Transmission Line Analyses with a Single Real Transformation Matrix – Non-symmetrical and Non-transposed cases

A. J. Prado, *Member, IEEE*, J. Pissolato Filho, *Member, IEEE*, S. Kurokawa, *Member, IEEE*,
L. F. Bovolato, *nonmember*

***Abstract*— In transmission line transient analyses, a single real transformation matrix can be used to obtain exact modes when the analyzed line is transposed. Other interesting characteristics of this matrix transformation are: frequency independent, line parameter independent, identical for voltage and current determination. Using, for example, Clarke's matrix, some mathematical simplifications are obtained as well as a model which can be applied directly in programs based on time domain. This model does not need the convolution procedures to deal with phase-mode transformation. In EMTP programs, Clarke's matrix can be represented by ideal transformers. With this representation, the electrical values at any line point can be accessed for phase domain or mode domain using Clarke's matrix or its inverse matrix.**

For non-transposed lines, the results of Clarke's matrix application are not exact. In this paper, non-symmetrical and non-transposed three-phase line samples are analyzed with the proposed matrix application (Clarke's matrix). This matrix application is analyzed using comparisons between its results and the exact eigenvalues. The Z_C (characteristic impedance) and γ (propagation function) are also used to determine the accuracy of proposed matrix application. For these analyses, the considered frequency range is from 10 Hz to 1 MHz. Basing on the comparison errors, the accuracy of the proposed matrix application is investigated for some transmission line designs, determining the types where the proposed method can be applied.

Keywords: Clarke matrix, eigenvector, eigenvalue, frequency-time transformation, mode domain, transformation matrix, transmission lines.

I. Introduction

THE mathematical representations of transmission lines have some difficulties in time domain because the longitudinal parameters are frequency dependent [1, 2, 3]. Some models apply phase-mode transformation, considering the problem in mode domain and improving the frequency dependent parameter representation [4, 5, 6]. So, the transformation matrices are also frequency dependent, because the Z and Y line matrices depend on the frequency. Some real transformation matrices have been used as an alternative to these analyses. For ideally transposed lines, a single real transformation matrix can be used, obtaining exact modes and diagonal mode matrices [4]. For a single real transformation matrix based on line geometrical characteristics and the Clarke matrix, an interesting model is created to analyze transmission line transients [7]. This model is applied to the transmission lines, without convolution procedures to deal with phase-mode transformation, and it works with a single, real, frequency independent, line parameter independent transformation. Because of this, electrical values can be determined in phase domain or mode domain at any line point using a simple matrix multiplication. For symmetrical non-transposed lines, the results of a single real transformation matrix application (called quasi-modes) are not exact modes and the errors can be calculated.

Considering typical non-transposed three-phase lines, the eigenvector and eigenvalue analyses are presented, using the Clarke transformation matrix, to lines with and without a vertical symmetry plane [8, 9, 10]. The exact modes are compared with the Clarke matrix obtained results, showing these quasi-modes errors are reasonably small for 440 kV three-phase line samples [11]. Besides the comparisons between exact eigenvalues and quasi-modes, the curves of Z_C and γ values that depend on the frequency are also shown. These results present small differences between exact values and quasi-mode results. In future, it will be investigate what is the best way to implement the proposed model in eletromagmetic transient analyses using EMTP type programs.

II. Mathematical Development

When transmission lines are submitted to electrical transients, the relations among transversal voltages (u_F) and longitudinal currents (i_F) in phase domain, can be expressed by the following equations (Z being the per unit length longitudinal impedance matrix and, Y, the per unit length transversal admittance matrix):

$$-\frac{du_F}{dx} = Z \cdot i_F \qquad and \qquad -\frac{di_F}{dx} = Y \cdot u_F \qquad (1)$$

The modal transformation matrices are identified by T_V and T_i and they determine the per unit length longitudinal impedance matrix (Z_{MD}) and transversal admittance matrix (Y_{MD}) in mode domain. In most cases, the T_V and T_I

This work was supported by FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo).

