Mutual Ground Impedances between Overhead and Underground Transmission Cables

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Abstract-- Overhead and underground cable transmission systems are often sharing a common right of way with another supply services as oil, gas, water and communication lines. When a phase-to-ground fault occurs, high transient induced over-voltages appear from short circuit currents to ground.

An accurate calculation of mutual impedances between overhead lines and underground cable systems, both with earth-return is required. A problem here is that mutual impedances are given by the Pollaczek coupling integral, which is highly irregular oscillatory and does not possess an analytic closed-form solution.

Recently, the author developed an algorithmic strategy to solve Pollaczek integral for calculating ground-return impedances of underground transmission cables. In this paper an extension of this algorithm is applied for the solution of the Pollaczek coupling integral at a wide range of applications.

Finally, the obtained result set is used in the assessment of approximated formulas issued by Lucca and CCITT.

Keywords: Mutual earth-return impedances, Pollaczek coupling integral, earth effects, underground transmission, aerial transmission.

I. NOMENCLATURE

ω	—	angular frequency,
μ_0	—	magnetic permeability of vacuum and air,
σ	—	soil conductivity,
d	—	Distance between cables,
x	—	horizontal distance between cables,
p	—	complex depth of the Skin Effect layer $p = 1/\sqrt{j\omega\mu_0\sigma}$,
h_1	—	Height of the aerial conductor,
h_2	—	depth of the underground cable,
ξ	—	cable depth *"h_2"* normalized by the Skin Effect layer thickness *"$\|p\|$"*,
η	—	horizontal distance *"x"* normalized by the cable depth *"h_2"*,
ζ	—	height of the aerial conductor *"h_1"* normalized by the cable depth *"h_2"*,

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II. INTRODUCTION

OFTEN overhead lines and underground transmission systems share right of way with another supply services such as oil, gas, water pipes, communication lines and electrified railway systems [1-4].

At transient as well as at steady state operation, power transmission cables induce voltages and currents on these other systems. Moreover, the aerial conductors in the proximity could be a part of a metallic fence and, then, the induced transient poses a safety hazard. To analyze induction levels, it is required to calculate the mutual inductances as functions of frequency between transmission cables and the other systems.

In 1926 Pollaczek postulated mathematical expressions for calculating electromagnetic fields inside an imperfectly conducting ground due to a buried thin filament of current [5]. In 1927 Pollaczek presented a formulation for calculating mutual earth impedances between overhead and underground transmission systems [2]. The latter case is the main interest in this paper.

Both Pollaczek solution sets are given in the form of highly oscillatory integrals that do not possess analytic closed-form solutions [2, 5-7]. The formula for calculating impedances of buried conductors has received far more attention than the one between buried and overhead conductors.

In the practice, power engineering analysts use approximate formulas. Problems with approximate solutions of Pollaczek integrals are that their accuracy levels are not well determined.

Recently, the author has proposed an algorithmic solution to the Pollaczek integral arising when evaluating mutual impedances of two buried conductors [6, 7]. This solution is numerically efficient, guarantees convergence and the error level is bounded [6, 7]. In this paper, an extension of the algorithmic solution is presented for the Pollaczek integral arising in the evaluation of the mutual impedance between a buried and an aerial conductor.

Finally, the extended algorithm is further applied at evaluating the accuracy levels of the approximate formulas by Lucca [3] and by CCITT [4].

III. COUPLING BETWEEN OVERHEAD AND UNDERGROUND TRANSMISSION SYSTEMS

Fig. 1 shows a hybrid transmission system composed by an overhead conductor and an underground power transmission cable. Assuming an imperfectly conducting ground, magnetic permeability in both air and ground equal to the vacuum

($\mu_1 \cong \mu_2 \cong \mu_0$), homogeneous ground, and that the displacement currents in the ground can be neglected, the mutual earth impedances are given by the following Pollaczek coupling integral [1, 2]:

$$Z_E = \frac{j\omega\mu_0}{2\pi}\int_{-\infty}^{+\infty} \frac{exp\left[-h_1 \cdot |\beta|\right]\cdot exp\left[-h_2\sqrt{\beta^2+1/p^2}\right]}{|\beta|+\sqrt{\beta^2+1/p^2}} \cdots$$
$$\times\, exp\left(j\beta\, x\right) d\beta \qquad (1)$$

