Amplitude, Phase and Frequency Estimation based on the Analytic Representation of Power System Signals

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Abstract—Power Quality problems have a significant impact in electrical power systems. Therefore, real-time techniques to detect and quantify these problems are getting much attention. This paper proposes a new approach based on the real wavelet transform of the analytic representation of distorted power signals. To generate the analytic signal, two methods are given. The first method is based on Fast Fourier Transform, whereas the second is Finite Impulse Response filter based.

The general structure of a DSP/FPGA based system used for the prototyping of the measurement device system and the experimental results are included.

Keywords: analytic signal, discrete wavelet transform, Hilbert transform, power quality.

I. NOMENCLATURE

APF: Amplitude-Phase-Frequency
AS: Analytic Signal
DWT: Discrete Wavelet Transform
DSK: DSP Starter Kit
DSP: Digital Signal Processor
FFT: Fast Fourier Transform
FPGA: Field-Programmable Gate Array
HT: Hilbert Transform
PQ: Power Quality

II. INTRODUCTION

IN modern electrical energy systems, voltages and especially currents become less stationary and periodicity is often complete lost due to the large numbers of non-linear loads and generators in the grid. More in particular, power electronic based systems such as adjustable speed drives, power supplies for IT-equipment high-efficiency lighting and inverters in systems generating electricity from distributed renewable energy sources are many sources of disturbances, being likely to worsen the waveform distortion on the power system. Distortions encountered are, for instance, harmonics, rapid amplitude variations (flicker) and transients, all being elements of Power Quality (PQ) problems. This development is going to continue and has a significant impact on the electrical energy flow, voltage profile and power quality for both customers and electricity suppliers in coming years.

In this situation, accurate measurement techniques to detect different related PQ problems in distorted environment are developed. This paper proposes different methods based on Amplitude-Phase-Frequency (APF) estimators to define the energy derivative of distorted power system signals. Among many APF algorithms suggested in the literature, the Analytic Signal (AS) procedure with the Hilbert Transform (HT) operator along with its related techniques is chosen for power quality assessment. Special emphasis has been put on a novel technique based on the real wavelet transform of analytic representation of given real signals. The new approach focuses on the Discrete Wavelet Transform (DWT) of the analytic representation of the signal. The wavelet transform provides a local representation (both in time and frequency) of the given signal. Therefore, it is suitable for analyzing signals where time information is needed, as in, for example, the study of transient (disturbance) events in power quality measurement. This temporal information is explicitly available from the wavelet coefficients. Since the real wavelet transform of a complex signal yields complex wavelet coefficients, the amplitude and phase information provided in the analytic representation is preserved.

First, the AS procedure is outlined, the output of which is employed as a basis for estimation of different related power quality problems algorithm. Then, depending on the AS approximation method, two algorithms wavelet transform based are discussed.

III. ANALYTIC SIGNAL PROCEDURE

A. General Overview

An Analytic Signal *s(t)* is a complex signal created by taking a signal *x(t)* and then adding in quadrature its HT

$$\hat{x}(t) = H\{x(t)\} = \frac{1}{\pi t} * x(t) \qquad (1)$$

It is also called the pre-envelope of the real signal and is defined as follows

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$$s(t) = x(t) + j\,\hat{x}(t) = a\,e^{j\phi(t)} \tag{2}$$

where its modulus *a(t)* and phase derivative can serve as estimates for the amplitude envelope and instantaneous frequency of *x(t)*. Thus, the instantaneous amplitude, phase, frequency is given by

$$\begin{aligned} a &= \sqrt{x^2 + \hat{x}^2} = |s| \\ \phi &= \arctan\left(\frac{\hat{x}}{x}\right) = Arg(s) \\ \omega &= \frac{\dot{\hat{x}}x - \dot{x}\hat{x}}{x^2 + \hat{x}^2} = \mathrm{Im}\left(\frac{\dot{\hat{x}}}{x}\right) \end{aligned} \tag{3}$$

There are a number of possible APF algorithms (e.g. Energy Separation Algorithm [1], [2]) and operators $H\{\cdot\}$ for computing the imaginary part of the signal but according to [1], only AS procedures with an HT operator satisfy conditions

- amplitude continuity and differentiability
- phase independence of scaling and homogeneity
- harmonic correspondence

This puts the AS procedure in a special position among APF estimators. For a full discussion the reader is referred to [1]. As mentioned above, the analytic representation of the input signal is derived by means of the HT operator. The HT operator is a linear one, capable of tracking the amplitude envelope and the instantaneous frequency respectively and in discrete case can be approximated in several ways.

