[ \dot{x}(t) = (A_i + \Delta A_i(x, t, \xi))x(t) + B_i u(t) \]

where \( \Theta_j (j=1, ..., p) \) are the premise variables, which are functions of state variables \( x, \mu_j (i=1, ..., r) \) are fuzzy sets, \( r \) is the number of fuzzy rules, and \( p \) is the number of the premise variables. \( \Delta A_i \) represents the uncertainties in system matrix. It is assumed that \( \Delta A_i \) admits the following form:

\[ \Delta A_i(x, t, \xi) = D_i Y_i(x, t, \xi) \]

where \( D_i \) and \( E_i \) are known real constant matrices of appropriate dimensions and uncertainty \( Y_i \) , an unknown matrix valued function of \((x, t, \xi)\) with a known bounded set [16].
Fuzzy blending of each individual model yields the overall fuzzy model as follows [16]:

\[
\dot{x}(t) = \sum_{i=1}^{r} \alpha_i(\theta) [(A_i + \Delta A_i(x,t,\xi))x(t) + B_iu(t)] \\
y(t) = Cx(t)
\]

(26)

where \( \alpha_i(\theta) = \frac{\alpha_i(\theta)}{\sum_{i=1}^{r} \alpha_i(\theta)} \) with constraints \( \alpha_i(\theta) \geq 0, \sum_{i=1}^{r} \alpha_i(\theta) = 1 \).

and \( \alpha_i \) (i = 1, ..., r) are the membership function of the systems belonging to plant rule i. The output equation is also added into the overall model for convenience, and it is assumed that all the triples \((A_i, B_i, C)\), \(i=1,\ldots,r\), are controllable and observable. It should be noted that the parameter uncertainty structure used in this paper has been widely used in the study of the problem of robust stability and stabilization of uncertain linear systems [20].

The first objective in this section is to design a robust proportional controller of the following form:

\[
u(t) = \sum_{i=1}^{r} \alpha_i(\theta)F_iy(t)
\]

(27)

where \( F_i(i=1,2,\ldots,r) \) are constant matrices; such that the closed loop system (26) and (27) is asymptotically stable. In other words, the feedback matrices \( F_i(i=1,2,\ldots,r) \) need to be obtained such that system is robustly stabilized. In this regard, the following theorem is given for the stabilization of system (26) with \( B_i=B \) by a proportional controller.

For HVDC systems, all of the matrices \( B_i \) are also identical.

**Theorem 1**: The system (26) can be robustly stabilized by controller (27) if there exist positive definite matrices \( F_i \) and positive numbers \( \varepsilon_i \) (i = 1, ..., r), such that the matrix inequalities in (28) hold.

\[
P\Delta_i^T + A_iP + P E_i^T E_i + \varepsilon_i^{-1} P E_i^T (P - P C^T C P) i=1,2,\ldots,r
\]

\[
+ \begin{bmatrix} B F_i + P C^T \end{bmatrix}^T \begin{bmatrix} B F_i + P C^T \end{bmatrix} \leq 0
\]

(28)

The proof of this theorem and its necessary lemmas are given in [16] and hence is omitted here. The solution of (28) is very complicated, and the matrix inequalities (28) can not be directly used to calculate the required feedback matrices \( F_i \). In [16], an iterative linear matrix inequality (ILMI) algorithm was developed to solve matrix inequalities (28).

The final objective in this section is to design a robust PI controller of the following form:

\[
u(t) = F_p y(t) + \int_{0}^{t} y(\tau)d\tau
\]

(29)

System (26) is considered again, but now a PI controller (29) is used instead of proportional controller (27). It can be easily shown that with a simple variable change \( z_1 = x \), \( z_2 = \int_{0}^{t} yd\tau \), the problem of PI controller design for system (26) is reduced to that of proportional controller design for the following system [16]:

\[
\dot{\bar{y}} = \sum_{i=1}^{r} \alpha_i(\theta) \begin{bmatrix} \bar{A}_i + \Delta \bar{A}_i(x,t,\xi) \end{bmatrix} \bar{y}(t) + \bar{B}_iu(t)
\]

\[
\bar{y} = \bar{C} \bar{z}
\]

(30)

where \( \bar{A}_i, \bar{B}_i, \bar{C}_i, \Delta \bar{A}_i \) are appropriate matrices in new system.

The feedback matrices \( F_p \) and \( F_i \) can be calculated by applying theorem 1 and the ILMI algorithm to system (30).