Now, after some algebraic manipulations, *(1)* is transformed into a more suitable form for its numerical evaluation [6, 7]. A normalized dimensionless parameter representation of the Pollaczek coupling integral is:

$$Z_E = \frac{j\omega\mu_0}{2\pi} J(\xi,\eta) \qquad (2a)$$

where

$$J(\xi,\eta) = -2j\int_0^{+\infty}\left[F(u) - u + j\cdot\, G(u)\right]\ldots$$
$$\cdot\, exp\left[-\xi\cdot\left(\zeta\cdot u + \, F(u)\right)\right]\times exp\left[-j\cdot\xi\, G(u)\right]\cdot cos\left(\xi\,\eta\, u\right) du \qquad (2b)$$

the dimensionless parameters $\xi = h_2/|p|$, $\eta = x/h_2$ and $\zeta = h_1/h_2$ are related to the physical configuration of the system in Fig. 1. Functions *F(u)* and *G(u)* are independent of the physical geometry of the system and are solutions of the primitive integrand function *"$(u^2+j)^{1/2}$"* given in [6] and in a detailed form in [7]. Other advantage of this representation is that integral *(2b)* is defined inside the rank $[0, \infty]$. Notice that both Pollaczek underground and coupling integrals are represented in four factors. The first two factors are of the damping type, while the second two are oscillatory. The first damping factor depends on functions *F(u)-u* and *G(u)*, which are decreasing monotonic. The second factor is a pure damping exponential function. The third and the fourth factors are irregular and regular oscillatory functions, respectively. Because of their function arguments are identical than for the Pollaczek underground integral case, their zero crossings can be identified by using the rules in the algorithmic strategy proposed in [6, 7].

The differences between both Pollaczek integral schemes can be summarized in the following relationship which is valid for the case $\zeta = 0$:

$$exp\left(-\xi\cdot F(u)\right) \cong exp\left(-\xi\cdot\left(\zeta\, u + F(u)\right)\right) \qquad (3a)$$

Right hand side of *(3a)* means that Pollaczek coupling integral is a particular case of Pollaczek underground earth impedance integral. When $\zeta \neq 0$ introduces and additional damping factor to the integrand as:

$$exp\left(-\xi\cdot\left(\zeta\, u\right)\right) \qquad (3b)$$

The compression level of *(2b)* depends on parameter ζ. The more general application case considered in this paper occurs when $\zeta = 0.1$. Values greater from this limit are damped sub-cases of the first one.

As an example, consider the integrand pattern of $J(\xi,\eta)$ shown in Fig. 2. Two values of $\zeta = 0.1$ and $\zeta = 1$ are compared with parameter values $\xi = 0.1$ and $\eta = 100$ inside the $u \leq 4$ range.

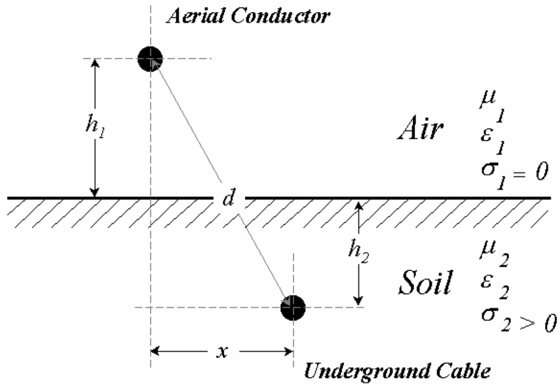

Fig. 1 Hybrid transmission system composed by an overhead aerial conductor and an underground power cable buried in an imperfectly conducting ground.

IV. Extension of the Algorithmic Solution of the Pollaczek Integral

The algorithmic solution of the Pollaczek integral for calculating underground cable earth-impedances [6] has been extended in this paper to solve the Pollaczek coupling integral in the following two steps.

First, consider the modified truncating criterion for the pure damping exponential function in the right hand side of *(3a)*. The second step is to include the modified regular and irregular oscillating factors in the new truncated range $0\text{-}u_{max}$.

