Condensed Transmission Line Model in EMTP-Type Program – Use of Complex Elements

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Abstract - This paper presents a method for condensing modal domain transmission line models based on matrix transformation. The proposed model takes advantage of some similarities between non-homopolar modes, applying orthogonal vectors and complex number theory to manipulate the state vectors in modal domain, reducing the vector dimension. This method is applied to a single three-phase system (500 kV line).

The resources used led to a reduction both in the number of operations per iteration and state allocation, which optimize the procedure and increase the processing simulation speed.

Keywords: Transmission Lines, Simulation Tools, Switching Surges, Solution Methods, Complex Numbers, Orthogonal Vectors.

I. Introduction

In this paper a method for compacting modal domain transmission line models is proposed. The proposed model takes advantage of some similarities between non homopolar modes, applying orthogonal vectors and complex number theory to manipulate the state vectors in modal domain reducing the vectors dimension.

The distributed parameter model [1] was the representation chosen to implement the Condensed Model, although it does not take into account the frequency dependence of longitudinal parameters. Aside from this approximation, due to its simplicity, it is still one of the most used transmission line representation in electromagnetic studies. The proposed methodology can be extended to more complete models, which properly represent frequency dependence of longitudinal parameters in modal domain.

This procedure is applied to a single three-phase 500 kV line assumed ideally transposed (that is to say, the length of the transposed section is much shorter than that of the wavelength of frequencies involved in the study).

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The above-mentioned resources and procedures, as well as their advantages (reduction of state vectors, and an increase in processing speed, which leads to a processing time reduction) are the contributions of this paper to line modeling.

II. Condensed Transmission Line Model

The transmission line modeling proposed is applied to ideally transposed transmission lines and lines with a vertical symmetry plane [2]. In these special cases the mode-to-phase transformation allows line representation through decoupled modal circuits that have the following characteristics: a well-defined homopolar mode and two modes with equal or very similar eigenvalues, which result in propagation characteristics identical or almost identical. There is an infinity linear combination of non homopolar modes or quasi-modes that could be used. For lines with a vertical symmetry plane, Clarke β-mode is an exact mode and Clarke α and homopolar components can be treated as quasi-modes, namely α quasi-mode and homopolar quasi-mode [2]. For this application Clarke transformation T_{Clk} [3] was adopted because it is a real transformation matrix, frequency independent and well adjusted to time domain programs.

As the non-homopolar propagation modes have similar (or equal) behavior, the objective is to use a single circuit to represent both non-homopolar modes. A research of a more condensed way to represent the state signal propagation for non-homopolar components (α and β) through a single circuit is the main purpose of this present work. The numerical solution of the differential equations related to non-homopolar propagation modes must process at the same time both signals (which can be different) related to these modes, recovering each one in the final processing without mixing. The following sections will present the basic idea implemented in a simple *RL* circuit, and the development required to solve modal equations of transmission lines.

II.1 Basic Concept of Model

A simple series *RL* circuit was considered in the initial study. Two voltage sources with distinct frequency and signal amplitude were applied to a single circuit. The model should give as output two current signals not mixed.

The voltage signals were converted into complex quantity through a Condenser Device and injected into a single circuit, Fig. 1. The purpose was to obtain a numeric processing of the differential circuit equations (1) – (2) without having data loss or data mix. For the initial test the sinusoidal sources were used and configured to have different angular frequency

$\omega_2 = 2\omega_1$ and different amplitude $E_2 = 2E_1$. The sources are respectively:

$S_1 = E_1 \cos(\omega_1 t + \phi_1)$

$S_2 = E_2 \cos(\omega_2 t + \phi_2)$

being $v_{comp}(t) = \Re\{S_1\} + j\Re\{S_2\}$.

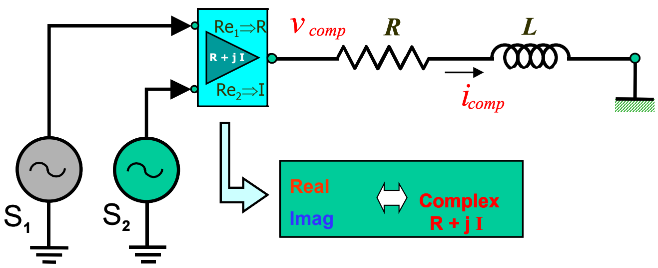

Figure 1. Initial test-circuit.

