

Detailed High Frequency Models of Various Winding Types in Power Transformers

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Abstract—In this paper, techniques are described which demonstrate how a highly detailed internal transformer model can be obtained systematically with Matlab and how it can be prepared for subsequent transient analysis. The input of such a model will mainly be the description of the cross section of a single transformer leg, including all turns and windings. From the input data a three phase equivalent circuit is calculated and corresponding nodal impedance matrices are found for appropriate frequencies. Vector fitting is then applied which produces an excellent rational function approximation to the nodal impedance matrix in the valid frequency range. From the approximation an equivalent circuit can be extracted and implemented in circuit simulation software. A small but novel step makes it possible to compute not only the voltage distribution but also the internal currents from the extracted equivalent circuit. Finally a new circuit extraction technique is proposed for vector fitted impedance matrices for more efficient computation.

Keywords: Power transformer, very fast transient overvoltages (VFTOs), vector fitting, Matlab.

I. NOMENCLATURE

R, L, C	Resistance, inductance and capacitance
M	Mutual inductance
A	Incidence matrix of a directed graph as defined in the appendix
Z	Impedance matrix as defined in the appendix
Y	Admittance matrix equal to the inverse of Z
n	Number of nodes in internal transformer model
N	Number of transformer terminals corresponding to the number of nodes in the external transformer model
P	Number of measurement probes equal to $n-N$

II. INTRODUCTION

THE high frequency content of power system transients due to fast breakers and power electronics makes high

This work was supported by Elkraft System, the power system operator of East Denmark, and by ABB A/S and ENERGI E2, Denmark.

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Presented at the International Conference on Power Systems Transients (IPST'05) in Montreal, Canada on June 19-23, 2005
Paper No. IPST05 - 100

frequency models of various power system components necessary in order to assess the risk of unexpected component failures. The use of fast vacuum breakers instead of SF₆-breakers due to environmental aspects is an example of a possible source of very fast transients. Transformers are an example of a component, of which the internal voltage distribution under transient voltage application can only be determined from a detailed high frequency model. Thus, easily applicable techniques have been gathered to assist in the making of detailed high frequency transformer models. The result is a mixture of well known and novel techniques.

III. MODELLING

A. Principles

The modelling principles applied are chosen in such a way that arbitrary connections between turns are allowed. This is necessary if one wishes to model the high frequency effects of interleaved windings, positions of taps etc. Thus each turn must be modelled individually. This is done using lumped elements, i.e. ideal R , L and C components together with mutual inductance M between all inductances. Each inductance represents a single turn. The resulting equivalent circuit of three turns is outlined in fig. 1. Losses in dielectrics and the transformer core are modelled using complex permittivity and complex permeability, giving complex values for capacitance and inductance.

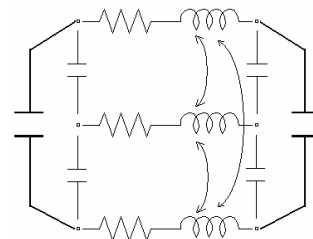


Fig. 1. Fundamental equivalent circuit for three arbitrary turns. Resistance and inductance of each turn are calculated, as well as the capacitance and mutual inductance between all turns. Each turn in the transformer has two nodes which, depending on the transformer design, can be connected to any other node.

As the high frequencies are in focus, linearity of the resulting equivalent circuit is at first assumed, i.e. superposition of currents and voltages can be applied. The large amount of parameters makes it impossible to produce exact analytical expressions for transfer functions in the transformer. And even though the model is detailed, it will still have a limited frequency range. Having realized these two

facts, it seems natural to compute the model numerically in the frequency domain for discrete frequencies. Transient analysis in the time domain can be performed on such a model using discrete Fourier transforms. However, the frequency response of the model can also be used for producing much smaller equivalent circuits which approximate the computed frequency response in the valid frequency range using vector fitting. Using these simplified circuits, time domain simulations can be performed with standard circuit analysis software.