Integrating the new second factor in the complete range yields the following partition of the *u* rank:

$$\int_0^{\infty} exp\left(-\xi\cdot\left(\zeta\, u + F(u)\right)\right) du \cong \ldots$$
$$\cong \int_0^{u_{max}} exp\left(-\xi\cdot\left(\zeta\, u + F(u)\right)\right) du + \varepsilon_r \qquad (4a)$$

the relative error of using this approximation is:

$$\varepsilon_r = exp\left(-\xi\cdot\left(\zeta+1\right) u_{max}\right) / \xi\cdot\left(\zeta+1\right) \qquad (4b)$$

For an arbitrary fixed value of ε_r, the corresponding u_{max} is:

$$u_{\boldsymbol{max}} = \lambda_e / \xi\cdot\left(\zeta+1\right) \qquad (4c)$$

where

$$\lambda_e = -log\left(\varepsilon_r\cdot\xi\left(\zeta+1\right)\right) \qquad (4d)$$

A truncating error ε_r can be found by refining the value of λ_e. A value of $\lambda_e = 10$, has been determined empirically satisfactory in this paper applications [6, 7].

The regular and irregular oscillations in *(2b)* depends on the magnitude of parameters η and ξ, respectively.

The former are introduced to *(2b)* by the cosine term. This term does not oscillate if :

$$\eta < \frac{\pi}{2}\frac{\left(\zeta+1\right)}{\lambda_e} \qquad (4e)$$

The latter are introduced to *(2b)* by the complex exponential function, when:

$$\xi > \frac{\pi}{2}\sqrt{2} \qquad (4f)$$

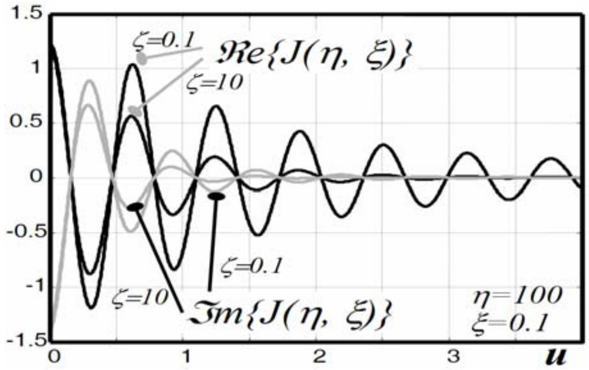

Fig. 2 Pollaczek coupling integrand compression dependency on parameter ζ inside the $u \leq 4$ range.

The total zero crossings of *(2b)* in the new truncated range $[0,\ u_{max}]$ can be identified sorting the regular and irregular oscillations due to *(4e)* and *(4f)* [6,7].

Now, the dimensionless variables ξ, η and ζ depends on the physical quantities h_1, h_2, ω, ε, μ, σ and x.

A broad application range for these physical quantities is shown in Table I. Most of practical engineering application cases lie within these variable ranges. Subsequently, these ranges are used to establish the dimensionless parameter ranges of ξ, η and ζ which are given in Table II.

The extended algorithmic methodology was applied to solve the Pollaczek coupling integral 10^3 *times* with 10^2 logarithmic spaced samples of ξ and *10* logarithmically spaced samples of η and the more general case for variable $\zeta=0.1$ it is considered. Any value greater than this limit are damped sub-cases of the first one.

The time required for doing this task on a *Pentium 4M™* at *1.6GHz* with *1.0 GB* of *RAM* running *MATLAB® V. 6. 5*, was slightly less than *1s* with $N=256$ (Number of samples per integration test). Figs. 3a and 3b depicts the obtained results of solving the integral $J(\xi,\eta)$ in *(2b)*. These results have been further tested by increasing N up to *1024*. The differences with $N=256$ are well below *0.01%*.

