Transient Analysis of Single Phase Non Uniform Lines with Frequency Dependent Electrical Parameters

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***Abstract*—In this work a new model for analyzing single phase non uniform transmission lines with frequency dependent electrical parameters is presented. The model is based on synthesizing an equivalent uniform transmission line and can be used when the non-uniformities are symmetric with respect to the center of the line. The frequency and the space dependence of the electrical parameters of the non uniform line are introduced into the equivalent uniform line by means of a transient resistance and a shunt transient conductance.**

***Index Terms*—Non uniform line, Frequency dependent parameters, method of characteristics, transient parameters.**

I. Introduction

For a number of practical cases, such as when one wants to take into account sagging effects or when nonlinear phenomena occurs on some points of a Transmission Line (TL), a knowledge of the voltage and currents on interior points is required. In those cases a discretization of the transmission line length is necessary. Although subdivision of the line in a number of sections for EMTP simulation is possible, this procedure is cumbersome and a great deal of experience is needed in order to define the optimal number of line sections. Moreover, oscillation problems have been reported when simulating nonlinear problems using Bergeron´s method [1].

The method of characteristics, one of several solution methods that discretize both time and distance, has been used successfully in calculating transients on transmission lines with non-uniformities and nonlinear effects. It has been reported that this method does not present the numerical oscillations that are very common in finite difference methods [2-5].

In a recent paper by Semlyen, a new model for time domain analysis of nonuniform multiconductor lines has been presented [6]. This work is based on the chain matrix and the key idea is the determination of the frequency domain propagation functions of an equivalent two-port model. For the case of a single-phase line this method requires the approximation of 10 parameters by rational functions (5 for each transmission line's end). Moreover in some cases the propagation functions can be non-smooth and damping techniques for time domain simulations should be used [7].

In this work the problem of electromagnetic transients in single phase non-uniform transmission lines (NULs) with frequency dependent electrical parameters is analyzed. In particular, non-uniformities with a point of symmetry at the center of the line are considered.

The model proposed in this work is based on synthesizing an equivalent uniform transmission line (UTL) from the chain matrix of the NUL. The frequency and the space dependence of the electrical parameters of the NUL are introduced into the equivalent UTL by means of a transient resistance and a transient conductance. This new model requires the approximation of only these two electrical parameters by rational functions.

Results obtained with the proposed method are compared with those obtained with a Numerical Laplace Transform program (NLT) [8-10], the ATP and a field experiment published elsewhere [16].

II. Cascaded Connection of Chain Matrices

The 2-port frequency domain representation of a single-phase transmission line segment of length Δx is as follows:

$$\begin{bmatrix} V(x+\Delta x, s) \\ I(x+\Delta x, s) \end{bmatrix} = \mathbf{\Phi}(\Delta x, s) \begin{bmatrix} V(x,s) \\ I(x,s) \end{bmatrix} \tag{1}$$

where $\mathbf{\Phi}(\Delta x, s)$ is the chain or ABCD matrix given by:

$$\mathbf{\Phi}(\Delta x, s) = \begin{bmatrix} \cosh(\gamma\,\Delta x) & Y_0^{-1}\sinh(\gamma\,\Delta x) \\ Y_0 \sinh(\gamma\,\Delta x) & \cosh(\gamma\,\Delta x) \end{bmatrix} \tag{2}$$

being γ the propagation constant:

$$\gamma = \sqrt{ZY} \tag{3a}$$

and Y_0 the characteristic admittance of the line segment:

$$Y_0 = \gamma / Z \tag{3b}$$

Z and Y are the longitudinal impedance and transversal admittance of the segment. Eq. (1) can be used to construct a model for non-uniform transmission lines. The procedure consists of (a) dividing the non-uniform line in several segments, (b) computing the chain matrix of each segment and (c) putting together all the chain matrices into an equivalent matrix for the whole line. After dividing the transmission line,