B. Approximation methods

By taking Fourier transform of both sides of (2)

$$S(\omega) = X(\omega) + j\left(-j\,\mathrm{sgn}(\omega)\,X(\omega)\right) \tag{4}$$

The first term is the Fourier transform of the signal *x(t)*, and the second term is the inverse HT. By rewriting, the well-known relation between spectra of a real signal and the associated AS results

$$S(\omega) = \begin{cases} 2X(\omega) & \text{if } \omega > 0 \\ 0 & \text{if } \omega < 0 \end{cases} \tag{5}$$

Therefore, for creating the AS, the signal spectrum should be restricted to positive frequencies. This can be accomplished by using a phase splitter [14]. First, the Fast Fourier Transform (FFT) of the input sequence is calculated. Then, the negative frequency components are suppressed, and an Inverse Fourier Transform (IFT) is performed. This algorithm is shown in Fig.1.

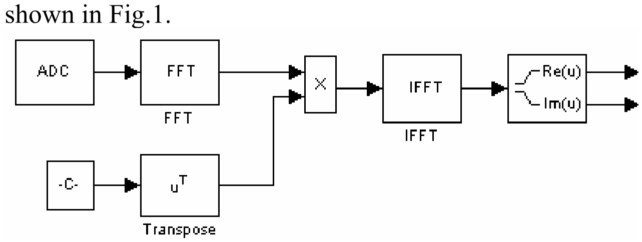

Fig. 1. Analytic Signal – Fast Fourier Transform approximation method

An alternative way is by using a Finite Impulse Response (FIR) filter. Such filter can be designed by the Park-McClellan algorithm, which uses the Remez exchange algorithm, and imposes a delay of N/2 on the input samples [13]. N is the length of the filter.

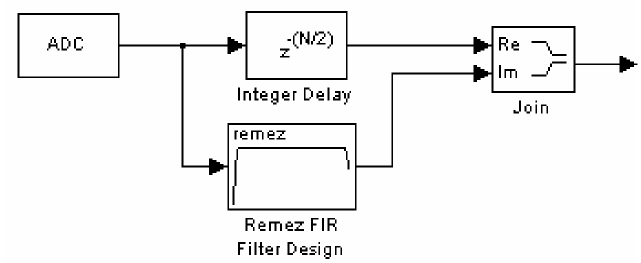

Fig. 2. Analytic Signal – Finite Impulse Response filter approximation method

To exemplify the above method, two input signals of measured voltage and current and their analytic representation are depicted in Fig. 3, Fig. 4, and Fig. 5 respectively.

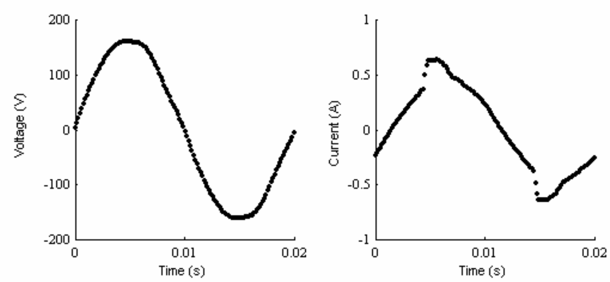

Fig. 3. Time domain representation of measured voltage and current

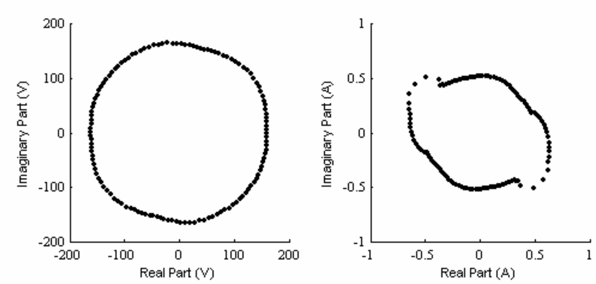

Fig. 4. Analytic representation of measured voltage and current FFT approximation method