TABLE I
APPLICATION RANGES FOR PHYSICAL VARIABLES

4	$\leq$	h_1	$\leq$	50	$[M]$
0.1	$\leq$	h_2	$\leq$	30	$[M]$
10^{-4}	$\leq$	x	$\leq$	400	$[M]$
2π	$\leq$	ω	$\leq$	$2\pi\times10^{6}$	$[rad/s]$
10^{-4}	$\leq$	σ	$\leq$	1	$[S/M]$

TABLE II
RANGES FOR NORMALIZED DIMENSIONLESS PARAMETERS

10^{-4}	$\leq$	ξ	$\leq$	10^{2}
10^{-4}	$\leq$	η	$\leq$	10^{2}
10^{-1}	$\leq$	ζ	$\leq$	10^{2}

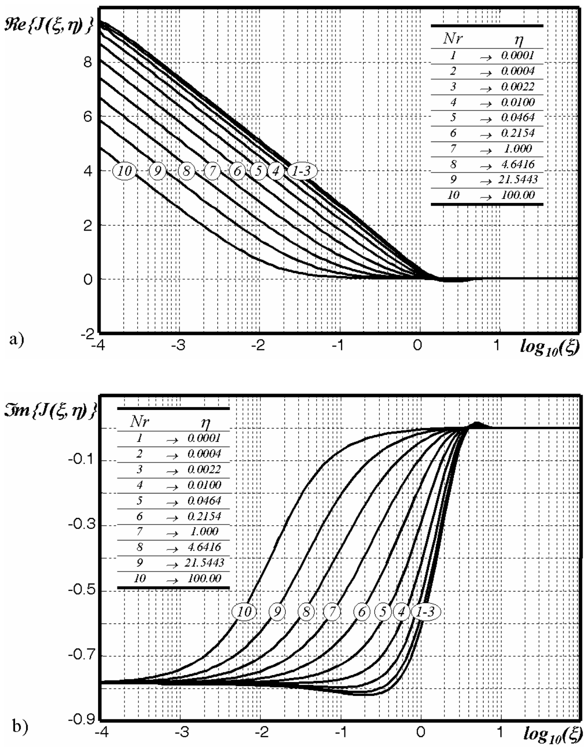

Fig. 3 Pollaczek induction integral solutions for ranges inside Table II.
a) Curves of $\Re\{J(\xi,\eta)\}$, b) Curves of $\Im\{J(\xi,\eta)\}$.

V. NORMALIZED COUPLING IMPEDANCES

The normalized coupling impedance concept is introduced now as [6, 7]:

$$Z_E \triangleq Z_E \cdot \left(2\pi / \omega \cdot \mu_0\right) \qquad (5a)$$

Thus, the Pollaczek coupling integral is re-defined as:

$$\begin{aligned} Z_E &= j \cdot J(\xi,\eta) \\ &= 2\int_0^{+\infty} \left[F(u) - u + j \cdot G(u)\right]\ldots \\ &\cdot exp\left[-\xi \cdot \left(\zeta \cdot u + F(u)\right)\right] \times exp\left[-j \cdot \xi\, G(u)\right] \cdot cos\left(\xi\, \eta\, u\right) du \end{aligned} \qquad (5b)$$

Approximation *(5b)* extracts dimensional physical variables, allowing the possibility to handle general application ranges for ξ, η and ζ.

The very intricate problem of solving both Pollaczek integrals has motivated the search for new closed-form approximations, which are preferred by engineering analysts than using numerical algorithms or methodologies, that sometimes are time consuming and cumbersome.

VI. ASSESSMENT OF CLOSED-FORM APPROXIMATIONS

The broad range solution set obtained from the normalized Pollaczek coupling integral *(5b)* it has been used in this paper section to assess two of the most often-used formulas for calculating mutual earth-return impedances between overhead and buried lines. The first one is the formula proposed by Lucca [3] and the second the one proposed by the CCITT [4].

LUCCA formulation

On a recent publication Lucca developed his well-known closed-form solution for calculating mutual impedances between overhead and buried lines, both with earth return [3]. Basically, Lucca considered a two step approximation. The first step is based in the utilization of the theory of images following the work by Wait and Spies [3]. The second step consists in the cancellation of the oscillatory exponential factor of the integrand. The latter step is very similar to the one proposed by A. Ametani for approximating underground cable earth impedances by using the Carson's integral [1].

