An Approach for Reduced Order Modeling of Nonlinear Power Systems

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Abstract--In this paper we present an approach for the reduced order modeling of nonlinear power systems. We describe a combined symbolic-numeric methodology for a local model order reduction which is based on the theory of singularly perturbed systems. Since the properties of power systems are taken into account, we were able to define an easy to use formula which eliminates the necessity of engineering insight into the mechanism of large scale systems. This leads to the possibility of implementation in automated systems like simulation and analysis programs. Thus, this approach can make a significant contribution to the efficiency of the computation of electrical power systems.

Keywords: nonlinear power systems, simulation, model order reduction, singular perturbations

I. Introduction

THE size of power grids and the increasingly stressed conditions under which power systems operate demand the use of computers for analysis and simulation whereas the calculation effort increases by leaps and bounds. So the economization or reduction of the models of large scale power systems plays a decisive role for the simplification of high dimensional grids with nonlinear loads. Since power systems possess a multiple time scale behavior [1], reduced order modeling by means of singular perturbation can be a useful tool to achieve this aim. Since the fast phenomena are mostly negligible while the slowest transients represent the imported properties, model order reduction techniques are applicable to electrical power systems. These methods lead to simpler subsystems which can be analyzed easier and allow faster simulations of power grids. Moreover, it is easier to interpret the behavior and properties of subsystems which possess a lower dimension as the original one.

While in general various model order reduction methods are a possibility [2], especially reduced order modeling by means of the theory of singularly perturbed systems is appropriate to simplify power systems [3]. In addition, the identification of subsystems by singular perturbations and time scales is relevant in many fields of application [4], whereas the properties of the systems wildly differ from one application area to another. Generally it is difficult to apply model reduction techniques to nonlinear dynamical systems, since this leads typically to heavy calculations [5]. Moreover, reduced order modeling by singular perturbation possesses another main drawback: in opposite to other techniques it needs detailed knowledge about the application. Since multiple time scales are directly coupled with stiff systems of first order ordinary differential equations (ODE) [6], it is obvious that the diagonalization of the linear part of nonlinear systems can be used to identify the fast and slow state variables for a local state space reduction. Since the linear part is given by a linearization at a fixed point, the reduced model is only valid in a restricted region of the state space. However, in practice this approach is rendered impractical: the change to modal coordinates leads to near singular and badly scaled transformation matrices.

Maas and Pope [7] suggested another method for the identification of slow and fast state variables which is based on a real Schur decomposition of the right hand side of the ODE. This methodology is reported to be quite powerful for the reduced order modeling of combustion problems in the context of partial differential equations, where the stiffness is based on big differences between the real parts of the eigenvalues. Unfortunately, electrical power systems are weakly damped: power systems are stiff because of the large imaginary parts of conjugate complex eigenvalues, whereas real eigenvalues are small. So the models of power systems have the character of a highly oscillatory system, which cannot be split by the technique introduced by Maas and Pope.

We close this gap and present a modified methodology which takes the properties of electrical power systems into account. By using the particular conditions of power systems we were able to find an easy to use formula for the model reduction of nonlinear power systems. Since our technique required no deeper insight into the behavior of particular problems, we have implemented our methodology as an automatic algorithm. So our technique can be used in simulation and analysis software. Furthermore, we discuss the practicability of error estimation [8], the choice of the splitting parameter [9] and the improvement of the results by asymptotic expansions [10]. In the end we demonstrate the capability of our methodology by means of a 9-dimensional model of a 110 kV power grid with a nonlinear load.