The differential equation for the circuit is written in terms of the current.

$$R\, i_{comp}(t) + L\frac{d i_{comp}(t)}{dt} = v_{comp}(t) \qquad (1)$$

Rewriting (1) in state equation form:

$$\dot{i}_{comp}(t) = -\frac{R}{L} i_{comp}(t) + \frac{1}{L} v_{comp}(t) \qquad (2)$$

and making $a = -\frac{R}{L}$, $b = \frac{1}{L}$, $x = i_{comp}(t)$ and $u = v_{comp}(t)$, we have

$$\dot{x} = ax + bu \qquad (3)$$

with nil initial conditions, $x(0) = x_0 = 0$.

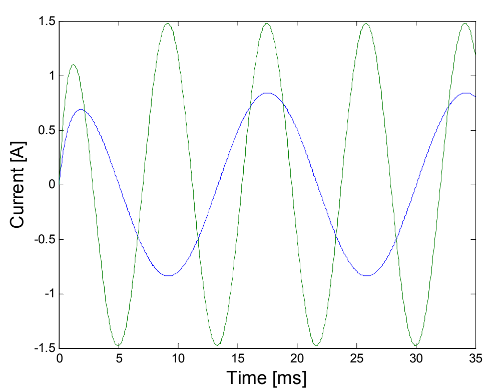

Figure 2. *RL*-circuit response (inductor current).

After applying trapezoidal integration in the solution of the differential equation for the *RL* circuit, together with handling injected signals through complex numbers, there were no signal distortions in the circuit response during the simulation process, Fig. 2. The two current signals were properly obtained and could be separated at anytime.

The operations carried out by means of complex quantity manipulation were accomplished without affecting the solution of the first-order differential equation of the test circuit. The same approach will be implemented to a modified transmission line model.

II.2 Expanding the New Concept to Transmission Line Equations

The application of the proposed methodology to transmission line's non homopolar modes will use the same approach. It is necessary to condense the two non homopolar voltages and currents in one single voltage and current signal and inject them in the single non homopolar circuit.

To perform the Condensation, a real phase-mode transformation matrix was applied in order to avoid any improper operation of the condensed vectors with respect to the transmission line theory. In the present research, Clarke transformation T_{Clk} [3] was adopted because it is a real transformation matrix and well adjusted to three-phase studies. After identifying the phase-mode transformation matrix it was necessary to define a matrix M, a non-quadratic matrix, composed of some complex elements that will condense the non-homopolar modes. Pre-multiplying the matrix T_{Clk} by the above mentioned M matrix leads to a new transformation matrix T_{mod} whose dimension is not square.

The model schematic diagram is presented in Fig. 3.

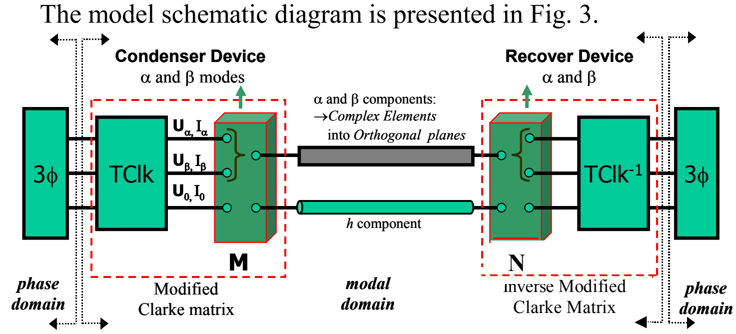

Figure 3. Condensed Transmission Line Model.

The final transformation matrix must have the 2 x 3 dimension for a three-phase system, as it has the objective of condensing the modal vectors from 3 x 1 dimension to 2 x 1. This condenser device performs α- and β-components compaction (related to non-homopolar modes) in a single complex element.

The modal components α and β are manipulated through orthogonal unitary vectors V_1 and V_2, resulting in a complex signal (this is the condenser device). The orthogonal axis can have an arbitrary angle θ related to a positive real axis (there is no restriction to θ) as depicted in Fig. 4.