The formula proposed by Lucca on its original physical variables as in Fig. 1 is [3]:

$$Z_{E-L} = \frac{j\omega\mu_0}{2\pi}\left\{ \boldsymbol{ln}\left(\frac{\overline{R}_{12}}{R_{12}}\right) - \frac{2\overline{y}}{3\gamma^3}\cdot\left[\frac{\overline{y}^2 - 3x^2}{\overline{R}_{12}^{\,6}}\right]\right\} \qquad (6a)$$

where

$$R_{12} = \sqrt{x^2 + (y_1 - y_2)^2}$$
$$k_e^{\,2} = -j\omega\mu_0\sigma$$
$$\overline{R}_{12} = \sqrt{\overline{y}^2 + x^2}$$
$$\gamma = j\cdot k_e$$
$$\overline{y} = y_1 - y_2 + \frac{2}{\gamma}$$

Transforming *(6a)* into the context of the dimensionless parameters in Table II, and of the normalized coupling impedances, we have:

$$Z_{E-L} = j\cdot\left\{ \boldsymbol{ln}\left(\sqrt{\frac{\lambda^2+\eta^2}{\eta^2+(\zeta-1)^2}}\right) + \frac{2\lambda\left[\lambda^2 - 3\cdot\eta^2\right]}{3\sqrt{j}\cdot\xi^3\left[\eta^2+\lambda^2\right]^3}\right\} \qquad (6b)$$

where, $\lambda = \zeta - 1 + 2\sqrt{j}/\xi$.

Expression *(6b)* is a function of the dimensionless variables ξ, η and ζ only.

CCITT recommended formula

The CCITT in [4] (a specialized paper concerning telecommunication lines protection), recommended the use of the following approximate formula for evaluating mutual impedances between overhead and buried lines:

$$Z_{E-C} = \frac{j\omega\mu_0}{2\pi}\left\{ \boldsymbol{ln}\left(\frac{1.851}{j\cdot k_e\cdot R_{12}}\right) + \frac{2j\cdot k_e(y_1+y_2)}{3}\right\} \qquad (6c)$$

Expression *(6c)* is presented on its original variables [4]. Now, it is convenient to represent *(6c)* into the dimensionless variables given in Table II.

According to the normalized impedance concept stated in *(5a)* we have:

$$Z_{E-C} = j\cdot\left\{ \boldsymbol{ln}\left(\frac{1.851\cdot\sqrt{j}}{\xi\cdot\sqrt{\eta^2+(\zeta-1)^2}}\right) - \frac{2j\cdot\sqrt{j}\,\xi(1+\zeta)}{3}\right\} \qquad (6d)$$

Expression *(6d)* also is a function of the dimensionless variables ξ, η and ζ only.

In order to test the normalized impedance approximate formulas *(6b)* and *(6d)*, first consider an application example reported in section 3 of reference [3]. In this case, the conductivity of the soil is $\sigma=0.01S/m$, the height of the aerial conductor is $h_1=15m$, the depth of the buried conductor is $h_2=1m$, the horizontal distance between conductors is linearly sampled *100times* for values inside $0<x\leq 2000m$ range. The frequency values for this test are $f_1=50Hz$, $f_2=500Hz$ and $f_3=5000Hz$.

The results obtained for $\boldsymbol{Z_E}$ in [3] have been reproduced in this paper through $\underline{\boldsymbol{Z}}_{\boldsymbol{E}}$ as a first application example. Fig. 4 depicts the real and imaginary curves for each frequency test using the Pollaczek algorithmic solutions and the Lucca formula [3]. The concordance between both set of curves are in a good agreement. Fig. 5 shows the relative percent error level (δ) calculated for the first application case along the horizontal distance between the aerial and the buried conductor. The relative error level is evaluated as:

$$\boldsymbol{\delta} = \left| 1 - \frac{f_{aprox}}{f_{exact}} \right| \times 100 \qquad (6e)$$

where f_{aprox} is the approximated function and f_{exact} is the function considered as exact for the calculation.