Afonso José do Prado (afonsojp@dee.feis.unesp.br), Sérgio Kurokawa (kurokawa@dee.feis.unesp.br) and Luiz Fernando Bovolato (bovolato@dee.feis.unesp.br) are with the Department of Electrical Engineering at FEIS/UNESP – University of São Paulo State, 15385-000, Ilha Solteira (SP), Brazil. José Pissolato Filho (pisso@dsce.fee.unicamp.br) is with the Department of Electrical Engineering, DSCE/UNICAMP – State University of Campinas, 13081-970, Campinas (SP), Brazil.

Presented at the International Conference on Power Systems Transients (IPST'05) in Montreal, Canada on June 19-23, 2005.
Paper No. IPST05 – 055

transformation matrices are different and they have complex elements. These elements depend on the frequency.

If the modal transformation is applied, the equation set (1), in phase domain, will change into next equation set in mode domain (index MD):

$$-\frac{d\left(T_V^{-1}\cdot u_{MD}\right)}{dx} = Z\cdot T_I^{-1}\cdot i_{MD}$$
$$and \qquad (2)$$
$$-\frac{d\left(T_I^{-1}\cdot i_{MD}\right)}{dx} = Y\cdot T_V^{-1}\cdot u_{MD}$$

The Z_{MD} and Y_{MD} matrices are described by:

$$Z_{MD} = T_V\cdot Z\cdot T_I^{-1} \quad and \quad Y_{MD} = T_I\cdot Y\cdot T_V^{-1} \qquad (3)$$

The proposed model changes these matrices into the Clarke transformation (T_{CL}) for three-phase lines. So:

$$Z_{MDCL} = T_{CL}\cdot Z\cdot T_{CL}^{-1} \quad and \quad Y_{MDCL} = T_{CL}\cdot Y\cdot T_{CL}^{-1} \qquad (4)$$

The Z_C (characteristic impedance) values are described by:

$$Z_{CMD} = \sqrt{\frac{Z_{MD}}{Y_{MD}}} \quad and \quad Z_{CMDCL} = \sqrt{\frac{Z_{MDCL}}{Y_{MDCL}}} \qquad (5)$$

The γ (propagation function) values are described by:

$$\gamma_{MD} = \sqrt{Z_{MD}\cdot Y_{MD}} \quad and \quad \gamma_{MDCL} = \sqrt{Z_{MDCL}\cdot Y_{MDCL}} \qquad (6)$$

The exact eigenvalues are:

$$\lambda = T_V\cdot Z\cdot Y\cdot T_V^{-1} = T_I\cdot Y\cdot Z\cdot T_I^{-1} \qquad (7)$$

The quasi-mode results are:

$$\lambda_{CL} = T_{CL}\cdot Z\cdot Y\cdot T_{CL}^{-1} \quad or \quad \lambda_{CL} = T_{CL}\cdot Y\cdot Z\cdot T_{CL}^{-1} \qquad (8)$$

For the ideally transposed lines, the quasi-mode matrices (equations (4 and 8)) are diagonal. In case of non-transposed lines, these are assumed diagonal matrices and the non-diagonal elements can be neglected, if the obtained errors are non-significant. With this assumption, the proposed transformation is applied to time domain without the need of convolution procedures, because it is real, frequency independent, and identical for voltage and current matrices. For non-symmetrical and non-transposed lines, it does not lead to exact modes. This paper shows the error of such transformations as quasi-modes is reasonably small for eigenvalues analyses.

III. Symmetrical Case

It is considered a line whose conductors are disposed in such a way that there is a vertical symmetry plane and two external phases not centered in such plane (this two phases being symmetrically disposed), as shown in Figure 1.

Suppose the application of Clarke transformation matrix, T_{CL}, to both Z and Y matrices in phase domain, at a generic point of the line. These matrices have similar structures and the transformation results are also similar. In the assumption of an ideal line transposition, it is considered that a transposition cycle length is reasonably shorter than a quarter wavelength, for the dominant frequency range of phenomena to study. The assumption of ideal transposition can be applied to most usual conditions, because the errors are reasonably small.