The linearity makes it possible to compute the wanted voltages and currents in the frequency domain using the extended Roth's diagram [1]. However, to avoid a model, which requires too much knowledge of topological matrices, the proposed model only uses one of the four topological matrices in Roth's diagram, the incidence matrix A which undoubtedly is the most simple one. An apparent disadvantage of discarding the remaining topological matrices is that no accessible path in Roth's diagram leads to the computation of *internal* currents if A is the only topological matrix available. However, this disadvantage can be overcome as it is still possible to calculate the internal currents by applying small modifications to A and the impedance matrix Z , a trick which apparently is not described in the literature.

The definitions of A and Z are given in the appendix. The necessary modifications needed, if some or all of the currents are wanted, are explained in section IV. The described modifications mean that only one path in Roth's diagram is required in order to calculate all internal currents and voltages, i.e. the path corresponding to the equation:

$$U_N = (A^t Y A)^{-1} \cdot I_N \quad (1)$$

Here, U_N and I_N are voltage and current vectors which hold the node voltages and node currents respectively. The matrix $(A^t Y A)$ and its inverse are denoted as the nodal admittance matrix NAM and the nodal impedance matrix NIM respectively.

B. Input data

The choice of input data for a model is a compromise between user friendliness and generality. In this paper, the geometry of a cross section of a single transformer-leg is chosen as input (see fig. 4) together with information about how the turns are connected and how the materials behave as functions of frequency. More specifically, the geometry is described by the type and position of each turn with respect to the core and surrounding dielectrics. Though this approach may not seem especially user friendly, this level of detail is necessary in order to cover highly non-uniform windings as the double-layer winding where the distances between turns varies a lot.

IV. COMPUTATION TECHNIQUES

Due to the huge number of nodes and components in the

model, the equation system must be set up systematically from the input data. Having done that, the computation of the model can begin. The fundamental steps are as follows:

A. Fundamental steps

- 1) Compute A from the connections of turns entered by the user.
- 2) Compute R , L , C and M components from geometry assuming vacuum. Resistances R are computed at dc. Their frequency dependent behaviour will be accounted for in step 4.
- 3) Start frequency loop.
- 4) Compute self and mutual impedances at the given frequency from R , L and M and place them in Z as if the capacitances did not exist. Y is simply the inverse of Z .
- 5) Calculate a temporary NAM as:

$$Y_{RLM} = A^t Y A \quad (2)$$
- 6) Take advantage of the simple interpretation of the nodal admittance matrix of a circuit without mutual couplings and the fact that NAMs can be added [2]. Thus, the computed capacitances should be converted to admittances at the given frequency and added to the existing values at the correct positions in Y_{RLM} to produce the NAM of the complete system, Y_{RLCM} . The NIM Z_{RLCM} can then be found by inversion.
- 7) Unlike Y_{RLCM} , the NIM Z_{RLCM} can be directly reduced. External components, including sources, must necessarily be connected to some of the nodes in order to perform a simulation. These nodes are in the following denoted terminals. Each row and column in Z_{RLCM} corresponds to a specific node, so in order to obtain a terminal model, the rows and columns corresponding to a terminal should be kept and the remaining nodes should be discarded by removing the corresponding rows and columns. This leaves the user with an N -by- N matrix describing the external behaviour of the transformer at the given frequency seen from the N terminals. Thus, an *external model* has been found at the given frequency.
- 8) Further more, if N more columns are extracted, the complete voltage distribution, consisting of n voltages, can be found by multiplying this extracted n -by- N sub matrix by a vector containing the N terminal currents.
- 9) Repeat step 4-8 for all relevant frequencies.

It is also possible to compute all internal currents. This can be done by adding a branch to the network for each current one wants to calculate. This branch should have one end floating and the other end should be connected to ground. As one end is floating, no current can run through it which means it does not affect the circuit. Instead it is affected *by* the circuit if it is mutually coupled to any other branch. If it is coupled to a single branch and the mutual impedance between them is

equal to unity independent of the frequency, the voltage at the floating terminal will, at any time, attain the same value as the current running in the branch to which it is coupled.