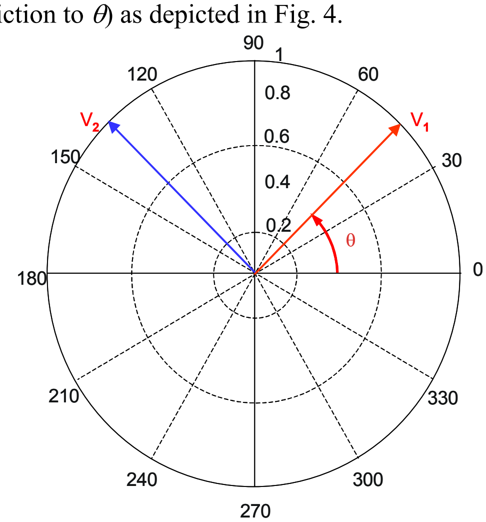

Figure 4. Unitary orthogonal vectors.

The matrix M composition is based on the chosen orthogonal vectors and is structured as follows:

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & V_1 & V_2 \end{bmatrix} \quad (4)$$

where

$$V_1 = \cos\theta + j\sin\theta \quad (5)$$

$$V_2 = \cos(\theta + 90^\circ) + j\sin(\theta + 90^\circ) = -\sin\theta + j\cos\theta \quad (6)$$

Which results in a variable transformation given by:

$$V^m = \underbrace{M\ T_{Clk}^{-1}}_{T_{mod}^{-1}}\ V^p = T_{mod}^{-1}\ V^p \quad (7)$$

The new modified transformation matrix has the following structure:

$$\left(T_{mod}^{-1}\right)^t = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & \sqrt{2}\,V_1 \\ 1 & (-\frac{1}{\sqrt{2}}V_1 + \frac{\sqrt{3}}{\sqrt{2}}V_2) \\ 1 & (-\frac{1}{\sqrt{2}}V_1 - \frac{\sqrt{3}}{\sqrt{2}}V_2) \end{bmatrix} \quad (8)$$

Applying the new transformation matrix, the modal vector now has a 2 x 1 dimension. The vector has a real element V_h^m representing the homopolar component and a complex element $V_{R,I}^m$ representing the multiplexed non-homopolar components, where:

$$\begin{bmatrix} V_h^m \\ V_{R,I}^m \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} c_1 \\ (c_2\cos\theta - c_3\sin\theta) + j(c_2\sin\theta + c_3\cos\theta) \end{bmatrix} \quad (9)$$

being

$$c_1 = V_a + V_b + V_c$$

$$c_2 = \sqrt{2}\,V_a - \frac{1}{\sqrt{2}}(V_b + V_c)$$

$$c_3 = \frac{\sqrt{3}}{\sqrt{2}}(V_b - V_c)$$

where V_a, V_b and V_c are scalar quantities in phase domain.

In order to return to phase components it is necessary to apply the transformation T_{mod}, where N is the pseudo-inverse matrix [4] of the matrix M (4) and (7). There comes

$$V^p = \underbrace{T_{Clk}\ N}_{T_{mod}}\ V^p = T_{mod}\ V^m \quad (10)$$

And matrix N can be calculated as in (11) .

$$N = \overline{M} \cdot (M \cdot \overline{M})^{-1} \quad (11)$$

where $\overline{M}$ is matrix M transposed conjugate.

$$N = \begin{bmatrix} 1 & 0 \\ 0 & \frac{\overline{V}_1}{2} \\ 0 & \frac{\overline{V}_2}{2} \end{bmatrix} \quad (12)$$

Matrix N is presented in (12) and the transformation matrix T_{mod} in (13) . The terms $\overline{V}_1$ and $\overline{V}_2$ correspond to the complex conjugate of vectors V_1 and V_2 , respectively.

$$T_{mod} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & \frac{\sqrt{2}\,\overline{V}_1}{2} \\ 1 & -\frac{\overline{V}_1}{2\sqrt{2}} + \frac{\sqrt{3}\,\overline{V}_2}{2\sqrt{2}} \\ 1 & -\frac{\overline{V}_1}{2\sqrt{2}} - \frac{\sqrt{3}\,\overline{V}_2}{2\sqrt{2}} \end{bmatrix} \quad (13)$$

The complex number theory applied to reduce the state vectors at line terminals was utilized because it allowed the multiplexed processing of different signals through a single circuit. By this means orthogonal unitary vectors were applied to propagate the states without mixing voltage and current α and β components.