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II. Quasi-Steady-State Assumption

The model of power systems can be written in the form

$$\dot{\boldsymbol{x}} = \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{p}, t), \tag{1}$$

where $\boldsymbol{x}$ is an n-dimensional state space vector corresponding to the electrical or mechanical quantities of our system and $\boldsymbol{h}$ is a nonlinear vector field. The vector $\boldsymbol{p}$ represents the parameters of the power grid while $\boldsymbol{h}$ also contains the nonlinear components. When (1) is called stiff, it has fast components which reach their steady state very quickly so that systems dynamics can be described in terms of the remaining slow components. Thus we should be able to split $\boldsymbol{x}$ to $(\boldsymbol{y},\boldsymbol{z})=(\boldsymbol{P}\boldsymbol{x},\boldsymbol{Q}\boldsymbol{x})$, where $\boldsymbol{P}$ and $\boldsymbol{Q}$ are projections on the dynamically slow and fast parts. Applying the projections we find

$$\begin{aligned} \boldsymbol{P}\,\dot{\boldsymbol{x}} &= \boldsymbol{P}\,\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{p}, t) \\ \boldsymbol{Q}\,\dot{\boldsymbol{x}} &= \boldsymbol{Q}\,\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{p}, t) \end{aligned}. \tag{2}$$

Under the so-called quasi-steady-state assumption (QSSA)

$$\boldsymbol{Q}\dot{\boldsymbol{x}} = \boldsymbol{0} \tag{3}$$

we get the slow model

$$\begin{aligned} \boldsymbol{P}\,\dot{\boldsymbol{x}} &= \boldsymbol{P}\,\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{p}, t) \\ \boldsymbol{0} &= \boldsymbol{Q}\,\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{p}, t) \end{aligned} \tag{4}$$

of (1) which represents a differential algebraic equation (DAE). The mathematical justification of the QSSA is given by the theory of singularly perturbed systems [3]. Applying this popular notation and using the approach of the geometric singular perturbation theory [11], first we have to detect a small parameter ε ($\varepsilon \ll 1$, $\varepsilon = p_k$) in $\boldsymbol{h}$ which transforms our given system into the so-called standard form of a singularly perturbed system

$$\begin{aligned} \dot{\boldsymbol{y}} &= \boldsymbol{f}(\boldsymbol{y}, \boldsymbol{z}, \varepsilon) \\ \varepsilon\dot{\boldsymbol{z}} &= \boldsymbol{g}(\boldsymbol{y}, \boldsymbol{z}, \varepsilon) \end{aligned}, \tag{5}$$

where $\boldsymbol{y} \in \mathbb{R}^m$, $\boldsymbol{z} \in \mathbb{R}^l$ and m+l=n. So we have to determine $\boldsymbol{P}$ and $\boldsymbol{Q}$ in dependency on ε. Under some additional conditions, (5) represents a dynamic system on a constrained manifold [12]. Under the assumption that the Jacobian $D_{\boldsymbol{z}}\boldsymbol{g}$ $(\boldsymbol{y},\boldsymbol{z},0)$ is invertible on a solution $\boldsymbol{g}(\boldsymbol{y},\boldsymbol{z},0)=\boldsymbol{0}$, the vector $\boldsymbol{z}$ represents the fast variables near the solution for small ε. For $\varepsilon=0$ we reduce the state space of (5) from n to m. We get

$$\begin{aligned} \dot{\boldsymbol{y}} &= \boldsymbol{f}(\boldsymbol{y}, \boldsymbol{z}, 0) \\ \boldsymbol{0} &= \boldsymbol{g}(\boldsymbol{y}, \boldsymbol{z}, 0) \end{aligned}, \tag{6}$$

which is called the degenerate system to (2). Now we can rewrite (6) in the form

$$\dot{\boldsymbol{y}} = \boldsymbol{f}(\boldsymbol{y}, \varphi(\boldsymbol{z}), 0), \tag{7}$$

where

$$\boldsymbol{z} = \varphi(\boldsymbol{y}) \;\Leftrightarrow\; \boldsymbol{g}(\boldsymbol{y},\boldsymbol{z}) = \boldsymbol{0}. \tag{8}$$

The second equation of (6) defines a slow manifold [12]

$$\Gamma_0 := \left\{(\boldsymbol{y},\boldsymbol{z}) :\; \boldsymbol{g}(\boldsymbol{y},\boldsymbol{z},0) = \boldsymbol{0}\right\}. \tag{9}$$

It is a necessary condition for (6) to have a solution at all that the initial value of (1) lies within (9) [8], i.e. at least a projection of the initial value is necessary to make (6) consistent. As shown in [13] not every system of the form (6) has a unique solution, even if the initial value lies on (9). In this case (6) has a higher index. In the following we assume that (6) is an index 1 problem.