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the entire equivalent chain matrix is obtained as the product, in the appropriate order, of the whole set of chain matrices, as follows [11, 12]:

$$\begin{bmatrix}\mathbf{V}(L,s)\\ \mathbf{I}(L,s)\end{bmatrix} = \mathbf{\Phi}^{(M)}\ldots\mathbf{\Phi}^{(i)}\ldots\mathbf{\Phi}^{(1)}\begin{bmatrix}\mathbf{V}(0,s)\\ \mathbf{I}(0,s)\end{bmatrix} \tag{4a}$$

being L the line length and $\mathbf{\Phi}^{(i)}$ the chain matrix for the i-th line segment. In compact form (4a) becomes

$$\begin{bmatrix}V(L,s)\\ I(L,s)\end{bmatrix} = \begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}V(0,s)\\ I(0,s)\end{bmatrix} \tag{4b}$$

III. TRANSIENT PARAMETERS OF A NON-UNIFORM LINE

For a symmetrical non uniform line, the values of A, B, C and D of Eq. (4b) can be defined as:

$$A = \cosh\left(\gamma_{nu}\, L\right) \tag{5a}$$

$$B = Y_{0,nu}^{-1}\,\sinh\left(\gamma_{nu}\, L\right) \tag{5b}$$

$$C = Y_{0,nu}\,\sinh\left(\gamma_{nu}\, L\right) \tag{5c}$$

$$D = A \tag{5d}$$

where the subscript nu denotes values corresponding to the complete non-uniform line. The propagation constant γ_{nu} is computed from (5a) as follows:

$$\gamma_{nu} = \operatorname{arccosh}\left(A\right)/\,L \tag{6}$$

while the characteristic admittance $Y_{0,nu}$ is computed from (5c) and (6):

$$Y_{0,nu} = C\,\operatorname{csch}\left(\gamma_{nu}\, L\right) \tag{7}$$

From Eq. (3b):

$$Z_{nu} = \gamma_{nu}\,/\,Y_{0,nu} \tag{8}$$

and from Eq. (3a) and (8):

$$Y_{nu} = \left(\gamma_{nu}\right)^2\,/\,Z_{nu} \tag{9}$$

Applying (8) and (9), a uniform line model equivalent to the non-uniform line can be proposed :

$$-\frac{dV(s)}{dx} = Z_{nu}I(s) \quad , \; -\frac{dI(s)}{dx} = Y_{nu}V(s) \tag{10a,b}$$

Z_{nu} and Y_{nu} can be written as follows:

$$Z_{nu} = R'(s) + sL_G \quad , \; Y_{nu} = G'(s) + sC_G \tag{11a}$$

where $R'(s)$ and $G'(s)$ represent the transient resistance and conductance of the line, while L_G and C_G are the geometric inductance and capacitance computed at a mean value of the line non-uniformity. From (11) the transient electrical parameters are given by:

$$R'(s) = Z_{nu}\,/\,s - L_G \quad , \quad G'(s) = Y_{nu}\,/\,s - C_G \tag{12a,b}$$

As explained in the next sections, these values can be included in time domain analysis by means of recursive convolutions using rational approximations.

IV. TIME DOMAIN ANALYSIS

The Telegrapher Equations of a single phase transmission line, including frequency dependence of the line electrical parameters, are defined as follows [13]:

$$-\frac{\partial v}{\partial x} = L_G\,\frac{\partial i}{\partial t} + \frac{\partial}{\partial t}\int_0^t r'(t-\tau)i(\tau)d\tau \tag{13a}$$

$$-\frac{\partial i}{\partial x} = C_G\,\frac{\partial v}{\partial t} + \frac{\partial}{\partial t}\int_0^t g'(t-\tau)v(\tau)d\tau \tag{13b}$$

where $r'(t)$ and $g'(t)$ are the time domain transient longitudinal resistance and shunt conductance, respectively. If $R'(s)$ and $G'(s)$ are synthesized using rational functions and applying the Leibnitz's rule [14], the line equations can be expressed as follows

$$\frac{\partial v}{\partial x} = D\frac{\partial i}{\partial t} + R_x\, i + \psi = 0 \tag{14a}$$

$$\frac{\partial i}{\partial x} = E\,\frac{\partial v}{\partial t} + G_x\, v + \phi = 0 \tag{14b}$$

