

# Abrupt Change Detection in Power System Fault Analysis using Wavelet Transform

Abhisek Ukil, *nonmember*, Rastko Živanović, *Member, IEEE*

**Abstract**-- This paper describes the application of the wavelets used to detect the abrupt changes in the signals recorded during disturbances in the electrical power network in South Africa. Main focus has been to estimate exactly the time-instants of the changes in the signal model parameters during the pre-fault condition and following events like initiation of fault, circuit-breaker opening, auto-reclosure of the circuit-breakers using the wavelet transform, particularly the dyadic-orthonormal wavelet transform. The key idea is to decompose the fault signals into effective detailed and smoothed version using the multiresolution signal decomposition technique based on discrete wavelet transform. Then we apply the threshold method on the decomposed signals to estimate the change time-instants, segmenting the fault signals. After segmenting the fault signal precisely into the event-specific sections, further signal processing and analysis can be performed on these segments, leading to automated fault recognition and analysis. In the scope of this paper, we focus on the first task i.e., segmentation of the fault signal into event-specific sections using the wavelet transform and threshold method. This paper presents application on recorded signals in the transmission network of South Africa.

**Keywords:** Abrupt change detection, Power system fault analysis, Wavelet transform.

## I. INTRODUCTION

**D**ETECTION of abrupt changes in signal characteristics is a much studied subject with many different approaches. It has significant role to play in failure detection and isolation (FDI) systems; one such domains viz., power systems fault analysis is the focus of this paper. In this paper, we propose the use of the wavelet transform, particularly the dyadic-orthonormal wavelet transform for estimating the time-instants of the abrupt changes in the power system fault signals.

Wavelet transform is particularly suitable for the power system disturbance and fault signals which may not be periodic and may contain both sinusoidal and impulse components. Also, for the power system fault analysis, time-frequency resolution is needed which states another reason for using the wavelet transform because it provides a local representation (both in time and frequency) of a given signal unlike the Fourier transform which provides a global

representation of a signal. In particular, the ability of the wavelets to focus on short intervals for high-frequency components and long intervals for low-frequency components improves the decomposition of the fault signals into finer and detailed scales, facilitating further effective signal processing and analysis.

In this paper, wavelet transform is used to transform the original fault signal into finer wavelet scales, followed by a progressive search for the largest wavelet coefficients on that scale [1]. Large wavelet coefficients that are co-located in time across different scales provide estimates of the changes in the signal parameter. The change time-instants can be estimated by the time-instants when the wavelet coefficients exceed a given threshold (which is equal to the '*universal threshold*' of Donoho and Johnstone [2] to a first order of approximation).

The remainder of this paper is organized as follows. In Section-II, wavelet transform is reviewed briefly. In Section-III, power system fault analysis as application domain is discussed. In Section-IV, signal decomposition using wavelet transform is discussed. Utilization of the threshold method for segmentation is explained in Section-V. Practical application results are presented in Section VI, and conclusions are given in Section VII.

## II. WAVELET TRANSFORM ANALYSIS

The Wavelet transform (WT) is a mathematical tool, like the Fourier transform for signal analysis. A wavelet is an oscillatory waveform of effectively limited duration that has an average value of zero. Fourier analysis consists of breaking up a signal into sine waves of various frequencies. Similarly, wavelet analysis is the breaking up of a signal into shifted and scaled versions of the original (or mother) wavelet. In Fig. 1, we show the basis functions for Fourier transform (Sine wave) and WT (*db10*: Daubechies 10 mother wavelet [3]).

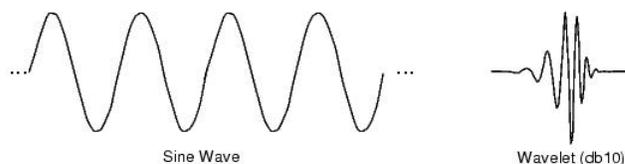


Fig. 1. Basis Functions for Fourier Transform & Wavelet Transform

Fourier analysis does not provide good results for the *non-stationary* signals, i.e., where the signal parameters change over the time unlike the *stationary* signal, because in

---

This work was supported in part by the National Research Foundation (NRF), South Africa.

Abhisek Ukil is with Tshwane University of Technology, Pretoria 0001, South Africa (e-mail: abhiseku\_17@yahoo.com).

Rastko Živanović is with Tshwane University of Technology, Pretoria 0001, South Africa (e-mail: zivanovr@yahoo.com).

transforming the complete signal to the frequency domain, the time information gets lost in Fourier analysis. This deficiency in the Fourier analysis can be overcome to some extent by analyzing a small section of the signal at a time - a technique called *windowing* the signal, first proposed by Dennis Gabor. This leads to an analysis technique called Short-Time Fourier Transform (STFT). But the drawback in STFT is that the size of the time-window is same for all frequencies. Wavelet analysis overcomes this deficiency by allowing a windowing technique with variable-sized regions, i.e., wavelet analysis allows the use of long time intervals where we want more precise low-frequency information, and shorter regions where we want high-frequency information. In Fig. 2, we show the time-domain (Shannon), frequency-domain (Fourier), STFT (Gabor) and wavelet views of signal analysis.