If we understand the manifold Γ_0 as a first approximation of an invariant manifold Γ_ε for $\varepsilon=0$, we can rewrite (9) in the form

$$\Gamma_\varepsilon :\; \boldsymbol{y} = \varphi(\boldsymbol{z},\varepsilon) = \xi_0(\boldsymbol{z}) + \varepsilon\xi_1(\boldsymbol{z}) + \ldots \tag{10}$$

By means of this asymptotic expansion [14] we are able to improve the precision of (7) [10].

III. Splitting Algorithm

The identification of a sufficient parameter ε and the determination of an adequate transformation to get (5) from (1) is a difficult task. While in the case of a linear system the calculation of an adequate transformation is well-known [12], the transfer to the standard form of a nonlinear dynamical system depends on user's skill. In order to get a suitable transformation for a nonlinear vector field one has to have a detailed knowledge about the application area and a sure mathematical instinct. In [15] we have compared different methods for the calculation of the standard form. All these techniques have the same drawback: they cannot be realized in a uniform way. That is why we present in the following a standardized procedure which is based on the properties of power systems. Starting from the well-known idea of time-scale separation by means of diagonalization, i.e. the calculation of the Jordan canonical form of the linear part of $\boldsymbol{h}$, we try to find the fast and slow state variables. Unfortunately, in the case of a singularly perturbed system the calculation of a Jordan form leads to a nearly singular and badly scaled transformation matrix [16]. In order to produce relief the well-known QZ-Algorithm, which is well-established in the field of singular systems [17]-[18], seems appropriate, at which the approach of a generalized eigenvalue problem is used. If a network model has the form of a DAE the QZ-Algorithm is suitable. Since we are starting from a model in form of an ODE we have taken the basic splitting scheme from Mass and Pope [7]. However, compared to [7] we have taken into account that power systems are weakly damped and expanded the identification of time-scales to highly oscillatory systems. For the realization of the splitting algorithm it is suitable to start from a complex Schur decomposition. To any real nxn-matrix $\boldsymbol{M}$ exists an orthogonal nxn matrix $\boldsymbol{Q}$, where every 1x1-matrix $\boldsymbol{C}_{ii}$ represents a real eigenvalue or a part of a conjugate complex eigenvalue:

$$\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{Q}=\boldsymbol{C}=\begin{bmatrix} C_{11} & C_{12} & \cdots & \cdots & C_{1\mathrm{n}} \\ 0 & C_{22} & \cdots & \cdots & C_{2\mathrm{n}} \\ \vdots & 0 & \ddots & & \vdots \\ \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & 0 & 0 & C_{\mathrm{nn}} \end{bmatrix} \tag{11}$$

Here, the matrix $\boldsymbol{M}$ represents the linearization of field $\boldsymbol{h}$. To get an ordering of the eigenvalues of C_{ii} according to their magnitude a Givens-rotation can be used [19]. Here we assume that $|\lambda(C_{11})|\geq|\lambda(C_{\mathrm{nn}})|$. So we are able to split the spectrum of $\boldsymbol{M}$ by means of the splitting parameter $\nu>0$ into two disjoint sets:

$$\begin{aligned} \sigma_{slow} &:= \{\lambda\in\sigma(\boldsymbol{C}):|\lambda|<\nu\} \\ \sigma_{fast} &:= \{\lambda\in\sigma(\boldsymbol{C}):|\lambda|\geq\nu\} \end{aligned} \tag{12}$$