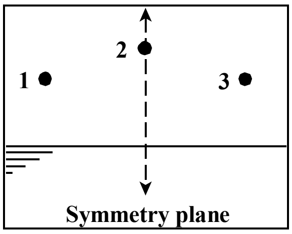

Fig. 1. Three-phase line symmetry plane.

The detailed discussion of this property is not within the scope of this paper. The Z and Y matrices are defined by:

$$Z = \begin{bmatrix} A & D & D \\ D & A & D \\ D & D & A \end{bmatrix} \quad and \quad Y = \begin{bmatrix} A' & D' & D' \\ D' & A' & D' \\ D' & D' & A' \end{bmatrix} \qquad (9)$$

Assuming that ground wires are considered implicitly, the Clarke transformation converts the three-phase line into three uncoupled mode circuits. The Z_{MDCL} matrix is:

$$Z_{MDCL} = Z_{MD} = \begin{bmatrix} A-D & 0 & 0 \\ 0 & A-D & 0 \\ 0 & 0 & A+2D \end{bmatrix} \qquad (10)$$

where A is the self phase impedance value and D is the coupling impedance value (average values). The Y_{MDCL} matrix has a similar structure to the Z_{MDCL} matrix.

Considering a non-transposed symmetrical line, the length longitudinal impedance matrix is described by:

$$Z_{NT} = \begin{bmatrix} B & G & H \\ G & F & G \\ H & G & B \end{bmatrix} \quad and \quad Y_{NT} = \begin{bmatrix} B' & G' & H' \\ G' & F' & G' \\ H' & G' & B' \end{bmatrix} \qquad (11)$$

The coupling impedance values are shown in Figure 2.

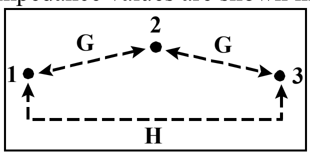

Fig. 2. Non-transposed symmetrical line coupling impedance values.

For this case, the Z_{MD} is described by:

$$Z_{MDCL} = \frac{1}{3}\cdot\begin{bmatrix} B+2F-4G+H & 0 & \sqrt{2}\cdot\left(F-B+G-H\right) \\ 0 & B-H & 0 \\ \sqrt{2}\cdot\left(F-B+G-H\right) & 0 & F+2B+4G+2H \end{bmatrix} \qquad (12)$$

The Y_{MDCL} matrix is similar to the Z_{MDCL} one. Considering T_I and T_V as the Z_{NT} eigenvector matrices, the eigenvalue matrix is calculated through equation (7). If the exact transformation matrices are changed into the Clarke transformation, the following will be obtained:

$$T_{CL}\cdot Y_{NT}\cdot Z_{NT}\cdot T_{CL}^{-1} = \lambda_{NTCL} \qquad (13)$$

The exact eigenvalues were compared with the results of equation (13), using the line structure shown in figure 3. The modulus ratio results are shown in Figure 4 where the greatest error is approximately 0.25 % for mode α about 10 kHz. The errors presented in Figure 4 are quite small and indicate that the Clarke transformation can be a good alternative for transient analyses of typical non–transposed three-phase lines with a symmetry plane in a range of frequency from 10 Hz to 1 MHz. In this comparison, only the main diagonal elements of λ and λ_{NTCL} matrices are used. These elements are identified as α, β and 0 modes or quasi-modes.

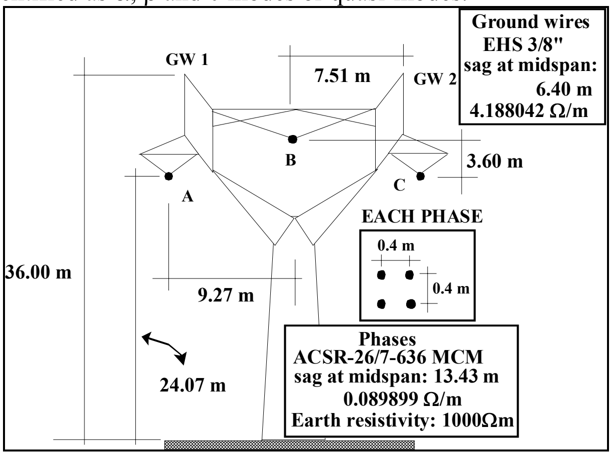

Fig. 3. Real three-phase line structure.