**V. SIMULATION**

To investigate system performance with the proposed robust nonlinear fuzzy controller, several simulations are performed. The design of HVDC controllers is strongly affected by the effective short circuit ratio (ESCR) at the converter stations. After optimizing these controllers for rated conditions of operation, the problem of operating them under different ESCR levels is critical, especially in the case of weak AC networks. In a weak AC system, and with an unsuitable control design, the AC/DC system may lose its stability under such disturbances. Such conditions can be created in two ways:

- During the operation of the system with a very weak AC network (very low ESCR) and/or operating the system with very small stability margin, changing one of the system reference values suddenly (e.g. step change of current order),
- Reducing the effective short-circuit ratio to a very low level by an AC network switching (switched reduction of ESCR).

In this study, for the switched reduction of ESCR, parallel AC transmission lines with switching capability are used. In order to obtain the different operating points, the nonlinear system equations (17) are solved at five different operating conditions. All of the required values, e.g. jacobian submatrices, dynamic state matrices, feedback matrices \( F_i = [F_{pi}\ F_{bi}], i=1,2,\ldots,5 \), are calculated off-line based on the previous section’s theories. There are many options to assign membership functions. For the sake of convenience in computation, triangular functions are selected as membership functions; some of which are depicted in Fig.3.

![Fig. 3 Some selected membership functions](image-url)
To account for the parameter perturbation and approximation error caused by linearization, $\Delta A_i (i=1,\ldots,5)$ equal to 0.15% times the corresponding $A_i$ is chosen. The vector control $u$ is shown in (31).

$$u = a_1 \int_{0}^{s} \left( F_{P1} y + F_{I1} \right) \, dr + a_2 \int_{0}^{s} \left( F_{P2} y + F_{I2} \right) \, dr + a_3 \int_{0}^{s} \left( F_{P3} y + F_{I3} \right) \, dr$$

(31)

The block diagram of the proposed controller is shown in Fig.4. The FFC block in Fig. 4 is a fuzzy system that adjusts the $\alpha_1,\ldots,\alpha_5$ constants based on the input state vector, $X$.

![Fig. 4 Block diagram of proposed robust nonlinear fuzzy controller](image_url)

The proposed and conventional controllers were simulated under different operating conditions and the results were compared. The comparison shows that for small disturbance and strong AC network, the performance of the proposed controller may not be much better than the conventional one. However, with a very weak AC/DC system (SCR<2), the proposed controller performs better and the system response is significantly improved after a sudden change in current order and/or a switched reduction in ESCR. In contrast, similar HVDC-system operation with a conventional controller brings the system into an unstable region. The behavior of the system during a 10% reduction of ESCR with conventional and proposed controller is shown in Fig. 5 and Fig. 6, respectively.

Regardless of first overshoot, the system becomes stable after about 0.4 second with the proposed controller. The existence of some oscillations in the waveforms is normal. This is due to operating the system with very weak AC systems. In this situation, the capability of system to retain a sinusoidal AC voltage is reduced and the harmonic distortion is increased.

VI. CONCLUSIONS

The design and stability of a new robust nonlinear fuzzy controller for HVDC systems has been studied in this paper. A new simplified nonlinear dynamic model has been developed for HVDC systems that can be used to design the controller. The proposed model decomposes into several linear systems around its important equilibrium points. The introduction of parameter uncertainties in the T-S fuzzy model can account for both approximation error of fuzzy model and actual parameter uncertainty in the HVDC system. Simulation results show that the transient stability can be improved by using proposed controller when large faults appear in the system. Also, the stable behavior of a very weak AC/DC system (SCR<2) with proposed controller is very significant when a sudden change in current order is applied. The same situation with conventional control brings the system into unstable region.

![Fig. 5 System performance with conventional controllers during a 10% reduction of ESCR](image_url)
VII. REFERENCES


VIII. BIOGRAPHIES

Hamid Reza Najafi was born in Mashad, Iran, on July 10, 1960. He obtained the degree, B.Sc and M.S (Eng) from University of Ferdowsi, Iran in 1981 and 1991, respectively and PhD from IUST, Iran in 2004. He has been a lecturer at Faculty of Engineering, University of Birjand since 1993, and he works currently as an assistant professor in Electric Power Group, at university of Birjand, Iran. His special fields of interest include: Power System modeling, HVDC and FACTS, reliability of power system, stability control of power system, application of neural networks and fuzzy system in power system.

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