With $\sigma_{\mathrm{fast}}=\sigma(\boldsymbol{S}_{11})$ and $\sigma_{\mathrm{slow}}=\sigma(\boldsymbol{S}_{22})$ we can rewrite (11) in the form

$$\boldsymbol{C}=\begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{S}_{12} \\ 0 & \boldsymbol{S}_{22} \end{bmatrix}, \tag{13}$$

where $\boldsymbol{S}_{11}$ and $\boldsymbol{S}_{22}$ are upper triangular matrices. In order to transform $\boldsymbol{C}$ into block-diagonal form we use the block diagonalization as shown in [19]. We determine a matrix $\boldsymbol{Z}$ in the nxn matrix $\boldsymbol{Y}$

$$\boldsymbol{Y}=\begin{bmatrix} \boldsymbol{E} & \boldsymbol{Z} \\ \boldsymbol{0} & \boldsymbol{E} \end{bmatrix} \tag{14}$$

so that

$$\boldsymbol{Y}^{-1}\boldsymbol{C}\boldsymbol{Y}=\begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}_{22} \end{bmatrix} \tag{15}$$

has the needed block-structure. The matrix $\boldsymbol{Z}$ is given by a generalized Lyapunov equation - the so called Sylvester equation

$$\boldsymbol{S}_{11}\boldsymbol{Z}-\boldsymbol{Z}\boldsymbol{S}_{22}+\boldsymbol{S}_{12}=\boldsymbol{0}\,. \tag{16}$$

So we arrive at

$$\boldsymbol{S}=\boldsymbol{Y}^{-1}\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{Q}\boldsymbol{Y}=\boldsymbol{T}^{-1}\boldsymbol{M}\boldsymbol{T}=\begin{bmatrix} \boldsymbol{S}_{11} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{S}_{12} \end{bmatrix}. \tag{17}$$

IV. Local State Space Reduction

By means of the shown splitting algorithm we can eliminate the main drawback of the reduced order modeling via singular perturbations: the difficult calculation of the standard form in the case of a nonlinear dynamical system. Now we want to present the procedure for a local state space reduction. The initial point of our approach is the following singularly perturbed system:

$$\begin{aligned} \frac{\mathrm{d}\boldsymbol{y}}{\mathrm{d}t} &= \boldsymbol{f}(\boldsymbol{y},\boldsymbol{z},t) \\ \varepsilon\frac{\mathrm{d}\boldsymbol{z}}{\mathrm{d}t} &= \boldsymbol{B}\boldsymbol{y}+\varepsilon\boldsymbol{g}(\boldsymbol{y},\boldsymbol{z},t) \end{aligned} \tag{18}$$

It can be shown that (18) has for $\varepsilon=0$, $\mathrm{Re}\{\lambda(\boldsymbol{B})\}<0$ and under additionally assumptions the integral manifold $\boldsymbol{z}\equiv\boldsymbol{0}$ [9]. Furthermore, if ε is sufficiently small, (18) possesses an integral manifold Γ_ε near $\boldsymbol{z}\equiv\boldsymbol{0}$. So (18) represents the standard form of a singularly perturbed system which can be reduced to a simpler subsystem. In order to transform (1) into (18), first a model of a power system has to expand with its linearization given by

$$\dot{\boldsymbol{z}}_{\mathrm{L}}=\boldsymbol{h}_0+\boldsymbol{J}_0(\boldsymbol{z}-\boldsymbol{z}_0), \tag{19}$$

where $\boldsymbol{z}_0$ is user-defined, $\boldsymbol{h}_0=\boldsymbol{h}(\boldsymbol{z}_0,t_0)$ and $\boldsymbol{J}_0$ is the Jacobian

$$\boldsymbol{J}_0=\left.\frac{\partial\boldsymbol{h}}{\partial\boldsymbol{z}}\right|_{\boldsymbol{z}_0,t_0} \tag{20}$$

evaluated at $\boldsymbol{z}_0$. So we find

$$\dot{\boldsymbol{z}}=\boldsymbol{z}_L+\boldsymbol{h}(\boldsymbol{x},t)-\boldsymbol{z}_L=\boldsymbol{h}_0+\boldsymbol{J}_0(\boldsymbol{z}-\boldsymbol{z}_0)+\tilde{\boldsymbol{h}}(\boldsymbol{z},t,\boldsymbol{z}_0,t_0), \tag{21}$$

with

$$\tilde{\boldsymbol{h}}(\boldsymbol{z},t,\boldsymbol{z}_0,t_0)=\boldsymbol{h}(\boldsymbol{z},t)-\boldsymbol{h}_0-\boldsymbol{J}_0(\boldsymbol{z}-\boldsymbol{z}_0)\,. \tag{22}$$