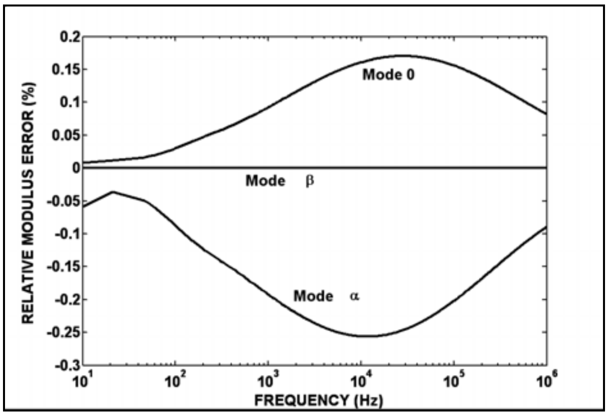

Fig. 4. Comparison between exact eigenvalues and quasi-mode results for symmetrical non-transposed three-phase line.

IV. Non-symmetrical Cases

Two designs are considered and shown in Figure 5. In the first case (Design I), the three phases are centered in the same vertical plane. The phase conductors are lined vertically and one or two ground wires can be used, depending on the typical tower structure of the analyzed system. The second case (Design II) does not present a vertical symmetry plane and the phase conductors are considered in a triangular distribution. The application of only one ground wire is more common to this case.

Considering the two designs shown in Figure 5, they can be represented by the same equations because the Z and Y matrices have similar structures for both cases. The Z and Y matrices can be described by equation (14).

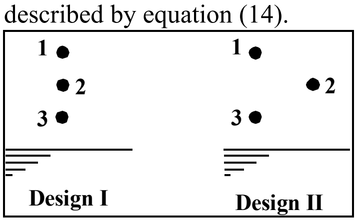

Fig. 5. Tree-phase line designs without vertical symmetry plane.

Representing the eigenvector matrices for these cases as T_I and T_V, the eigenvalue matrix is calculated by equation (7) and represented by λ.

$$Z_{NP} = \begin{bmatrix} P & S & T \\ S & Q & W \\ T & W & R \end{bmatrix} \quad and \quad Y_{NP} = \begin{bmatrix} P' & S' & T' \\ S' & Q' & W' \\ T' & W' & R' \end{bmatrix} \qquad (14)$$

The Z_{MDCL} matrix is described by:

$$Z_{MDCL} = \begin{bmatrix} Z\alpha & Z\alpha\beta & Z\alpha 0 \\ Z\alpha\beta & Z\beta & Z\beta 0 \\ Z\alpha 0 & Z\beta 0 & Z0 \end{bmatrix} \qquad (15)$$

If the exact transformation matrices are changed into the Clarke transformation, the following will be obtained:

$$T_{CL} \cdot Y_{NP} \cdot Z_{NP} \cdot T_{CL}^{-1} = \lambda_{NPCL} \qquad (16)$$

The exact eigenvalues (λ) were compared with the quasi–mode results of equation (16), using the line structures shown in Figure 5 and the modulus ratio values are shown in Figure 6 for design I and in Figure 7 for design II.

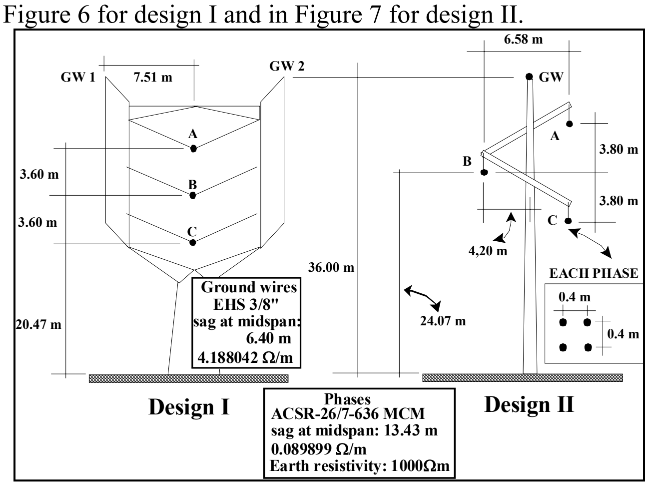

Fig. 5. Two designs for non-symmetrical three-phase line analyses.