where:

$$\psi = -\sum_{i=1}^{N_1} k_i p_i \int_0^t e^{-p_i(t-\tau)}\, i(\tau)\, d\tau\,, \tag{15a}$$

$$\phi = -\sum_{i=1}^{N_2} m_i q_i \int_0^t e^{-q_i(t-\tau)}\, v(\tau)\, d\tau\,, \tag{15b}$$

$$R_x = \sum_{i=0}^{N_1} k_i\,, \quad G_x = \sum_{i=0}^{N_2} m_i \tag{15c), (15d}$$

$$D = k_\infty + L_G\,, \quad E = m_\infty + C_G\,, \tag{15e), (15f}$$

Besides, k_i and p_i are the poles and residues of the rational approximation of $R'(s)$, while m_i and q_i are the poles and residues of the rational approximation of $G'(s)$. These poles and residues are computed using the technique known as vector fitting [15].

A. Numerical treatment of the recursive convolutions.

If $\Psi(s)$ is the Laplace domain spectrum of the recursive convolution ψ given in (15a), the rational approximation of $\Psi(s)$ with complex conjugate pairs of poles and residues is given by the following expression:

$$\Psi(s) = -\sum_{i=1}^{N_1} \Psi_i\,, \quad i = 1, 2, \ldots, N_1 \tag{16}$$

where N_1 is the order of the approximation and:

$$\Psi_i = \left(\frac{a_i\, s + b_i\, c_i}{s^2 + d_i\, s + b_i}\right) I(s) \tag{17}$$

where:

$$a_i = k_i\, p_i + k_i^*\, p_i^* \tag{18}$$

$$b_i = p_i\, p_i^*\;,\; c_i = k_i + k_i^*\,, \quad d_i = p_i + p_i^* \tag{19a,b,c}$$

Coefficients a_i, b_i, c_i, and d_i are always real. Eq. (17) can be written as:

$$s^2\,\Psi_i + d_i\, s\,\Psi_i + b_i\,\Psi_i = a_i\, s\, I(s) + b_i\, c_i\, I(s) \tag{20}$$

or in the time domain:

$$\frac{d^2\psi_i}{dt^2}+d_i\,\frac{d\psi_i}{dt}+b_i\,\psi_i=a_i\,\frac{d\,i(t)}{d\,t}+b_i\,c_i\,i(t) \tag{21}$$

Applying the central differences rule to (21) and in accordance to (16), the total convolution can be written as follows:

$$\psi_{n+1}=-\sum_{i=1}^{N_1}\frac{\Delta t}{1+d_i\,\Delta t/2}\left[\frac{a_i}{2}\left(i_{n+1}-i_{n-1}\right)+b_i\,c_i\,\Delta t\,i_n + \frac{\psi_{i,n}\left(2-b_i\,\Delta t^2\right)-\psi_{i,n-1}\left(1-d_i\,\Delta t/2\right)}{\Delta t}\right] \tag{22}$$

and similarly for ϕ

$$\phi_{n+1}=-\sum_{i=1}^{N_2}\frac{\Delta t}{1+h_i\,\Delta t/2}\left[\frac{e_i}{2}\left(v_{n+1}-v_{n-1}\right)+f_i\,g_i\,\Delta t\,v_n + \frac{\phi_{i,n}\left(2-f_i\,\Delta t^2\right)-\phi_{i,n-1}\left(1-h_i\,\Delta t/2\right)}{\Delta t}\right] \tag{23}$$

where the real coefficients e_i, f_i, g_i, and h_i are given by:

$$e_i=m_i\,q_i+m_i^*\,q_i^*\quad,\quad f_i=q_i\,q_i^* \tag{24a,b}$$

$$g_i=m_i+m_i^*\,,\qquad h_i=q_i+q_i^* \tag{24c,d}$$

B. Method of characteristics.

Equations (14a) and (14b) can be represented as:

$$\frac{\partial}{\partial x}\mathbf{U}+\mathbf{A}\frac{\partial}{\partial t}\mathbf{U}+\mathbf{B}\,\mathbf{U}+\mathbf{W}=\mathbf{0} \tag{25}$$

where:

$$\mathbf{U}=\begin{bmatrix}v\\ i\end{bmatrix},\quad \mathbf{A}=\begin{bmatrix}0 & D\\ E & 0\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}0 & Rx\\ Gx & 0\end{bmatrix},\quad \mathbf{W}=\begin{bmatrix}\psi\\ \phi\end{bmatrix}$$