Now we apply the complex Schur decomposition with the block-diagonalization

$$\boldsymbol{T}^{-1}\boldsymbol{J}_0\boldsymbol{T}=\tilde{\boldsymbol{C}}=diag(\boldsymbol{S}_{11},\boldsymbol{S}_{22}) \tag{23}$$

to (21):

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t}=\boldsymbol{T}^{-1}\boldsymbol{h}_0+\tilde{\boldsymbol{C}}\boldsymbol{u}+\boldsymbol{T}^{-1}\tilde{\boldsymbol{h}}(\boldsymbol{z}_0+\boldsymbol{T}\boldsymbol{u},t,\boldsymbol{z}_0,t_0) \tag{24}$$

With the coordinate transformation $\boldsymbol{z}=\boldsymbol{z}_0+\boldsymbol{T}\boldsymbol{u}$ and by choosing a suitable splitting parameter ν we arrive at

$$\begin{aligned} \dot{\boldsymbol{u}}_1 &= \boldsymbol{h}_{0,1}+\boldsymbol{S}_{11}\boldsymbol{u}_1+\bar{\boldsymbol{h}}_1(\boldsymbol{u},t,\boldsymbol{z}_0,t_0) \\ \dot{\boldsymbol{u}}_2 &= \boldsymbol{h}_{0,2}+\boldsymbol{S}_{22}\boldsymbol{u}_2+\bar{\boldsymbol{h}}_2(\boldsymbol{u},t,\boldsymbol{z}_0,t_0) \end{aligned}. \tag{25}$$

If we set $\nu=\varepsilon^{-1}$, (25) has the form

$$\begin{aligned} \dot{\boldsymbol{u}}_1 &= \boldsymbol{h}_{0,1}+\boldsymbol{S}_{11}\boldsymbol{u}_1+\bar{\boldsymbol{h}}_1(\boldsymbol{u},t,\boldsymbol{z}_0,t_0) \\ \varepsilon_\nu\dot{\boldsymbol{u}}_2 &= \varepsilon\boldsymbol{h}_{0,2}+\bar{\boldsymbol{S}}_{22}\boldsymbol{u}_2+\varepsilon\bar{\boldsymbol{h}}_2(\boldsymbol{u},t,\boldsymbol{z}_0,t_0) \end{aligned}, \tag{26}$$

where $\varepsilon_\nu\boldsymbol{S}_{22}=\bar{\boldsymbol{S}}_{22}$. If we set

$$\begin{aligned} \boldsymbol{f}(\boldsymbol{u}_1,\boldsymbol{u}_2,t) &= \boldsymbol{h}_{0,1}+\boldsymbol{S}_{11}\boldsymbol{u}_1+\bar{\boldsymbol{h}}_1(\boldsymbol{u},t,\boldsymbol{z}_0,t_0) \\ \boldsymbol{g}(\boldsymbol{u}_1,\boldsymbol{u}_2,t) &= \boldsymbol{h}_{0,2}+\bar{\boldsymbol{h}}_2(\boldsymbol{u},t,\boldsymbol{z}_0,t_0) \end{aligned}, \tag{27}$$

(26) has the same structure as (18). For a detailed justification refer [9].

V. Global Model Order Reduction

In order to apply the shown local state space reduction to a global analysis, we have to integrate the described procedure into a numerical ODE solver. As shown in [8] a numerical integration of an initial value problem (IVP) within a one-step descretization consists of a sequence of integration steps (Fig. 1). In every step a single integration is performed, where the solution of the last one is the initial value of the next one.