Only minor modifications must be applied to A and Z in order to insert this branch. This is done by enlarging the number of rows and columns in both A and Z by one for each current wanted and then adding the value 1 in the appropriate places. This is shown in fig. 2 and 3.

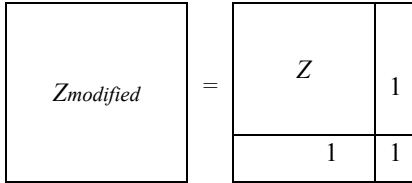


Fig. 2. Sketch on how to modify Z in order to compute an internal current. Besides the diagonal element, 1 should be placed in the two elements corresponding to the line one wants to measure the current in. The current of interest will thus induce a voltage across the added branch which is identical to the current running in the line.

B. Example

Consider a three-phase two-winding 10/0.4 kV transformer containing a total of 1500 turns per leg including the low voltage winding. This yields a total of 4500 turns. Thus the model will contain just as many R and L components together with around 20 million mutual inductances and, in worst case, just as many capacitances.

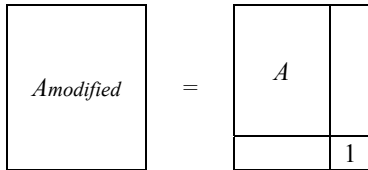


Fig. 3. Sketch on how to modify A in order to compute an internal current. The value 1 should simply be placed in the new corner element.

Z_{RLCM} is now approximately¹ a 4500-by-4500 matrix. On a PC with Pentium® 4 CPU 2.60 GHz, inversion of such a matrix took approximately 2 minutes using Matlab. The inversions are by far the most time consuming calculations in the frequency loop, and as the loop contains two inversions, the total computing time per frequency is approximately four minutes. The computation of the R , L , C and M elements may also be quite time consuming but depends very much on the chosen formulas/finite element methods and the symmetry in the transformer.

For each frequency the external model should be stored. The transformer has six terminals and when the external model is found (as described under step 7), the user is left with 36 elements compared to the original 20 million. However, if the complete voltage distribution is wanted, 6 columns of Z_{RLCM} should also be stored, giving a total of 27000 elements. If all currents also need to be computed, the

number of elements is approximately doubled. This number of elements can be reduced significantly, if only some of the voltages and currents are of interest.

V. COMPARISON WITH MEASUREMENTS

The model is programmed in Matlab but is still being developed, so the parameter estimations are not that accurate yet. Still the modelling technique can be roughly tested in the frequency domain by comparing it to measurements performed on a single phase of a dry type three-phase transformer with removed core. The experimental results are taken from [3] and were simply carried out by applying a sinusoidal voltage at the terminal of a grounded winding and measuring the voltage response at one of the centre-taps of the winding.

The parameters of the model, to which these experiments are compared, are based on the cross section of the actual winding as shown in fig. 4. The actual cross section is shown together with an output of a temporary graphical user interface added to the model. It can be seen that the turns are stacked in what may be denoted as columns, 6 per section. The model consists of one inductance per turn and takes into account that there are a different number of turns in each column as well as in each section. Simple formulas for the parameters have been given as a first approximation.

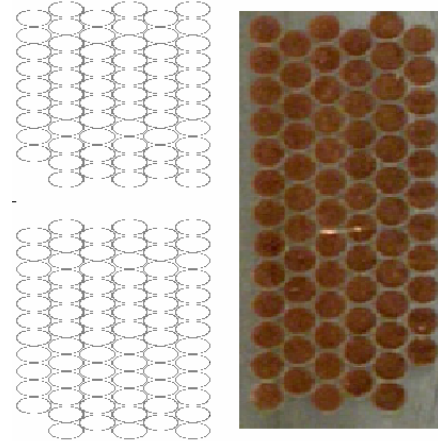


Fig. 4. Winding cross section of a dry type transformer: At left a sketch of two of 16 sections each winding consists of. Each section contains 6 columns of turns. At right a photo of a single section. Note the varying number of turns per section.