Two distinct modal circuits were used, one for the homopolar propagation mode and another for both non-homopolar propagation modes. In this last circuit, multiplexed signals of manipulated components α and β were injected. Each component magnitude is recovered at the process end without signal mixing, because α and β parcels are related to orthogonal planes. Therefore, the performed operations (trapezoidal integration and complex number manipulation) with reduced state vectors do not numerically compromise the signal integrity.

A non-square transformation matrix, T_{mod} , as well as its pseudo-inverse, were used in the operations of similarity transformation to compose the modal vector and afterwards the return to the phase domain.

It is important to reinforce that the proposed procedure can be applied to different transmission line methodologies that properly represent the frequency dependence of unitary parameter in modal domain. However, in the case study of the following section, the proposed model was applied to the distributed parameter model (a simple formulation but widely known and used). We notice that it is assumed that the phase-mode transformation matrix T_{Clk} of the Condensed Line Model is composed of real elements. However, this condition is not strictly necessary, and some more general approaches can be used, with adequate modification of algorithms. At least when Clarke's quasi-modes are enough accurate, it does not appear necessary to use more general approaches.

III. CASE STUDY

In order to present a practical application of the proposed model, the structure of the Condensed Transmission Line Model was inserted into the distributed-parameter line model.

III.1 System Description

The simulated system consisted of a 500 kV single three-phase transmission line with the following characteristics: fundamental frequency 60 Hz, 300 km long. The line is assumed transposed with 70 % reactive shunt compensation. All data of the system case, placed on Table I, refer to the high voltage side. The data related to the transmission line (the 60 Hz derived parameters) are on Table II. The single-line diagram of the test system (with source, short-circuit equivalent impedance, transformer, circuit breaker, transmission line and shunt reactor compensation) is presented in Fig. 5.

Table I - System Data Simulation

Source	***Transformer***	***Shunt reactor***
S_{sc} (rms) = 500 kV Req_{sc} = 4.189 Ω Xd_{sc} = 143.911 Ω	$R\,tr$ = 1.507 Ω X_{tr} = 150.74 Ω	X_{sr} = 935.91 Ω

Table II - Basic Line Unitary Parameters

components	***longitudinal*** (Ω/km)	***transversal*** (μS/km)
non homopolar	$0.0244 + j\,0.3219$	$j\,5.088$
homopolar	$0.3221 + j\,1.352$	$j\,2.78$

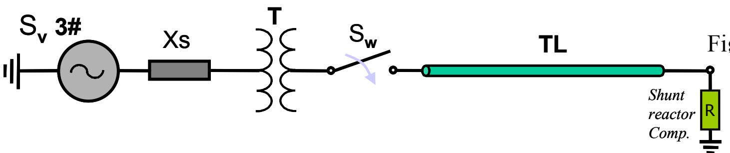

Figure 5. Energization – Single-line diagram.

Several cases were simulated to observe the Model behavior in a system with transmission lines which have distinct terminal conditions of α and β modes. The cases are:

a) Line energization with all switch-breaker poles closed at the same instant $t = 0$.
b) Line energization with all switch-breaker poles closed at the same instant $t = 0$, supposing a single phase to ground fault at the receiving end.
c) Line energization with all switch-breaker poles closed at the same instant $t = 0$, supposing a two-phase fault (not involving ground) at the receiving end.
d) Line energization with all switch-breaker poles closed at the same instant $t = 0$, supposing a two-phase to ground fault at the receiving end.

The results obtained in the simulation (a) are presented in Figs. 6 and 7. The simulation was repeated for different values of θ angle (used in the structure of the modified modal decomposition matrix), always leading to the same result.

The results obtained in the simulation (b) are presented in Figs. 8 and 9. In this case the homopolar mode appears after the non homopolar traveling waves reach the single phase-to-ground fault at the line end. After one travel time all modes are present in the transient. The Condensed Model represented properly the phenomenon.