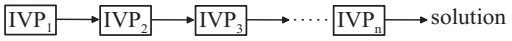

Fig. 1. Typical numerical integration scheme of an initial value problem within one-step discretization

In order to apply the local state space reduction to a global analysis, the classical integration scheme has to be replaced by the sequence shown in Fig. 2. Before a numerical integration is performed, a local state space reduction (LSSR) is necessary in order to reduce the order of the model as well as the step-size of the numerical integration. Especially the reduction of the step-size leads to decreasing calculation effort. Furthermore, the complexity of the solver can be reduced which leads to lower computing time. In the first view, this procedure seems to be lengthy. However, only by means of this structure it is guaranteed that the influence of the nonlinearity on the behavior of the state variables is taken into account. Against this background it is obvious why the efficiency of the splitting algorithm is important.

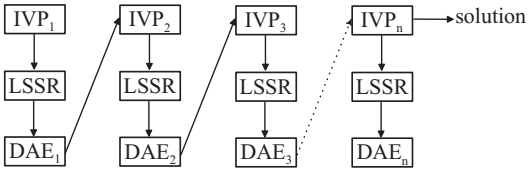

Fig. 2. Numerical integration scheme with local state space reduction (LSSR)

A critical role in our approach plays the choice of the splitting parameter ν. In [8] is suggested to choose the order of the slow subsystem by means of an error estimation. This approach, which is based on [20], is motivated by the fact that sometimes – especially in the case of stiff systems with high differences in the magnitude of the real parts of the eigenvalues – the QSSA leads to dramatic errors [21]. For highly oscillatory systems error estimation is not suitable for the calculation of the order of the slow system. In this case model order reduction has a character like "averaging" of the transients. When we neglect the highly oscillatory parts of transients, which causes on a perturbation of the power grid at $t=t'$, an "averaged" solution possesses a high error in the neighborhood of t'. Thus error estimation can provide an indication for the amplitude of highly oscillations, but it is not suitable for calculation of ν.

For reduced order modeling of power systems it must be our goal to eliminate as many as possible of such eigenvalues which have imaginary parts. How many we can eliminate is given by the dimension of the subsystem which leads to valid LSSR. To verify that (26) possesses an invariant manifold for $\varepsilon=\varepsilon_\nu$ for single eigenvalues the conditions

$$\frac{|\boldsymbol{S}_{11}|}{\nu}<1\,,\quad \frac{|\boldsymbol{h}_{0,2}|}{\nu}<d \tag{28}$$

must be fulfilled, where d defines a small ball around $\boldsymbol{z}_0$ in the original state space and $|\cdot|$ denotes a matrix norm induced by the Euclidean vector norm. So we are able to estimate the minimum order of the slow system. The parameter d is given by the step size while the step size can approximately be calculated via the magnitude of the eigenvalues of the reduced system. The mathematical justification of (28) is given in [9].

If the reduction leads to high errors, on the one hand it is possible to choose a higher order of the reduced system. However, from this follows the necessity of a lower step-size. On the other hand it is also possible to improve the approximation by an asymptotic expansion as shown above. For (18) the calculation of an asymptotic expansion – a so called higher order QSSA – leads to

$$\boldsymbol{z}=\varphi_0-\varepsilon\boldsymbol{B}^{-1}\boldsymbol{g}(\boldsymbol{y},\varphi_0,t,0)+O(\varepsilon^2)\,. \tag{29}$$

A detailed description of the power of asymptotic expansions and the proof of (29) is given in [10].

VI. Case Study

In order to demonstrate our approach by an example, we have decided to take a simplified model of an 110 kV power grid with a nonlinear resistive load (Fig. 3), whereas the characteristic of the nonlinearity is shown in Fig. 4.