(26a), (26b), (26c), (26d)

The eigenvalues of **A** are given by:

$$\lambda_{1,2}=\pm\sqrt{DE}\,, \tag{27}$$

and the eigenvectors are given by:

$$\mathbf{M}_L=\begin{bmatrix}1 & Z_W\\ 1 & -Z_W\end{bmatrix},\quad \mathbf{M}_R=\begin{bmatrix}1 & 1\\ Y_W & -Y_W\end{bmatrix} \quad (28a), (28b)$$

where:

$$Z_W=\sqrt{D/E}\,,\quad Y_W=Z_W^{-1} \quad (29a), (29b)$$

By left multiplying (14a) and (14b) times $\mathbf{M}_L$ and applying (27) and (29):

$$\begin{bmatrix}\left(\frac{\partial}{\partial x}+\lambda_1\frac{\partial}{\partial t}\right)v+Z_W\left(\frac{\partial}{\partial x}+\lambda_1\frac{\partial}{\partial t}\right)i+R_x\,i+Z_W\,G_x\,v+\psi+Z_W\phi\\ \left(\frac{\partial}{\partial x}+\lambda_2\frac{\partial}{\partial t}\right)v-Z_W\left(\frac{\partial}{\partial x}+\lambda_2\frac{\partial}{\partial t}\right)i+R_x\,i-Z_W\,G_x\,v+\psi-Z_W\phi\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix} \tag{30}$$

Along the characteristic curves defined by $\lambda=\pm\,dt/dx$, the following equivalence can be applied:

$$\left(\frac{\partial}{\partial x}+\lambda_1\frac{\partial}{\partial t}\right)\Leftrightarrow\frac{d}{dx}\quad\text{and}\quad\left(\frac{\partial}{\partial x}+\lambda_2\frac{\partial}{\partial t}\right)\Leftrightarrow\frac{d}{dx} \quad (31a), (31b)$$

Using (31) in (30) it can be written:

$$\begin{bmatrix}d\,v+Z_W\,d\,i+R_x\,i\,dx+Z_W\,G_x\,v\,dx+\left(\psi+Z_W\phi\right)dx\\ d\,v-Z_W\,d\,i+R_x\,i\,dx-Z_W\,G_x\,v\,dx+\left(\psi-Z_W\phi\right)dx\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix} \tag{32}$$

C. Numerical Solution - Internal point

A numerical form of (32) can be found using finite differences. Assuming that v and i are known at points R and S as shown in Fig. 1, it can be written:

$$\left(v_L-v_R\right)+Z_W\left(i_L-i_R\right)+\frac{R_X\,L}{4}\left(i_L+i_R\right)+\frac{Z_W\,G_X\,L}{4}\left(v_L+V_R\right)+\frac{L}{4}\left(\psi_L+\psi_R\right)+\frac{Z_W\,L}{4}\left(\phi_L+\phi_R\right)=0 \tag{33a}$$

$$\left(v_L-v_S\right)-Z_W\left(i_L-i_S\right)-\frac{R_X\,L}{4}\left(i_L+i_S\right)+\frac{Z_W\,G_X\,L}{4}\left(v_L+V_S\right)-\frac{L}{4}\left(\psi_L+\psi_S\right)+\frac{Z_W\,L}{4}\left(\phi_L+\phi_S\right)=0 \tag{33b}$$