In Figs. 10 and 11 the results for simulation (c) are presented. In this case the two non homopolar modes appear after the first reflection at the two phase fault (not involving ground) at the line end.

In Figs. 12 and 13 the results for the simulation (d) are presented. In this case all modes appear after the first reflection at the two phase to ground fault at the line end.

In all cases simulated the model represented properly the transient phenomenon, propagating each mode wave correctly in its circuit, an actual one for the homopolar mode and a multiplexed one for non homopolar modes.

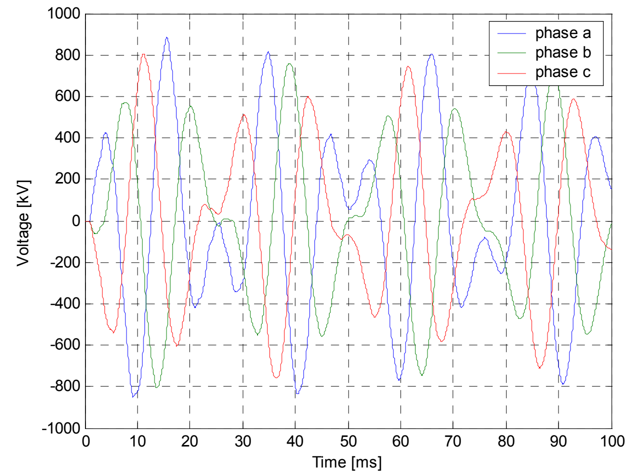

Figure 6. Successful line energization - Voltage at receiving end.

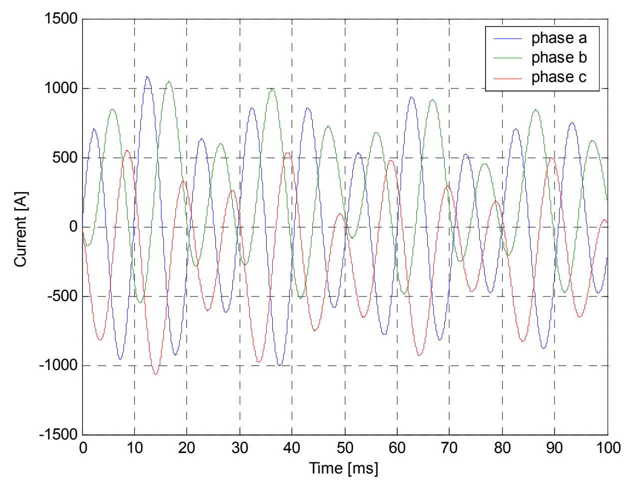

Figure 7. Successful line energization - Current at sending end.

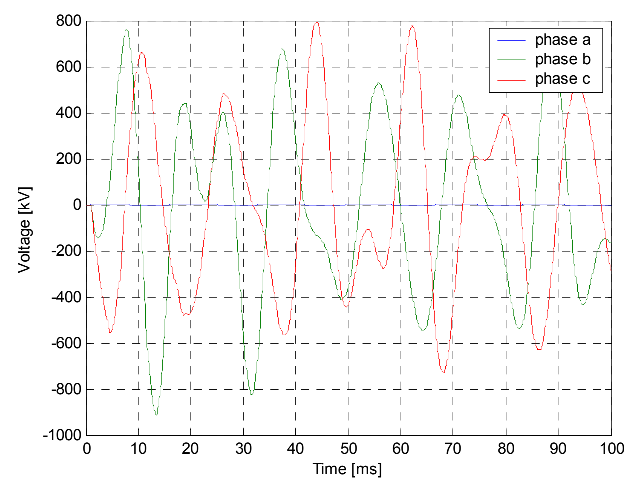

Figure 8. Line energization with single phase to ground fault (Ag) at receiving end - Voltage at receiving end.

III.2 Model Performance Analysis

The distributed parameter model and the Condensed Model were implemented in the same platform (Matlab®), in order to perform process time comparison. The simulation time for all the cases was 250 ms and the time step 50 μs.