Resistance, self- and mutual impedance have been distributed correctly but the capacitances between columns are placed between the upper turns of each column. The turns of the low voltage side is, in this rough comparison, assumed to be perfectly grounded, though the low voltage side should of course be modelled precisely in order to get good results in the MHz-range. Capacitances between turns in a single column have not been included yet. The comparison between measurements and model is shown in fig. 5 and fig. 6.

The agreement between model and experiments are for both magnitude and phase angle found to go from excellent to poor as the frequency approaches the MHz-range. At around 2

¹ The dimension of the square matrix Z_{RLCM} is given by the number of nodes, so the actual dimensions of Z_{RLCM} may be slightly smaller than the number of turns when interleaved windings are present.

MHz the magnitude and phase angle of the model explode, and beyond 2 MHz there is absolutely no agreement between model and experiments. It is expected that good agreement can be obtained at several MHz when the parameter estimation has been fully developed.

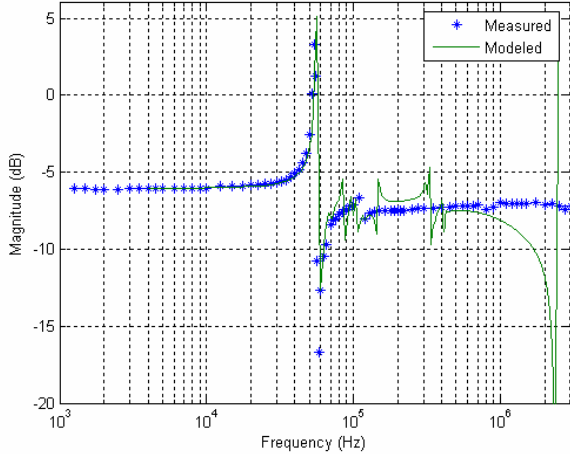


Fig. 5. Measured and modelled magnitude of the voltage ratio between centre-tap and terminal as a function of frequency.

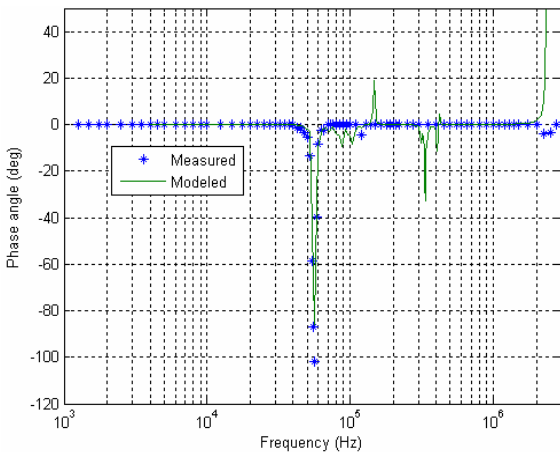


Fig. 6. Measured and modelled phase angle of the voltage ratio between centre-tap and terminal as a function of frequency.

VI. IMPLEMENTATION ISSUES

In the previous sections focus has been on the behaviour in the frequency domain. However, with respect to applications to transient behaviour, time domain simulations are needed. This can, in many situations, be overcome by using Fourier techniques, but in order to simulate more complex systems with several transformers and surrounding nonlinear components, direct time domain simulations are more practical. One solution is to create a linear electrical network equivalent, which solely consists of frequency independent R , L and C components and behaves approximately as the complete transformer model in the frequency domain. This can be done using vector fitting as described in [4] and [5]. Though these papers focus on measured transformer models with respect to their external behaviour, the vector fitting technique may just as well be applied on theoretical models in order to avoid huge models or frequency dependent R , L and

C components. It must be said though, that the corresponding equivalent circuit may contain negative parameters and in general it has no physical interpretation. Fortunately most software for transient analysis of electrical circuits can handle negative parameters including ATP EMTP and the blockset “SimPowerSystems” (SPS) for Simulink/ Matlab.