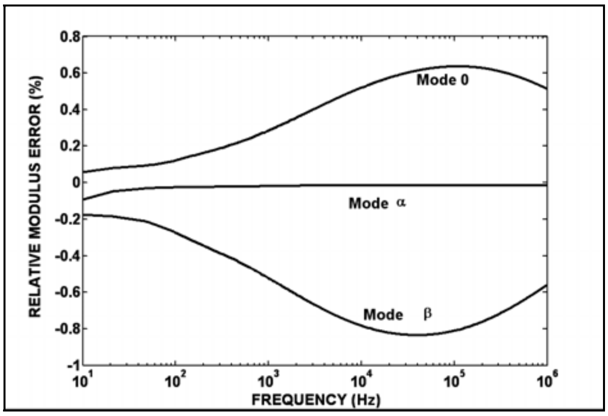

Fig. 6. Comparison between exact eigenvalues and quasi-mode results for design I (without vertical symmetry plane, non transposed case).

The results of Figure 6 demonstrate the Clarke transformation can also be a good alternative for transient analyses. These results present differences in modulus between the exact eigenvalues and the Clarke transformation results are lower than 0.9%, considering a wide frequency range from 10 Hz to 1 MHz. In this case, the highest difference, about 0.9%, is associated to mode β.

In Figure 7, the greatest errors are also associated to the mode β for each frequency value. The difference modulus is not higher than 1% in the same frequency range of Figure 6. Based on results of the last two figures, the Clarke matrix can be applied as a good approximation to the eigenvector matrices of non-transposed typical three-phase transmission lines without symmetry plane. The obtained quasi-modes present acceptable errors for exact eigenvalues.

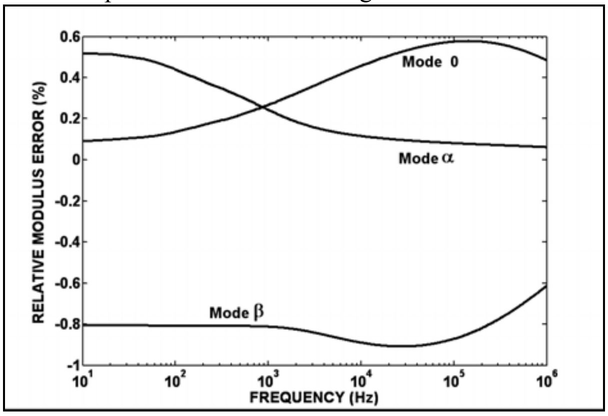

Fig. 7. Comparison between exact eigenvalues and quasi-mode results for design II (with the three phases centered in the same vertical plane, non transposed case).

The results related to designs I and II demonstrate that the eigenvector matrices can be change into the Clarke matrix (T_{CL}), because the errors are reasonably small. Considering electromagnetic transient simulations, adequate models should be investigated for application to these transmission line types. Depending on the applied on the applied model, the errors generated by Clarke transformation can be increase because the interaction between the model and the Clarke matrix. In the other hands, the elements of Clarke matrix can be used as initial values for calculation of eigenvectors and eigenvalues through Newton-Raphson methods. Synthetic circuits (modified π-circuits) have been checked in association to the Clarke transformation. These synthetic circuits represent frequency dependence of line longitudinal parameters using RL parallel branches. However, the effects of increase of the number of these branches are not studied hardly yet and for design II, some electromagnetic transient simulations lead inaccurate results. Probably, with the increase of number of RL parallel branches, better results can be got using the association between the Clarke transformation and the synthetic circuits.