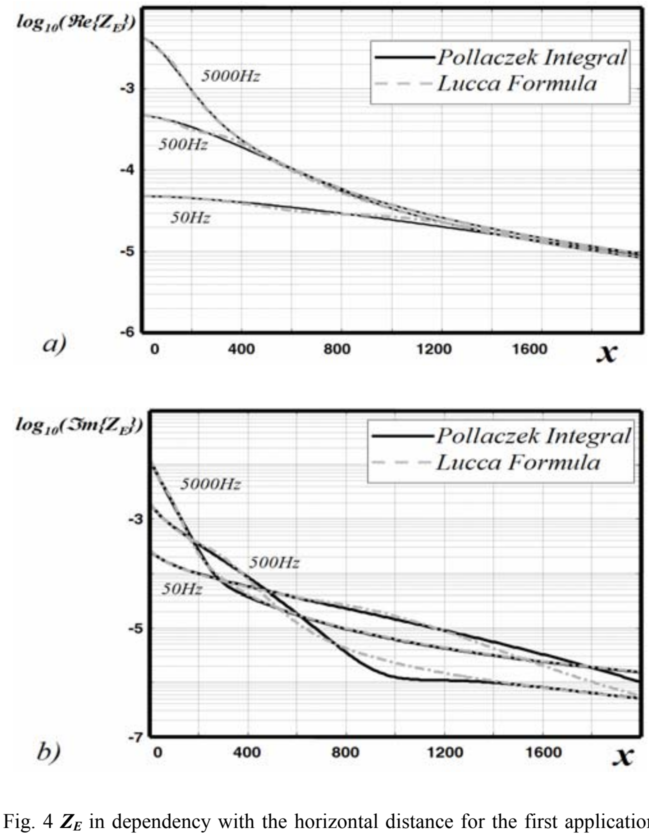

Fig. 4 $\boldsymbol{Z_E}$ in dependency with the horizontal distance for the first application example [3]). a) Curves of $\Re e\{\boldsymbol{Z_E}\}$, b) Curves of $\Im m\{\boldsymbol{Z_E}\}$.

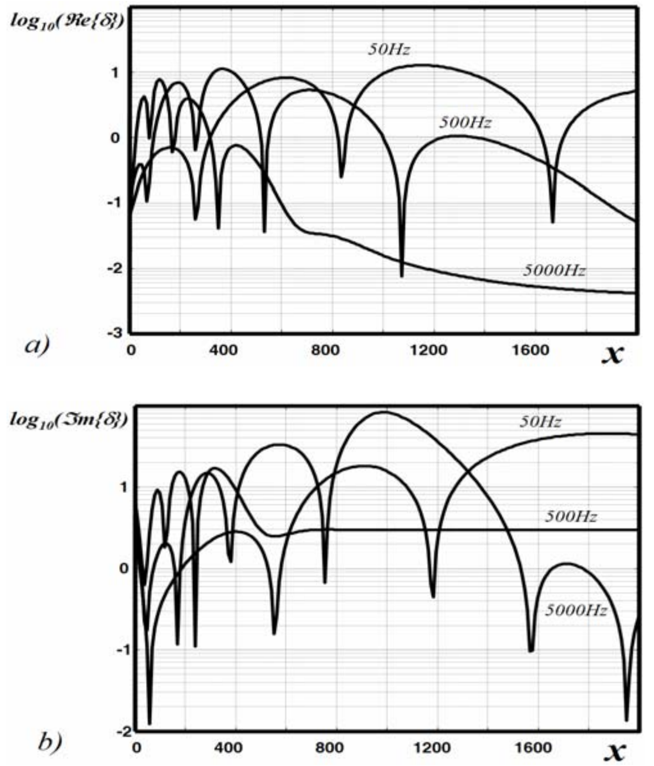

Fig. 5 Percent relative error level for the first application case between Pollaczek algorithmic solution and the formula proposed by Lucca [3]. a) Curves of $\Re e\{\delta\}$, b) Curves of $\Im m\{\delta\}$.

It can be noticed from Fig. 5a and 5b that the maximum error level occurs in the imaginary component for distances higher than *600m*.