The relation defined as Condensed Model processing time over Original EMTP Distributed Transmission Line Model execution time was 0.76, resulting in a gain of approximately 24.2 % in the TL processing time. A total time simulation reduction was obtained, as expected, based on the reduction of approximately ¼ of the total number of operations related to the transmission line. The total time reduction was 7.7 % considering the specific case simulated. The time reduction shall be more significant in actual system studies where more than just one transmission line is present, as is the case of the case study.

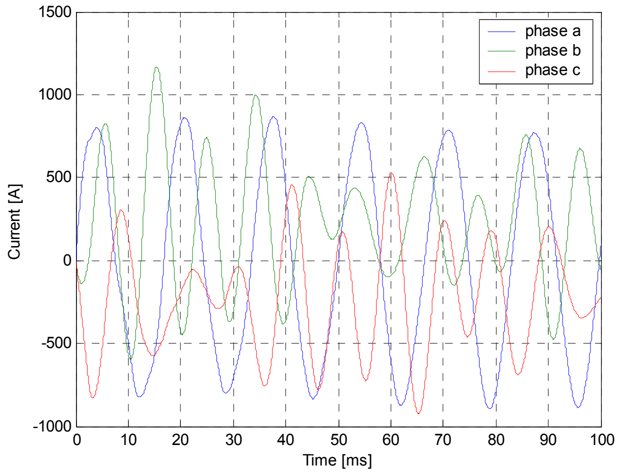

Figure 9. Line energization with single phase to ground fault (Ag) at receiving end - Current at sending end.

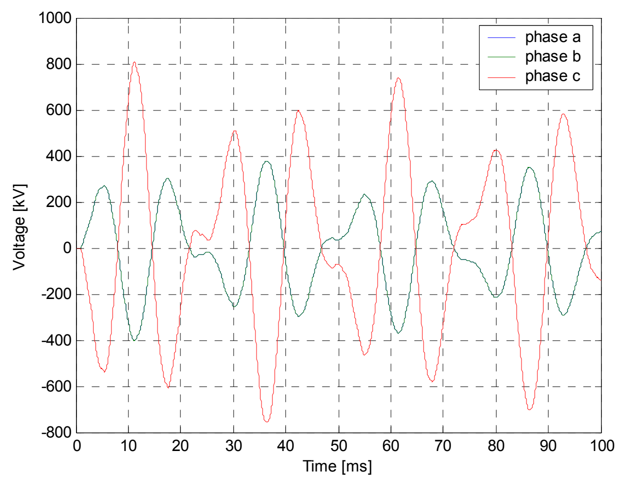

Figure 10. Line energization with two-phase fault (AB) at receiving end - Voltage at receiving end.

The simulator built for this study is quite simple, as the objective was not the development of simulation software, but the implementation of the proposed methodology. The example is also simple enough to demonstrate the computational processing time saving. The proposed procedure can be applied to transmission lines with proper representation of frequency dependence longitudinal unitary parameters. The lines can have distinct switching conditions represented in phase domain, which will result in distinct switching conditions of α and β modes. As shown on the simulated cases, the model can be properly applied to transmission lines, which have distinct terminal conditions of α and β modes.

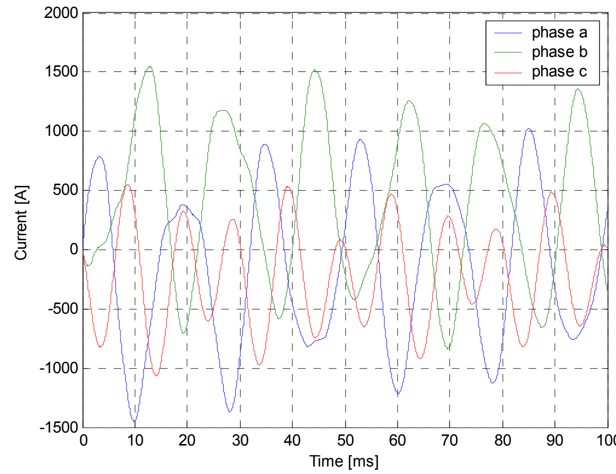

Figure 11. Line energization with two-phase fault (AB) at receiving end - Current at sending end.