Combining (33a) and (33b), expressions for the voltage and current at point L can be found:

$$v_L=\frac{1}{2G_1'}\left[-G_2'\,v_{L-\Delta t}-G_3'\,v_{L-2\Delta t}+G_2\left(v_R+v_S\right)+Z_2\left(i_R-i_S\right)-\frac{L}{4}\left(\psi_R-\psi_S\right)-\frac{Z_W\,L}{4}\left(2\phi_L'+\phi_R+\phi_S\right)\right] \tag{34a}$$

$$i_L=\frac{1}{2Z_1'}\left[-Z_2'\,i_{L-\Delta t}-Z_3'\,i_{L-2\Delta t}+G_2\left(v_R-v_S\right)+Z_2\left(i_R+i_S\right)-\frac{L}{4}\left(2\psi_L'+\psi_R+\psi_S\right)-\frac{Z_W\,L}{4}\left(\phi_R-\phi_S\right)\right] \tag{34b}$$

where

$$G_1=1+\frac{Z_W\,G_X\,L}{4}\,,\quad G_2=1-\frac{Z_W\,G_X\,L}{4} \quad (35), (36)$$

$$Z_1=Z_W+\frac{R_X\,L}{4}\,,\quad Z_2=Z_W-\frac{R_X\,L}{4} \quad (37), (38)$$

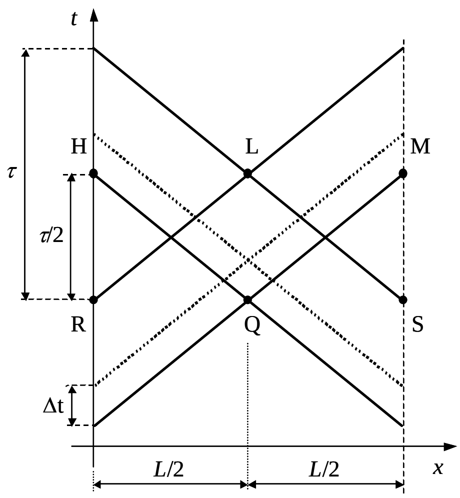

Figure 1. Characteristics grid.

$$\phi_L^{'} = -\sum_{i=1}^{N_2} \frac{\phi_{i,L-\Delta t}\left(2 - f_i\,\Delta t^2\right) - \phi_{i,L-2\Delta t}\left(1 - h_i\,\Delta t/2\right)}{1 + h_i\,\Delta t/2} \quad (39)$$

$$G_1^{'} = G_1 - \frac{Z_W\,L\,\Delta t}{4}\sum_{i=1}^{N_2}\frac{e_i}{2 + h_i\,\Delta t} \quad (40a)$$

$$G_2^{'} = -\frac{Z_W\,L\,\Delta t^2}{2}\sum_{i=1}^{N_2}\frac{f_i\,g_i}{1 + h_i\,\Delta t/2} \quad (40b)$$

$$G_3^{'} = \frac{Z_W\,L\,\Delta t}{2}\sum_{i=1}^{N_2}\frac{e_i}{2 + h_i\,\Delta t} \quad (40c)$$

$$\psi_L^{'} = -\sum_{i=1}^{N_2} \frac{\psi_{i,L-\Delta t}\left(2 - b_i\,\Delta t^2\right) - \psi_{i,L-2\Delta t}\left(1 - d_i\,\Delta t/2\right)}{1 + d_i\,\Delta t/2} \quad (41)$$

$$Z_1^{'} = Z_1 - \frac{L\,\Delta t}{4}\sum_{i=1}^{N_1}\frac{a_i}{2 + d_i\,\Delta t} \quad (42a)$$

$$Z_2^{'} = -\frac{L\,\Delta t^2}{2}\sum_{i=1}^{N_1}\frac{b_i\,c_i}{1 + d_i\,\Delta t/2} \quad (42b)$$

$$Z_3^{'} = \frac{L\,\Delta t}{2}\sum_{i=1}^{N_1}\frac{a_i}{2 + d_i\,\Delta t} \quad (42c)$$

The subscript L-$n\Delta t$ denotes values corresponding to n time steps backward from point L.