Another methodology proposed here to evaluate relative error levels per ranges on using closed-form approximations is through the use of contour maps [6, 7]. The maps are graphical tools generated for universal application cases, because of they are referred to the dimensionless parameters ξ, η and ζ. For the first here considered application example, the value of $\zeta=15$ has been applied varying parameters ξ and η according frequency and horizontal distance is moving in the test according data provided in this section. Fig. 6a depicts the real component for the contour map, while Fig. 6b shows the corresponding imaginary one. These maps classify four error regions inside $0\leq\delta<10\%$. From Fig.6b it can be noticed that an error of *10%* is more noticeable as $\boldsymbol{R_4}$ almost in the entire plot.

As a second application example, consider the normalized parameter ranges given in Table II. For a more general case the Pollaczek algorithmic solution, the Lucca [3] and CCITT [4] closed-form approximations, have been compared here through contour error maps. Results of these comparisons are plotted per components in Fig. 7 and 8, respectively.

In Fig. 8a the real component of CCITT [4] formula appears to have an error higher than *10%* in almost the entire plot. On the other hand the imaginary components between both formulas are very similar. Concretely, $\boldsymbol{R_1}$ and $\boldsymbol{R_4}$ presented more coincidences in Fig. 7b and 8b.

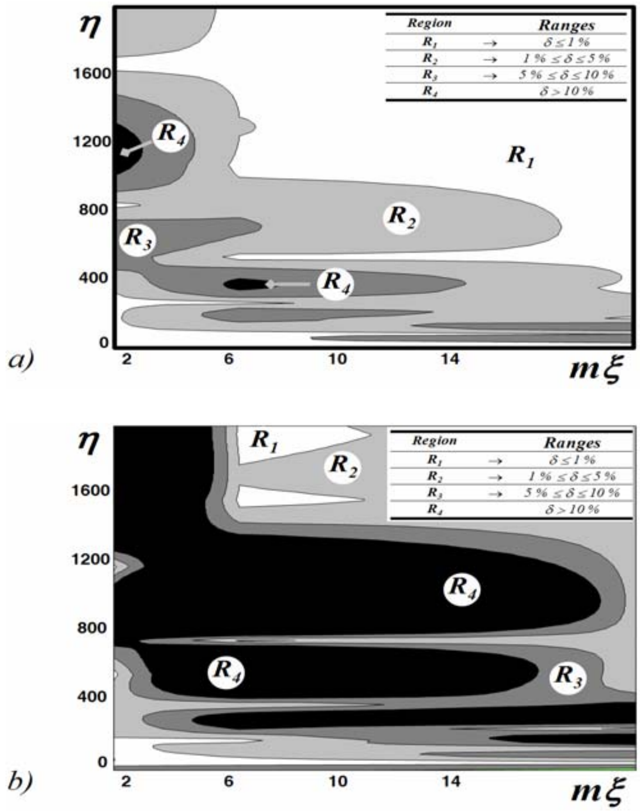

Fig. 6 Contour relative error maps for the first application example reported by Lucca in [3]. a) $\Re e\{\delta\}$, b) $\Im m\{\delta\}$.

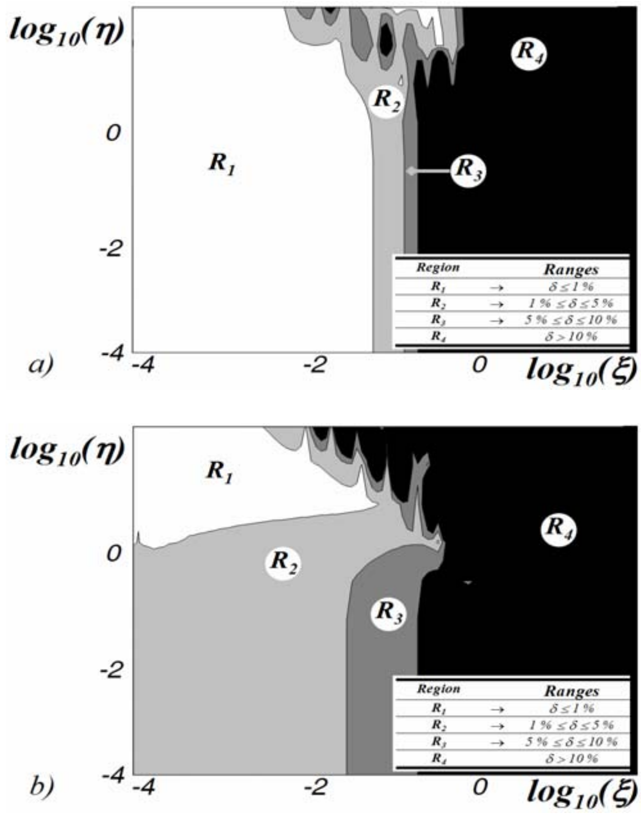

Fig. 7 Broad range contour relative error maps of Lucca formula [3].
a) $\Re e\{\delta\}$, b) $\Im m\{\delta\}$.