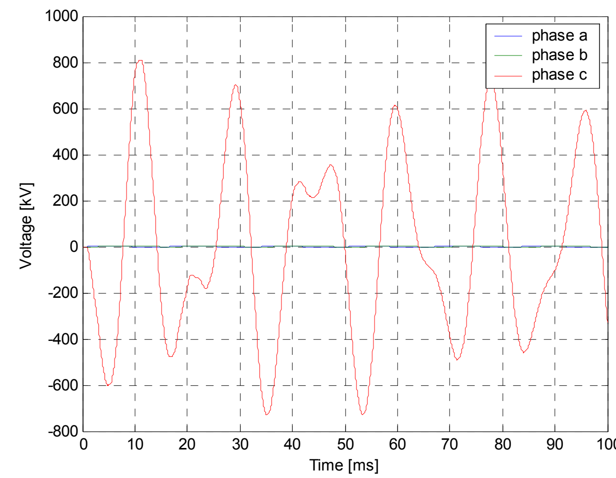

Figure 12. Line energization with two-phase to ground fault (ABg) at receiving end - Voltage at receiving end.

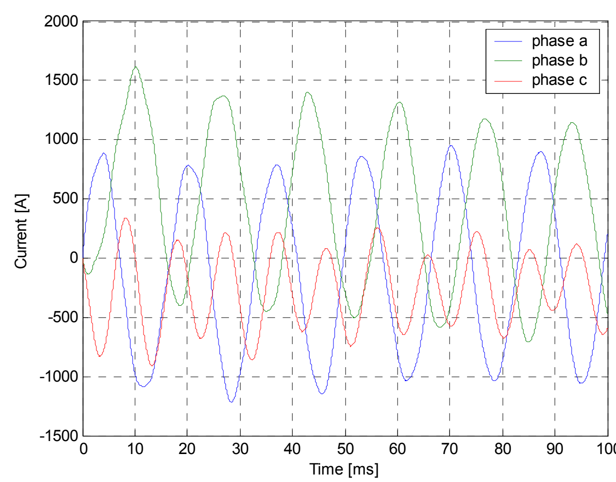

Figure 13. Line energization with two-phase to ground fault (ABg) at receiving end - Current at sending end.

IV. Conclusions

The paper describes a method for compacting modal domain transmission line models in EMTP-type programs. The proposed model takes advantage of some similarities between non homopolar modes, applying orthogonal vectors and complex number theory to manipulate the state vectors in modal domain, reducing the vectors dimension.

The proposed methodology can be applied to any model that works in the modal domain (whether using Clarke's transformation or even to any relationship of similarity that leads to the separation of the propagation modes). We notice that it is assumed that the phase-mode transformation matrix T_{Clk} of the Condensed Line Model is composed of real elements. However, this condition is not strictly necessary, and some more general approaches can be used, with adequate modification of algorithms. At least when Clarke's quasi-modes are enough accurate, it does not appear necessary to use more general approaches.

The use of non-square modal decomposition matrix leads to the application of its pseudo-inverse in the operations of similarity transformation, to compose the modal vector and afterwards, the return to the phase domain.

In the paper the model was applied to the worldly known distributed parameter model. As stated, the Condensed Model is applicable to any modal domain transmission line model, which considers properly the longitudinal parameter frequency dependence.

The use of the proposed procedure is pertinent in cases when the non-homopolar modes are equal (ideally transposed lines) or so similar (non-transposed lines with vertical symmetry plane) that they can be considered identical in specific studies, in which the error of this approximation is acceptable.

The Condensed Model can be applied to transmission lines with distinct terminal conditions of α and β modes, as presented, and distinct switching conditions of α and β modes.

The Condensed Transmission Line Model can be implemented in the code of EMTP-programs. This alternative resource provides a reduction in the number of operations related to transmission lines carried out by iteration. This reduction in processing time is relevant and the methodology can be applied to real time simulation tools.

V. References

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VI. Biographies

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J. Pissolato was born in Campinas, São Paulo, Brazil, in 1951. He received a PhD. degree in electrical engineering from Université Paul Sabatier, France, 1986. Since 1979 he has been at the School of Electrical and Computing Engineering of the University of Campinas (UNICAMP, Brazil) where he is presently a titular professor. Head of the High Voltage Laboratory of UNICAMP, and his main research interests are in high voltage engineering, electromagnetic transients and electromagnetic compatibility.

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