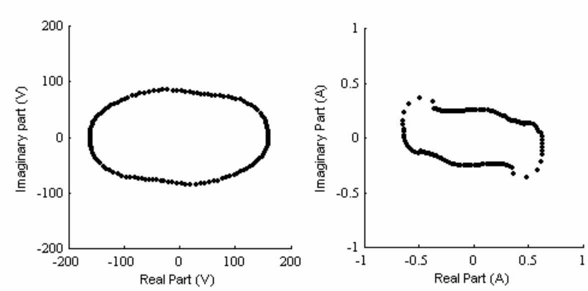

Fig. 5. Analytic representation of measured voltage and current FIR approximation method

As it can be seen from the above figures, the analytic representation of measured signals using those two approximation methods, FFT and FIR, respectively provides some disagreements. In order to evaluate these differences both implementations are tested.

IV. WAVELET-BASED MEASUREMENT TECHNIQUE

Detection of power quality events [3], [8]-[9] the measurement of the power flow [4]-[6], [10]-[11] these are just some applications in which the wavelet transform is given much consideration. The subject is covered by many publications and textbooks, therefore, the discussion of the wavelet transform is kept short and the reader must refer to e.g. [12] for an in-depth treatment.

A. Theory

The wavelet transform is a mathematical tool that analyses a given signal by convolution with a set of basis functions. These basis functions, or *wavelet* functions, are obtained from a *mother wavelet* by scaling and shifting operations. The mother wavelet is a function of zero average and is defined in discrete-time as:

$$\Psi_{m,n}\left(t\right) = 2^{-m/2}\,\Psi\left(\frac{t-n2^m}{2^m}\right) \qquad (6)$$

Thus, the dyadic-orthonormal (discrete) wavelet transform of x at the time n and scale m is given by:

$$DWT_x\left(m,n\right) = \left\langle x, \Psi_{m,n}\right\rangle = \int_{-\infty}^{\infty} x\left(t\right)\Psi^*_{m,n}\left(t\right)dt \qquad (7)$$

The wavelet transform balances between frequency and time resolution. In consequence, the low frequencies are covered with a high *frequency resolution* and a low *time resolution.* The opposite is true for high frequencies, where the event can be sharply localized in time, with an inherent uncertainty on the frequency. Therefore, as mentioned above, the wavelet transform is suitable for analyzing signals where time information is needed, as in, for example, the study of transient (disturbance) events.

Another interesting property is related to the orthonormality which implies, among other things, that the energy content of a signal is preserved through the wavelet transform:

$$\left\|x\right\|^2 = \left\langle x,x\right\rangle = \left\langle DWT_x, DWT_x\right\rangle = \left\|DWT_x\right\|^2 \qquad (8)$$

In the (real) wavelet transform this is done by selecting orthonormal wavelets, as i.e. the Daubechies wavelets.

B. Real wavelet transform of analytic signals

This section focuses on the DWT of the analytic representation of the signal *x(t)* which can be related to voltage or current. Since the (real) wavelet transform of a complex signal, yields complex wavelet coefficients, the amplitude and phase information provided in the analytic representation is preserved.

Let *s(t)* be the analytic representation of the signal *x(t)*. Then

$$DWT_s\left(m,n\right) = \left\langle s, \Psi_{m,n}\right\rangle = \left(A, \Phi\right) \qquad (9)$$

defines the DWT of the analytical representation of the signal at time n and scale m. Since this is a complex number, it is possible to denote A the amplitude and Φ the phase angle.

V. MEASUREMENT RESULTS

A. Measurement system architecture

After validation through simulation, all proposed methods for estimating the amplitude and frequency of signals are verified in an own-developed real-time measurement system [7]. Thanks to the rapid-prototyping setup of the system, this is a straightforward operation, which uses little time in the overall design process.

The algorithm is converted from MATLAB/Simulink to ‘C’-code using Real-Time Workshop. The code is then executed on a Texas Instruments ‘C6711 Digital Signal Processor (DSP). An FPGA daughtercard on top of the ‘C6711 DSK board provides an interface to the voltage and current measurements. The modules are chained so that they only occupy one FPGA board expansion slot. Optional PC performs monitoring, control and other activities. The test voltages are synthesized with a digital generator California Instruments 3001iX.