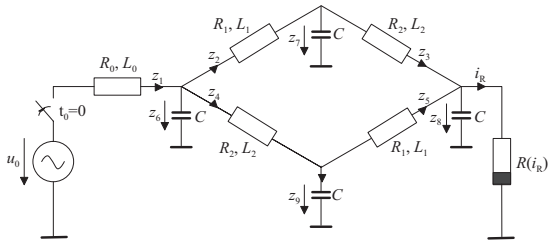

Fig. 3. Simplified model of a 110 kV power grid with nonlinear load

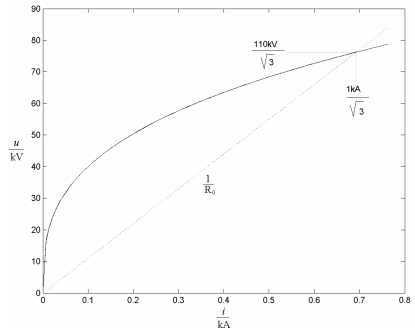

Fig. 4. The nonlinear load $R(i_R)$ is approximated by $u_R=\gamma\sqrt[3]{i_R}$ with $\gamma\approx 77$ and the auxiliary value R_0=110Ω

We dimension as follows: for $\omega_0=100\pi$/s we choose $R_0=0.007\Omega$, $L_0=0.07/(100\pi)$H, $R_1=0.8\Omega$, $L_1=4/(100\pi)$H, $R_1=2R_1$, $L_1=2L_2$ and $C=150$nF. $R_{1,}$ L_1 and $R_{2,}$ L_2 represent lines of 10 and 20 km length, respectively. The sinusoidal voltage source is given by $u_0=110\text{kV}/\sqrt{3}\cos(\omega_0 t+\varphi)$, where $\varphi=\pi/4$. By Kirchhoff's Law we find a set of nine first order ODEs, whereas $|\lambda_{max}|/|\lambda_{min}|\approx 2700$ for $t=[0,20\text{ms}]$. Because of the high stiffness, one-step solvers are impractical. Typically, solvers for stiff ODEs like numerical differentiation formulas (NDF) are used which have a big computational effort [22]. In order to decrease the computation time, reduced order modeling seems to be suitable. However, the nonlinearity excludes mostly a global model order reduction. So we use our approach of a local state space reduction to calculate the time depending currents and voltages. In Fig. 5 and Fig. 6 we show the solution for the current z_1 and the voltage z_8 of the complete system as well as the solution of reduced systems with and without a first order asymptotic expansion. As shown in Fig. 5 and 6 the use of our approach without asymptotic expansion leads to disastrous results. To avoid big discrepancies between the reduced and full model a higher order of the reduced model as well as an asymptotic expansion is possible, whereas the latter has a lower computational effort. It turns out that reduced order modeling with asymptotic expansions leads to a higher accuracy of the results, whereas the computational effort is moderate.

As shown in Fig. 5 and 6, the state variables possess near the start-up big highly oscillatory parts, which leads to the need of multi-step solvers and low step-sizes. By means of our algorithm we can eliminate these components. So we are able to use the classical Runge-Kutta method, whereas we have performed the calculation with an approximately 20 times lower step-size. In comparison with the original system the calculation time of the reduced models is approximately 30% lower. It turns also out that the solution of the averaged system has in the case of a state variable, which is perturbed by highly oscillatory parts, the character of an averaged solution. If these perturbations decay, the solution of the reduced system follows the solution of the original system. In order to eliminate the high oscillatory parts of the solution, we have tried to find reduced systems at which eigenvalues possess a minimum magnitude. Since the nonlinearity change the Jacobian, the eigenvalues depend on the nonlinearity. So the order of the reduced systems changes between two and three (Fig. 6). Especially during start-up there is a rapid change between subsystems of different orders. The reader should also note that in this example a transformation by the Jordan canonical form is very difficult. The transformation matrix has a condition of approximately $8.6\ 10^{-4}$. In contrast, by using our approach which is based on a complex Schur decomposition, $\boldsymbol{T}$ possesses in the domain of interests a condition of 0.1. For the estimation of the condition we have used the *rcond* routine of LAPACK [23] which is also in Matlab implemented.