For Figures 4, 6 and 7, the eigenvectors and the eigenvalues are calculated by a Newton-Raphson method using the Clarke matrix elements as initial values for the first frequency value of the considered range (10 Hz to 1 MHz). For each frequency range value, the equations (7, 13 and 16) are applied and the differences between quasi-modes and exact values are calculated.

V. The Z_C and γ Parameters

The Z_C (characteristic impedance) parameter has similar characteristics to the eigenvalue error analyses. Using the line structure of Figure 3, the modulus and the angle of Z_C parameter for each mode is calculated through equation (5). Figure 8 shows the Z_C modulus results for symmetrical line considering non-transposed case (Figures 2 and 3). Figure 9 shows the Z_C angles for the same case.

The next two figures demonstrated that the Z_C parameter present similar characteristics to the eigenvalue analyses. The exact Z_C parameters are coincident to the Clarke transformation ones for mode β. This result is similar to the result presents in Figure 4 where the errors for mode β are null. The results for modes α and 0 are also similar when Figures 4, 8 and 9 are compared.

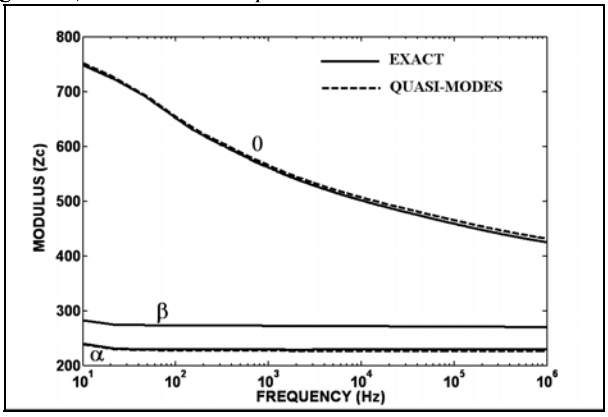

Fig. 8. The Z_C modulus – symmetrical line.

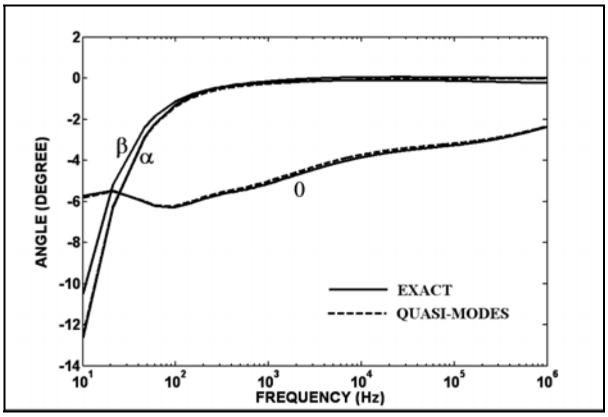

Fig. 9. The Z_C angle – symmetrical line.

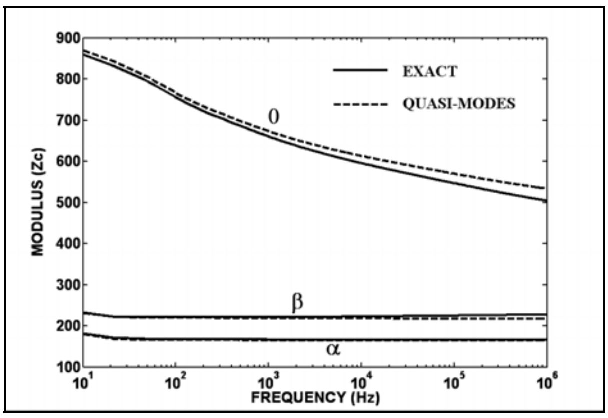

Fig. 10. The Z_C modulus – design II.

If the design II considering non-transposed case is used, the previous conclusions are confirmed. So, the Z_C results of this case, summarily presented in Figure 10, are similar to the eigenvalue analyses (see Figure 7).

The γ parameter analyses show different results to the eigenvalue error analyses. For all analyzed structure lines in this paper, the exact γ parameters are equal to the ones obtained with the Clarke transformation. The next three figures summarized these results.