B. Experimental Results

In order to evaluate the above proposed techniques, the experimental study involves two parts. A first part focuses on detection and analysis of a signal consisting of a fundamental with a sudden notch while the second part of the study is related to the measurement of other different power quality characteristics.

An orthonormal Daubechies wavelet, with four detail levels and one approximation level is chosen. Such choice gives the opportunity of including the fundamental in the subband located at the lowest frequency.

1) Signal consisting of a fundamental with a sudden notch

The case introduced in Fig. 6 and Fig. 7 represents a 50 Hz signal with a spike. First, the results obtained by applying the wavelet transform of the analytic representation of measured voltage, with AS approximation via FFT, is highlighted. The sampling frequency is 6400 Hz and the number of samples per window is set at 128.

Fig. 6b shows the envelope amplitude estimated according to (3), whereas in Fig. 6c the wavelet transform is added as part of the algorithm. The results prove that both estimation algorithms perform well. However, the latter, who includes in the tracking process the wavelet transform, carried out more accurate results. On the other hand, the computational complexity is higher an issue which thanks to the latest DSP architecture becomes trivial.

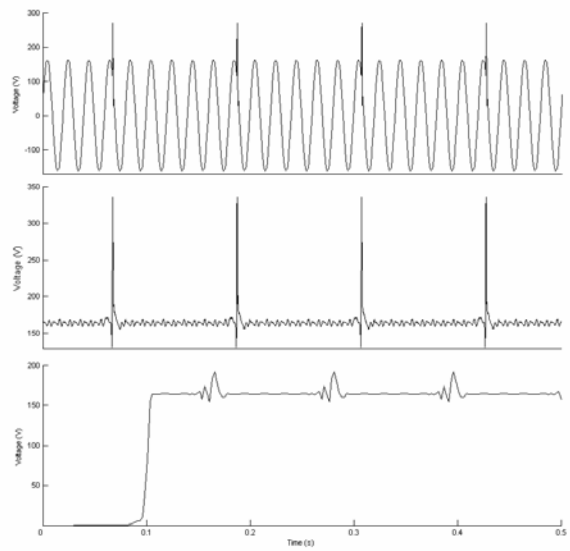

Fig. 6. Analytic representation of measured voltage FFT approximation method. (a) Signal with a disturbance. Envelope tracking using (b) AS-FFT. (c) Wavelet-AS-FFT (level 1).

As stated above, another advantageous property of adding the wavelet transform to the algorithm is related to the time localization. The idea is exemplified in Fig. 7, where DWT analysis is displayed.

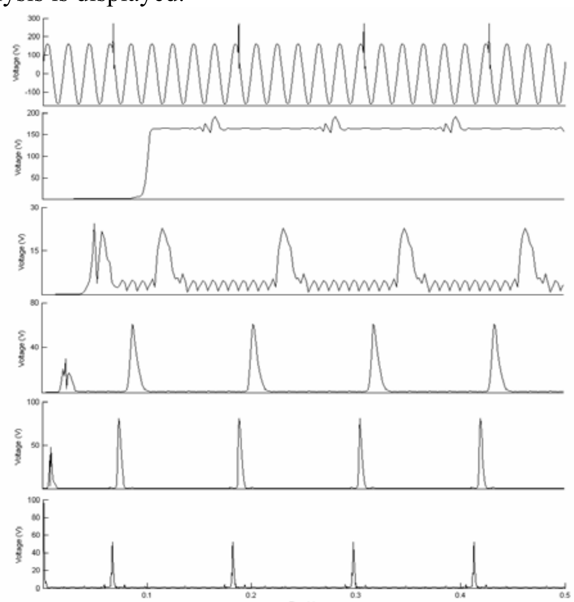

Fig. 7. Wavelet transform of analytic representation of measured voltage FFT approximation method. (a) Signal with a disturbance. DWT analysis (b) level 1 (lowest frequency). (c) level 2. (d) level 3. (e) level 4. (f) level 5 (highest frequency).

As mentioned above, besides creating the AS via FFT, an alternative way proposed is based on FIR filter. In the case study presented in Fig. 8, the envelope amplitude tracked by AS procedure is obtained for different sampling rates. The input signal is the same as the one applied in Fig. 6. The AS-FIR based algorithm performs well for lower sampling frequencies, as it can also be seen in Figure 8.