As described in [6], Matlab computer code which can perform vector fitting on a matrix can be downloaded from the internet. In fig. 7-8 it can be seen that the code gives an excellent fit to the computed 1-by-1 impedance matrix of the transformer coil, described in section V. Passivity has been enforced using the simplistic approach described in [7], in order to obtain stable time domain simulations.

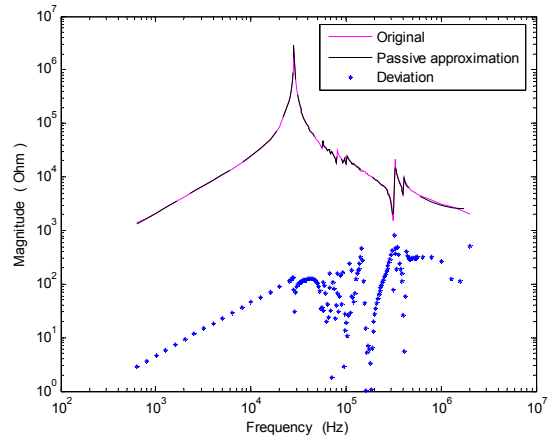


Fig. 7. Magnitude of transformer impedance as a function of frequency together with a 25 pole vector fit and the deviation between them. All three curves were computed for the same amount of frequencies, though only the deviation is plotted as points in order to visualize the applied frequencies.

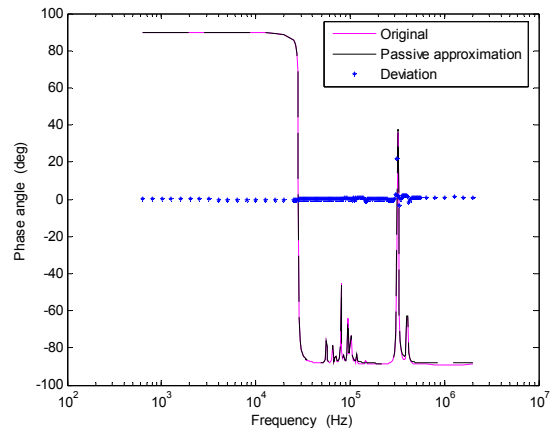


Fig. 8. Phase angle of transformer impedance with 25 pole fit and deviations.

The code described in [6] is also capable of extracting an equivalent circuit from a fitted NAM. By interpreting the transformer impedance as an admittance, an equivalent circuit was extracted from the fit and tested in SPS. The comparison is shown in fig. 9. The SPS-results were obtained for each frequency by measuring the voltage when injecting a current of 1 Ampere. In fig. 9 it can be seen, that the implementation is excellent except at frequencies close to the limit of the fitted frequency range. If a high accuracy is needed close to the limiting frequency, this can be achieved by extending the

fitted frequency range as demonstrated in fig. 10.

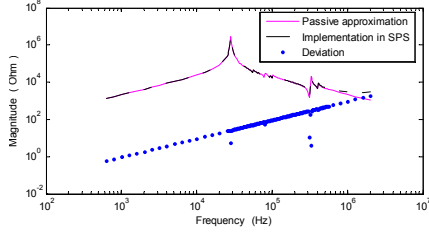


Fig. 9. Comparison between passive approximation and SPS-implementation. The SPS results were obtained by repetitive phasor solutions.

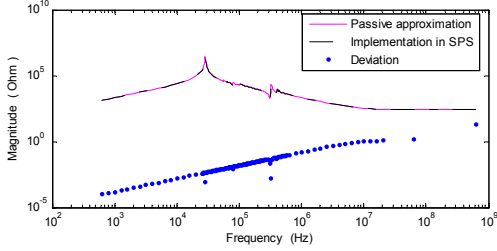


Fig. 10. Comparison with extended fitting range.