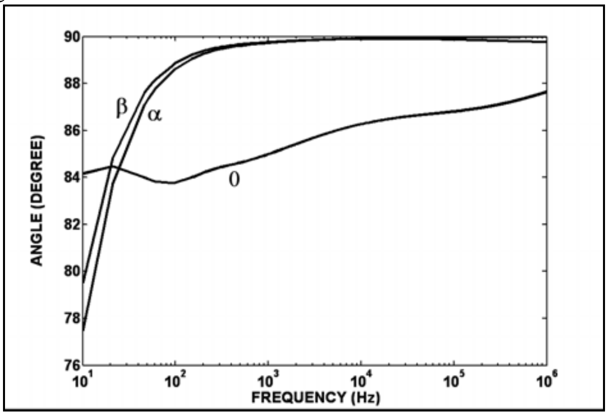

Fig. 11. The γ angle – symmetrical line.

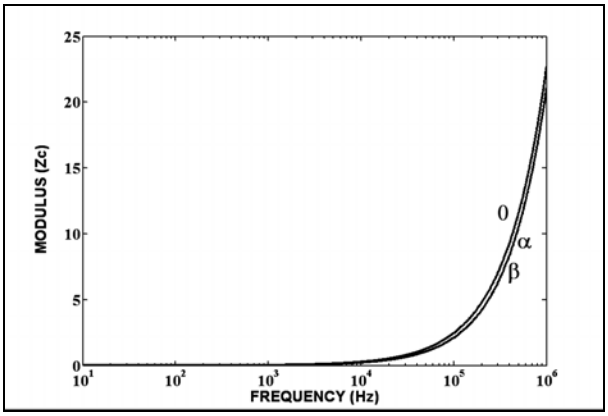

Fig. 12. The γ modulus – design II.

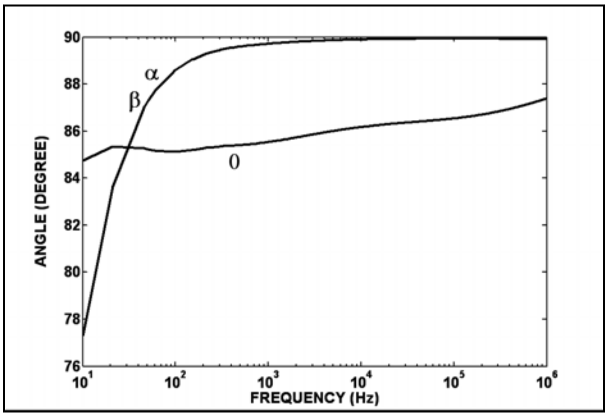

Fig. 13. The γ angle – design II.

The Z_C and γ analyses show different results when compared to the eigenvalue errors analyses. Applying the Clarke transformation associated to the Z_C and γ parameters, the errors are only related to the Z_C parameters. This characteristic could lead to more simple and more efficient error minimization routines, because the error are concentrated on only one parameter. The future development will check if it is possible.

VI. Conclusions

For typical three-phase transmission lines, eigenvector, eigenvalue, characteristic impedance (Z_C) and propagation function (γ) analyses were made in this paper, using a single real transformation matrix (the Clarke matrix). This allows the determination of electrical values, such as voltage and current, at any line point in phase domain or in mode domain with a simple matrix multiplication. For transposed lines, with or without symmetry plane, the Clarke matrix is an eigenvector matrix and the quasi-mode results are exact. The advantages of the Clarke matrix are: single, real, frequency independent. The Clarke matrix is not influenced by line parameters and it is identical to the voltage and the current. These all advantages can be considered in applications to typical non-transposed three-phase transmission lines without symmetry plane. In these cases, the errors between exact eigenvalues and quasi-modes are reasonably small. This is confirmed through λ error analyses in this paper.

The Z_C and γ parameter analyses show that the errors of the Clarke matrix application are only concentrated on the Z_C parameters. More simple and more efficient error minimization routines could be obtained using this relation between the proposed model errors and the Z_C parameters. Detailed analyses about the association between the single real transformation matrix and electromagnetic transient models can be performed based on the results of this paper. The objective of these detailed analyses will be to determine what associations can lead to the lowest errors for electromagnetic transient simulations.