However, no matter of the sampling frequency used, the algorithm produces a delay, which is equivalent to the half of the filter length. The study uses a filter length of 60.

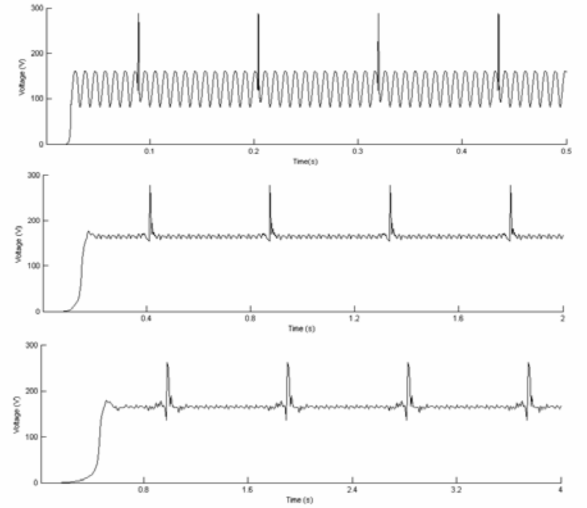

Fig. 8. Analytic representation of measured voltage FIR approximation method. Envelope tracking using (a) AS-FIR (F_s=6.4kHz). (b) AS-FIR (F_s=1.6kHz). (c) AS-FIR (F_s=800Hz)

2) Different PQ events

The techniques presented above are employed to track a signal with successive PQ events, like voltage sags and swells, and momentary interruptions. The sampling rate is fixed at 6400 Hz and 128-sample window length for FFT approximation method and 1600 Hz and 60-sample window length for FIR method.

The results when the wavelet transform of analytic representation of measured voltage (FFT approximation) is applied are presented in Fig. 9 and Fig. 10, respectively.

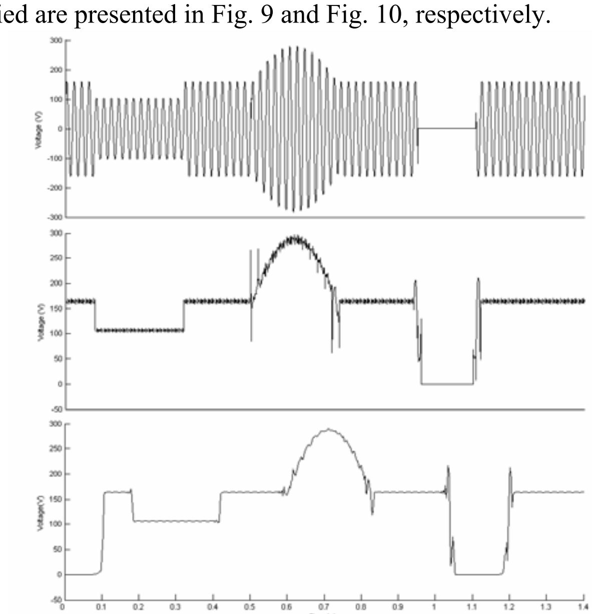

Fig. 9. Analytic representation of measured voltage FFT approximation method. (a) Signal with different PQ events. Envelope tracking using (b) AS-FFT. (c) Wavelet-AS-FFT (level 1-lowest frequency).

The scenario is the same as the one introduced in previous section. As it can be seen, Fig. 9 shows a signal with a succession of different PQ events. The envelope amplitude of the signal is tracked correctly by both methods, AS procedure (Fig. 9-b) and wavelet transform-AS procedure (Fig. 9-c). However, the latter gives again a more accurate result but introduces a very short delay. Fig. 10 shows the DWT analysis.