In [6] it was explained, how a circuit can be extracted from a vector fitted NAM. However, it may sometimes be advantageous to fit the NIM instead. When dealing with the internal behaviour of transformers, this is normally the case as the number of internal voltages n one wishes to compute is usually much bigger than the number of terminals N . Let us denote the nodes, which are not connected to external objects, probes. The number of probes is then $P = n - N$. A nice property of the NIM is then, that all NIMs which hold the N columns extracted under step 8 in section IV, has the same behaviour seen from the n nodes, when the system described by the NIM is only connected to external components through the N terminals. This leaves p^2 elements of the NIM free to vary. As a result, the computational cost of fitting the NIM is of the order $O(n)$ when p is much greater than N , which should be compared to the order $O(n^2)$ which is the computational cost of fitting the NAM. Another advantage is, that passivity does only need to be enforced on the terminal part of the NIM, in order to assure input-output stability.

In [2] it was explained how vector fitted impedances could be synthesised. But to extend the synthesis of impedances to the multi-terminal case, i.e. a vector fitted NIM, is not straight forward, as NIMs (unlike NAMs) can not in general be added. However, in the special case of a tree, NIMs can be added. This fact can be realised from equation 3 which is only valid when the incidence matrix A is square. This condition is only fulfilled when A describes a tree.

$$(A'(Z+\Delta Z)^{-1}A)^{-1} = A^{-1}(Z+\Delta Z)A^{-t} = A^{-1}ZA^{-t} + A^{-1}\Delta ZA^{-t} \quad (3)$$

In the special case where A equals a unit matrix, the NIM $(A'YA)^{-1}$ becomes identical to the impedance matrix Z , and A represents a rake equivalent, i.e. a tree where all n branches meet in the ground node. Thus, all n diagonal elements of the NIM can be interpreted as branch-impedances in such a rake

equivalent whereas the remaining elements can be interpreted as mutual impedances between the branches. In [2] it was shown how vector fitted impedances can be synthesised and thus, what remains is a description on how to synthesise vector fitted *mutual* impedances, which is given in the following text:

The outcome of a vector fit is, for each matrix element, a sum of rational functions which approximates the actual element. The form of this sum is given in equation 4.

$$d + s \cdot e + \sum_{m=1}^k \frac{r_m}{s - a_m} \quad (4)$$

Here, s equals $j \cdot \omega$, k is the number of poles and the remaining symbols denote the coefficients of the fit. Only the poles a_m and the residues r_m are allowed to be complex and when that happens, they always come in complex conjugate pairs.

In order to synthesise mutual impedances given by (4), standard transformer-models are applied as well as positive and negative R , L and C elements. SPS includes transformer models where the inductive part or the resistive part of the magnetising branch can be removed. As the induced open-circuit voltage on the secondary side of a 1:1 transformer model equals the voltage drop across the magnetising branch, the d and e terms of equation 4 can be synthesised by a resistive and inductive magnetising branch respectively.

Further more, a purely resistive magnetising branch can be chosen to equal 1Ω . This can be utilized when the contribution from the remaining r and a terms are synthesised:

Imagine a branch which, when seen from outside the branch, has a frequency dependent magnetic coupling to another branch. By opening the branch, two sub-branches in parallel, with impedances Z_A and Z_B , are revealed. If sub-branch A is coupled by 1 Ohm to another complete branch C , the voltage induced from branch A to branch C will numerically be identical to the current running in sub-branch A , if SI-units are applied. Thus, the problem of finding a mutual impedance which can be described on the form of (4), has been reduced to finding the ratio between the current in sub-branch A and the total current $I_{total} = I_A + I_B$ as shown in (5).

$$Z_g = \frac{U_C}{I_{total}} = \frac{I_A}{I_{total}} \cdot 1\Omega = \frac{Z_B}{Z_A + Z_B} \cdot I\Omega \quad (5)$$

The synthesis problem is then solved, if the fraction to the right $Z_B/(Z_A + Z_B)$ can be written on the form of (4) for both complex and real-valued poles. In order to represent a set of conjugate complex poles, the fraction must equal:

$$\frac{Z_B}{Z_A + Z_B} = \frac{r_1 + j \cdot r_2}{s - (a_1 + j \cdot a_2)} + \frac{r_1 - j \cdot r_2}{s - (a_1 - j \cdot a_2)} \quad (6)$$