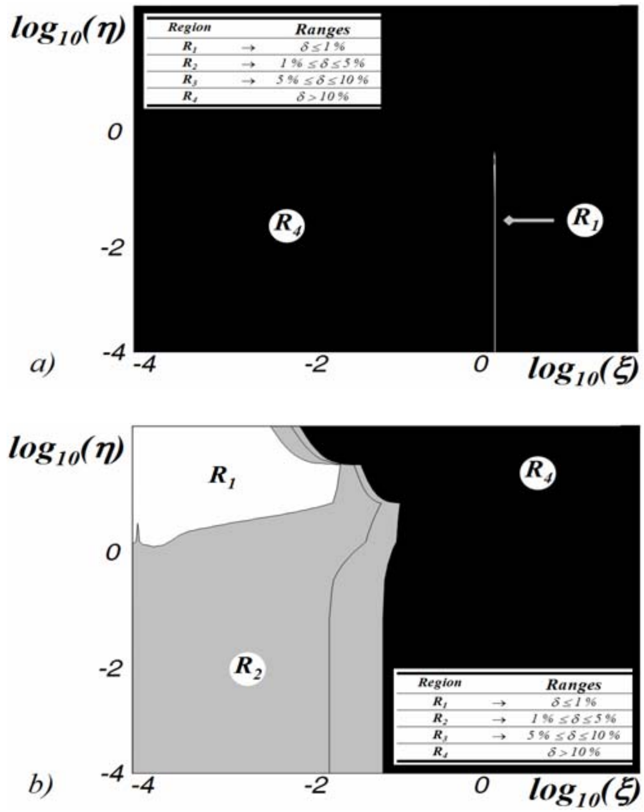

Fig. 8 Broad range contour relative error maps of CCITT formula [4].
a) $\Re\{\delta\}$, b) $\Im m\{\delta\}$.

Advantages of using contour maps as graphical tools for analyzing error levels of closed-form approximations, is that permits the analysis per ranges for each application case. This means that for any system configuration it is possible to determine by direct substitution on $\xi=h_2/|p|$, $\eta=x/h_2$ and $\zeta=h_1/h_2$ the incurred error by using a specific formula.

VII. Conclusions

The Pollaczek coupling integral for calculating mutual impedances between overhead and buried conductors, both with earth-return, has been solved in this paper at a broad range of applications [2].

The presented solution is based on an extension of a previously developed accurate algorithm for solving the Pollaczek integral for calculating earth-return impedances of buried conductors [6, 7]. It has been stated in this paper that Pollaczek coupling integral is a particular damped case of the Pollaczek buried conductors integral.

Through using the concept of normalized impedances and the dimensionless parameters ξ, η and ζ it has been possible to obtain broad range comparisons with two well-known closed-form approximations proposed by Lucca [3] and by the CCITT [4].

Finally, as a graphical tools it has been proposed in this paper the use of contour error maps based on the percent relative error criterion. The maps represent an aim in the evaluation of incurred errors per ranges of closed-form and series approximations of formulas for calculating mutual impedances between overhead and buried conductors, both with earth-return.

VIII. References

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IX. Biography

Felipe Alejandro Uribe was born in Guadalajara, México in July 31, 1970. He received the B. Sc. and M. Sc. Electrical Engineering degrees from The University of Guadalajara, in 1994 and 1998, respectively. In November 15 2002, he obtained the Ph.D. degree in Electrical Engineering from Cinvestav, Guadalajara. Now, Dr. Uribe is with the Electrical Engineering Graduate Program at The Autonomous University of Nuevo León, San Nicolas de los Garza, Nuevo León, México.