D. Numerical Solution - Boundary points

Consider an ideal voltage source $v_H = f(t)$ connected to the initial boundary point H (x = 0), as shown in Fig. 1. In this case (34b) can be solved for for i_H :

$$i_H = \frac{1}{Z_1^{'}}\left[-\frac{Z_2^{'}i_{H-\Delta t} + Z_3^{'}i_{H-2\Delta t}}{2} + G_1 v_H - G_2 V_Q + Z_2\,i_Q - \frac{L}{4}\left(\psi_H^{'} + \psi_Q\right) + \frac{Z_W\,L}{4}\left(\phi_H + \phi_Q\right)\right] \quad (43)$$

where

$$\psi_H^{'} = -\sum_{i=1}^{N_1} \frac{\psi_{i,H-\Delta t}\left(2 - bi\,\Delta t^2\right) - \psi_{i,H-2\Delta t}\left(1 - d_i\,\Delta t/2\right)}{1 + d_i\,\Delta t/2} \quad (44)$$

For the final boundary point M ($x = L$), as shown in Fig. 1, the connection of a resistive load R_L is considered:

$$i_M = v_M / R_L \quad (45)$$

Considering (45), from Eq. (34a) it can be written:

$$v_M = \frac{R_L}{G_1^{'}R_L + Z_1^{'}}\left[-v_{M-\Delta t}\left(\frac{G_2^{'}R_L + Z_2^{'}}{2R_L}\right) - v_{M-2\Delta t}\left(\frac{G_3^{'}R_L + Z_3^{'}}{2R_L}\right) + G_2 V_Q + Z_2 i_Q - \frac{L}{4}\left(\psi_M^{'} + \psi_Q\right) - \frac{Z_W\,L}{4}\left(\phi_M^{'} + \phi_Q\right)\right] \quad (46)$$

where

$$\psi_M^{'} = -\sum_{i=1}^{N_1} \frac{\psi_{i,M-\Delta t}\left(2 - bi\,\Delta t^2\right) - \psi_{i,M-2\Delta t}\left(1 - d_i\,\Delta t/2\right)}{1 + d_i\,\Delta t/2} \quad (47a)$$

$$\phi_M^{'} = -\sum_{i=1}^{N_2} \frac{\phi_{i,M-\Delta t}\left(2 - f_i\,\Delta t^2\right) - \phi_{i,M-2\Delta t}\left(1 - h_i\,\Delta t/2\right)}{1 + h_i\,\Delta t/2} \quad (47b)$$

V. Application Example.

A. Sagging Between Towers.

A single line 600m long with a sagging between towers is analyzed. The line maximum and minimum heights are 28m at the towers and 8m at the middle span. A unit step voltage source is connected to the sending node, while the receiving node is left open. Fig. 2 shows the voltage at the receiving end of the line, comparing the results obtained with the Numerical Laplace Transform, the ATP and the method of characteristics. Results when the line presents no sagging (UL) are also included. Figs. 3 and 4 show a comparison of frequency spectrums of the transient resistance and conductance of the NUL, computed using Eq. (12), against those spectrums obtained with the vector fitting technique.

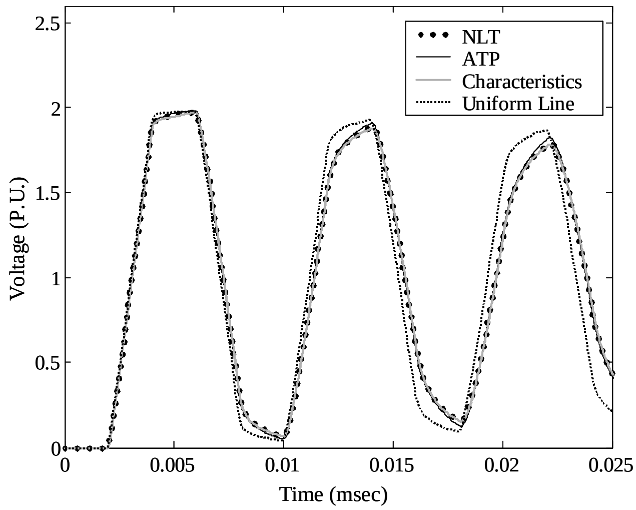

Figure 2. Voltage at the receiving end of the line.