VII. Appendix

The Clarke transformation matrix can be described by:

$$T_{CL} = \begin{bmatrix} -1/\sqrt{6} & 2/\sqrt{6} & -1/\sqrt{6} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ -1/\sqrt{3} & -1/\sqrt{3} & -1/\sqrt{3} \end{bmatrix} \qquad (A.1)$$

Each line of the T_{CL} matrix corresponds to one mode. The first line of the T_{CL} matrix is related to the mode α, the second line is associated to the mode β and the third line corresponds to the mode 0 (homopolar mode). The TCL matrix is applied to equations (4, 8, 13 and 16).

VIII. References

[1] H.W. Dommel, Electromagnetic Transients Program - Rule Book, Oregon, 1984.

[2] Microtran Power System Analysis Corporation, Transients Analysis Program Reference Manual, Vancouver, Canada, 1992.

[3] A. Morched, B. Gustavsen and M. Tartibi, "A Universal Model for Accurate Calculation of Electromagnetic Transients on Overhead Lines and Underground Cables", IEEE Trans. on Power Delivery, vol. 14, no. 3, pp. 1032-1038, July 1999.

[4] J.A. Brandão Faria and J. Briceño Mendez, "On the Modal Analysis of Asymmetrical Three-phase Transmission Lines using Standard Transformation Matrices", in 1997 IEEE Winter Meeting, 97WMIEEE-PES.

[5] L. M. Wedepohl. "Application of Matrix Methods to the Solution of Travelling-wave Phenomena in Polyphase Systems", Proceedings IEE, vol. 110, pp. 2200-2212. December, 1963.

[6] A. Semlyen, A. Deri, "Time Domain Modelling of Frequency Dependent Three-phase Transmission Line Impedance", IEEE Trans. on Power Apparatus and Systems, vol. 104, no. 06, pp. 1549-1555, June, 1985.

[7] E. Clarke, Circuit Analysis of AC Power Systems, vol. I, Wiley, New York, 1950

[8] L. M. Wedepohl, H. V. Nguyen, G. D. Irwin, "Frequency –dependent transformation matrices for untransposed transmission lines using Newton-Raphson method", IEEE Trans. on Power Systems, vol. 11, no. 3, pp. 1538-1546, August 1996.

[9] T. T. Nguyen, H. Y. Chan, "Evaluation of modal transformation matrices for overhead transmission lines and underground cables by optimization method", IEEE Trans. on Power Delivery, vol. 17, no. 1, January 2002.

[10] B. Gustavsen, "Frequency-dependent transmission line modeling utilizing transposed conditions", IEEE Transactions on Power Delivery, vol. 17, no. 3, pp. 834-839, July, 2002.

[11] A. J. Prado, J. Pissolato Filho, M. C. Tavares, C. M. Portela, "Modal transformation applied to transients in double three-phase transmission lines", 2001 IEEE Power Engineering Society Summer Meeting, Vancouver, BC, Canada, 15-19 July, 2001.

IX. BIOGRAPHIES

Afonso José do Prado - Electrical engineering (1991) at FEIS/UNESP - Paulista State University, Brazil, M.Sc (1995) at FEIS/UNESP and D.Sc. at UNICAMP - State University of Campinas (2002). His main research interests are in electromagnetic transients of transmission lines.

José Pissolato Filho received the PhD. degree in electrical engineering from Université Paul Sabatier, France, 1986. Since 1979, he has been with Department of Energy and Control of UNICAMP. His main research interests are in high voltage engineering, electromagnetic transients and compatibility.

Sérgio Kurokawa have been Assistant Professor at FEIS/UNESP since 1994. He received D.Sc. degree from UNICAMP (2003). His main research interests are electromagnetic transients in power electric systems.

Luiz Fernando Bovolato – M.Sc. in Electrical engineering (1983) at UFRJ - Federal University of Rio de Janeiro, Brazil, Brazil, D.Sc (1993) at USP – State University of São Paulo. His main research interests are in transient analyses and line parameter calculations. Presently researcher at FEIS/UNESP.