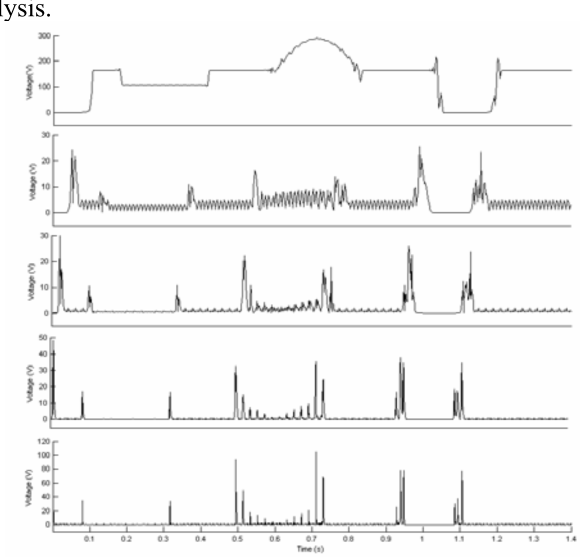

Fig. 10. Wavelet transform of analytic representation of measured voltage FFT approximation method for a signal with different PQ events. DWT analysis (a) level 1 (lowest frequency). (b) level 2. (c) level 3. (d) level 4. (e) level 5 (highest frequency).

As mentioned above, the AS-FIR method performs better at a lower sampling frequency.

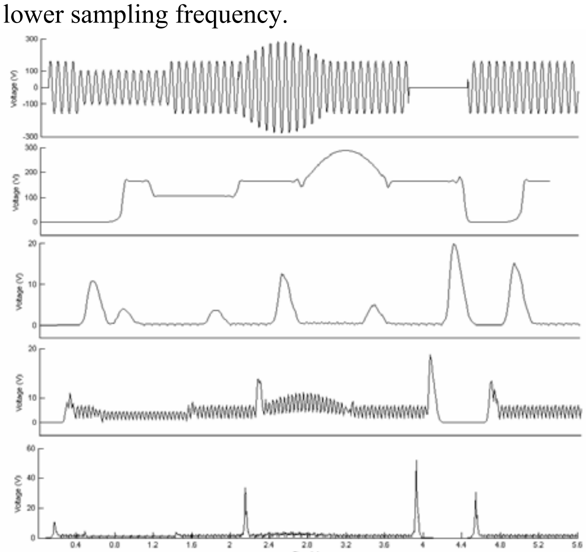

Fig. 11. Wavelet transform of analytic representation of measured voltage FIR approximation method for a signal with different PQ events. (a) Signal with different PQ events. F_s=1.6kHz. DWT analysis (b) level 1 (lowest frequency). (c) level 2. (d) level 3. (e) level 4.(highest frequency).

Although the sampling frequency used to analyze the signal in Fig. 11 by use of the FIR approach is eight times lower, it can be seen that the results are satisfactory. The analytical representation of the signal, seen in Fig. 11-b, yields the correct amplitude, which displays less of the high-frequency peaks which are apparent in the FFT approach. Because of the low sample frequency, only three levels of detail are used in the wavelet transform. This gives a further advantage to the computational efficiency, while retaining the accurate time-localization property of the highest level.

VI. Conclusions

A new method of analyzing power quality phenomena is presented. This method proposes to analyze the real wavelet transform of analytic signals, in order to get amplitude, phase and frequency information. In order to obtain an analytical representation of the input signals, two methods are possible. One is based on the property of non-negativity in the Fourier transform of analytical signals, and uses the Fourier transform. The other method uses the well-known Hilbert transform. Both methods are introduced and the results are compared. It can be concluded that the FFT method works better with high sampling frequencies, while the HT (FIR) method is more suited for lower sampling frequencies. The FFT method is less efficient in terms of computational complexity.

The experimental measurements show that the algorithms proposed are accurate, easy to implement and that the computational burden is not preventing the algorithms to run in real-time on a modern DSP based platform.

The results also prove that the proposed APF estimators are capable of detecting different power quality problems making them valuable tools in assessing power quality problems. In addition, the wavelet transform of analytic signals approach is also used in the measurement of the power flow.

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VIII. Biographies

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Ronnie Belmans received the M.S. degree Electrical Engineering in 1979 and the Ph.D. degree in 1984, both from the Katholieke Universiteit Leuven, Belgium, the Special Doctorate in 1989 and the Habilitierung in 1993, both from the RWTH, Aachen, Germany. Currently, he is a full professor with the K. U. Leuven. His research interests include variable speed drives, distributed power, power quality and renewable energy in grid. He is also performing research on the system aspect of the liberalization of the electricity market. Dr. Belmans is a fellow of the IEE (U.K.), senior member of IEEE and member of the Koninklijke Vlaamse Ingenieursvereniging (kVIV).