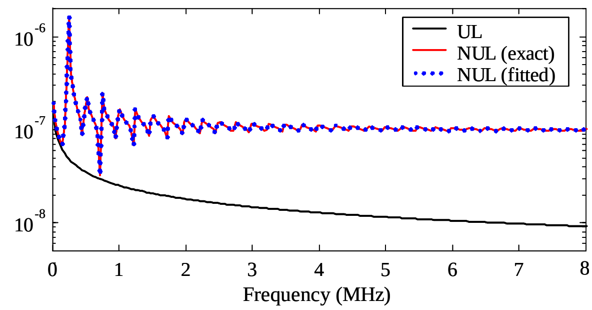

Figure 3. Transient resistance.

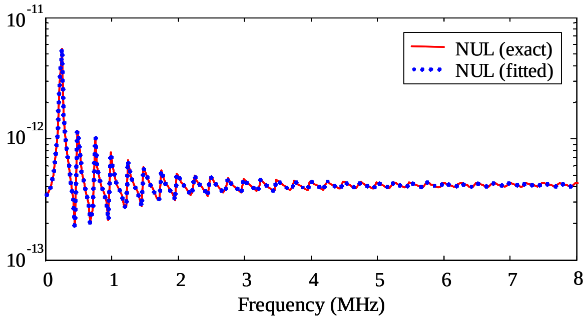

Figure 4. Transient conductance.

B. *Simulation of a Field Experiment.*

As second application example, the proposed method is applied to the simulation of a field experiment performed by Wagner, *et al.* [16]. The experiment consists on injecting a step like wave at one end of a 2185.4m long line divided in 7 equal segments, as shown in Fig. 5. Each segment has a length of 312.2m. The line maximum and minimum heights are 26.2m at the towers and 15.24m at the middle span. The 3 line conductors are ACSR with radius of 2.54cm. The injected wave is applied simultaneously to the 3 conductors at the sending node, while the receiving node is left open. For the simulation, the line is represented by a single-phase equivalent.

The voltage at the receiving end of the line is shown in Fig. 6, comparing the experimental results and those obtained with the method of characteristics. Waveforms were plotted as half of their actual magnitude, as done in [16], to remove the doubling due to the open circuit.

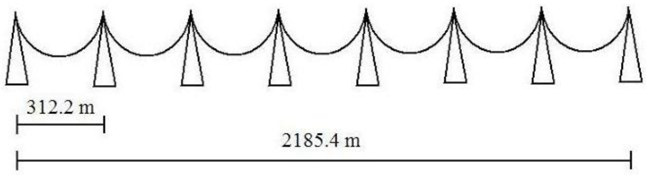

Figure 5. Configuration of the non-uniform line.

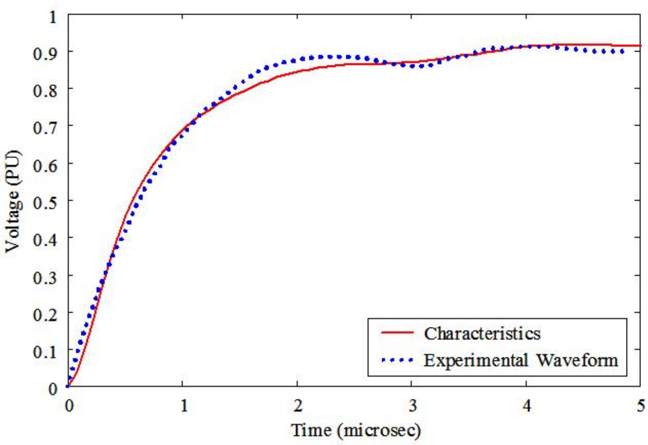

Figure 6. Voltage at the receiving end of the line.

VI. CONCLUSIONS

A time domain model for analyzing single phase non uniform transmission lines with frequency dependent electrical parameters has been presented. The model is based on synthesizing an equivalent uniform transmission line from the chain matrix of the NUL. The application examples show very good agreement between the results obtained with the proposed method and those produced by the Numerical Laplace Transform program, ATP and a field experiment.

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