By insertion it can be derived that the above equation is

fulfilled when Z_A is a series RLC-branch and Z_B is a series RC-branch. The R, L and C elements can then be found as:

$$L_A = l, R_A = -2(r_1 + a_1), R_B = 2 \cdot r_1, \\ C_A = \frac{l}{a_1^2 + a_2^2 + 2(a_1 \cdot r_1 + a_2 \cdot r_2)}, C_B = \frac{-l}{2(a_1 \cdot r_1 + a_2 \cdot r_2)} \quad (7)$$

For real poles, a_2 and r_2 must equal zero and the fractions of equation 6 reduces to a single fraction on the form:

$$\frac{r_1 + j \cdot 0}{s - (a_1 + j \cdot 0)} + \frac{r_1 - j \cdot 0}{s - (a_1 - j \cdot 0)} = \frac{r}{s - a}, \quad r = 2r_1, \quad a = a_1 \quad (8)$$

The parameters of a real pole are thus:

$$L_A = l, R_A = -(r + 2a), R_B = r, \\ C_A = \frac{l}{a^2 + a \cdot r}, C_B = \frac{-l}{a \cdot r} \quad (9)$$

As linear transformers does not contain nonreciprocal mutual impedance, the NIM becomes symmetrical and we do not have to worry about the mutual impedance from the secondary to the primary side. Finally it must be said, that the circuit elements for the synthesis of mutual impedance necessarily will influence on the involved branch impedances. This influence can be cancelled by series connecting corresponding impedances with opposite sign.

VII. DISCUSSION

The presented modelling technique shows good results when compared to the measurements. However, the parameter estimation is not fully developed yet and consequently, the accuracy is still not acceptable in the MHz-range. It seems likely that detailed estimation of the parameters will lead to good agreement in the MHz-range.

For time domain simulation, circuits can be extracted from vector fitted NAMs with software available on the internet. However, with respect to voltage distributions and internal currents it is much more efficient to extract the circuit from a partly vector fitted NIM as the number of elements which need to be fitted and thus the number of circuit elements are proportional to n when $N \ll n$ and not n^2 like the standard procedure.

VIII. CONCLUSIONS

A modelling procedure for power transformers has been presented in order to obtain reasonable accuracy in the MHz range. The model is very flexible, hereby giving the possibility of a large variety of transformer-types being modelled, e.g. foil windings and double-layer windings.

The modelling technique itself is based on topological matrices in order to treat the large amount of components systematically. The treatment given here has shown that it is

sufficient to understand only a single equation (eq. 1) to compute not only internal voltages but also internal currents, by applying some modifications to A and $Z=Y^{-1}$.

Finally a method for circuit extraction of vector fitted nodal impedance matrices has been presented which is more efficient than the standard procedure when a large amount of internal voltages and currents must be computed.

IX. APPENDIX

The definitions given in this appendix correspond to those found in [1].

A. Definition of the A -matrix

The A -matrix contains R rows corresponding to the number of lines in the given system. The number of columns corresponds to the number of ungrounded nodes in the system. All elements of each row are zero except the elements corresponding to the two nodes to which the line is connected. These elements hold a 1 and a -1. The sign defines the orientation of the lines with respect to the line currents. The current is defined positive from 1 to -1.

B. Definition of the Z -matrix

The diagonal of the Z -matrix contains the self-impedances of all the lines in the system, e.g. $R + j \cdot \omega \cdot L$. Mutual impedances $j \cdot \omega \cdot M$ are placed in the off-diagonal elements. The Z -matrix should be symmetrical. Note that the signs of the mutual impedances depend on the chosen line orientations with respect to the physical orientations of the coils or components.

X. ACKNOWLEDGMENT

The authors would like to thank R. E. Jensen (ENERGI E2, Denmark) for access to valuable results from [3].

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