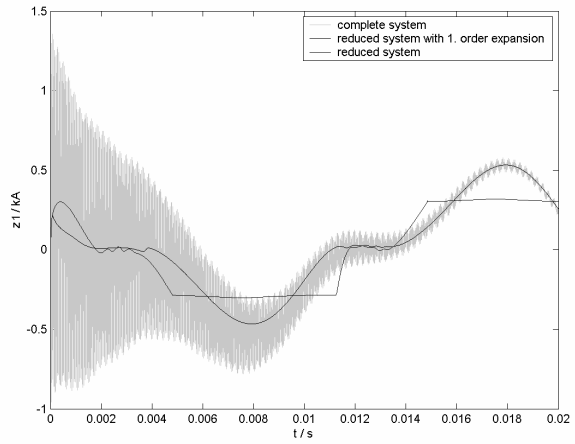

Fig. 5. Solution of the highly perturbed current z_1 of the complete model as well as "averaged" solutions of reduced systems.

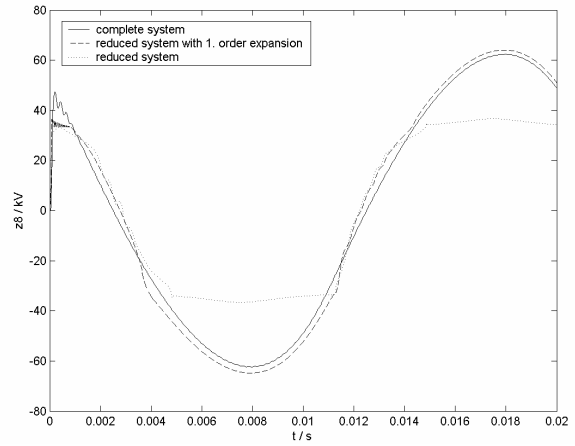

Fig. 6. The voltage z_8 possesses only high oscillatory parts during start-up which decay very fast.

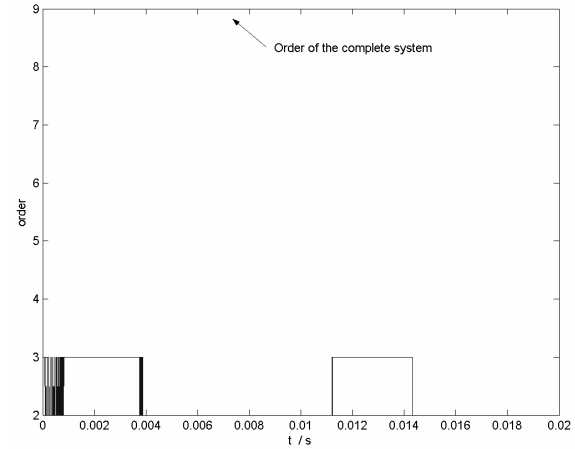

Fig. 7. Dimension of reduced systems during the first period.

Of course, we have demonstrated only fundamentals of an approach for local reduced order modeling. In order to increase the efficiency several improvements are needed. During the validity of a local slow manifold as many as possible integration steps must be performed. Furthermore, especially in the case of very high dimensional systems scaling [19] as well as balancing methods [24] are necessary, in order to guarantee the quality of the reduction.

VII. Conclusions

In this paper we have presented a systematic procedure for a local state space reduction of nonlinear electrical power systems. Because of its simple formula reduced order modeling now can be used in practice with a minimum of knowledge about the underlying mathematical methods and the properties of particular nonlinear power systems. By using this methodology it is possible to benefit the opportunities which are provided by model reduction methods: the calculated subsystems can be analyzed easier and simulated faster than the complete model. This follows from lower step-sizes and no need for multi-step solvers as well as the dramatic reduction of the number of equations. So the implementation of this approach in simulation and analysis software is possible. Of course, for large scale systems this approach should be improved of scaling and balancing methods as well as clustering techniques for the identification of relevant groups of eigenvalues. However, the most substantial reduction of the computational effort can only be achieved, if even more knowledge about power systems is an integral part of this methodology.

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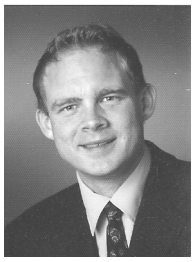

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