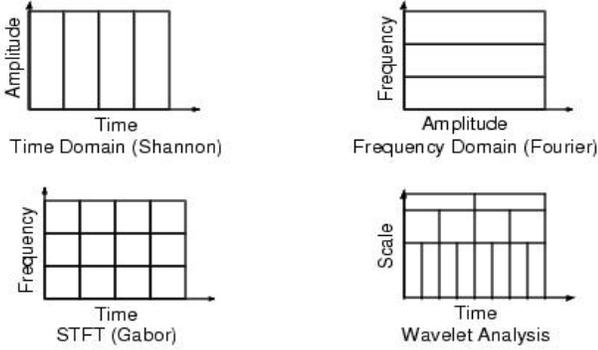


Fig. 2. Time, Frequency, STFT, Wavelet views of signal analysis

While detail mathematical descriptions of WT can be referred to in [3], [4], a brief mathematical summary of WT is provided in the following sections in relation to the application domain within the scope of this paper.

### A. Continuous Wavelet Transform

The continuous wavelet transform (CWT) is defined as the sum over all time of the signal multiplied by scaled and shifted versions of the wavelet function  $\psi$ . The CWT of a signal  $x(t)$  is defined as

$$CWT(a, b) = \int_{-\infty}^{\infty} x(t) \psi_{a,b}^*(t) dt, \quad (1)$$

where

$$\psi_{a,b}(t) = |a|^{-1/2} \psi((t-b)/a). \quad (2)$$

$\psi(t)$  is the *mother* wavelet, the asterisk in (1) denotes a complex conjugate, and  $a, b \in R, a \neq 0$ , ( $R$  is a real continuous number system) are the *scaling* and *shifting* parameters respectively.  $|a|^{-1/2}$  is the normalization value of  $\psi_{a,b}(t)$  so that if  $\psi(t)$  has a unit length, then its scaled version  $\psi_{a,b}(t)$  also has a unit length.

### B. Discrete Wavelet Transform

Instead of continuous scaling and shifting, the mother wavelet maybe scaled and shifted discretely by choosing  $a = a_0^m, b = na_0^m b_0, t = kT$  in (1), where  $T=1.0$  and  $k, m, n \in Z$ , ( $Z$  is the set of positive integers). Then, the discrete wavelet transform (DWT) is given by

$$DWT(m, n) = a_0^{-m/2} \left( \sum x[k] \psi^*[(k - na_0^m b_0)/a_0^m] \right). \quad (3)$$

By careful selection of  $a_0$  and  $b_0$ , the family of scaled and shifted mother wavelets constitutes an orthonormal basis. An orthonormal basis is a basis that consists of a set of vectors  $\mathbf{S}$  such that  $\mathbf{u} \cdot \mathbf{v} = 0$  (here ‘ $\cdot$ ’ indicates the dot product) for each distinct pair of  $\mathbf{u}, \mathbf{v} \in \mathbf{S}$ . We can choose  $a_0 = 2$  and  $b_0 = 1$  to constitute the orthonormal basis to have the WT called a *dyadic-orthonormal* WT. The implications of the dyadic-orthonormal WT is that due to the orthonormal properties there will be no information redundancy in the decomposed signals. Also, with this choice of  $a_0$  and  $b_0$ , there exists a novel algorithm, known as *multiresolution signal decomposition* [5] technique, to decompose a signal into scales with different time and frequency resolution.

### C. Multiresolution Signal Decomposition and Quadrature Mirror Filter

The *Multiresolution Signal Decomposition* (MSD) [5] technique decomposes a given signal into its detailed and smoothed versions. Let  $x[n]$  be a discrete-time signal, then MSD technique decomposes the signal in the form of WT coefficients at scale 1 into  $c_1[n]$  and  $d_1[n]$ , where  $c_1[n]$  is the smoothed version of the original signal, and  $d_1[n]$  is the detailed version of the original signal  $x[n]$ . They are defined as

$$c_1[n] = \sum_k h[k - 2n] x[k], \quad (4)$$

$$d_1[n] = \sum_k g[k - 2n] x[k], \quad (5)$$

where  $h[n]$  and  $g[n]$  are the associated filter coefficients that decompose  $x[n]$  into  $c_1[n]$  and  $d_1[n]$  respectively. Downsampling is done in the process of decomposition so that the resulting  $c_1[n]$  and  $d_1[n]$  are each  $n/2$  point signals. Thus, for the original  $n$  point signal  $x[n]$ , after the decomposition we have  $n$  point signal together with  $c_1[n]$  and  $d_1[n]$ , not  $2n$  point.

The next higher scale decomposition will be based on  $c_1[n]$ . Thus, the decomposition process can be iterated, with successive approximations being decomposed in turn, so that the original signal is broken down into many lower resolution components. This is called the *wavelet decomposition tree* [4].

MSD technique can be realized with the cascaded *Quadrature Mirror Filter* (QMF) [4] banks. A QMF pair consists of two finite impulse response filters, one being a

low-pass filter (LPF) and the other a high-pass filter (HPF). The QMF pair divides the input signal into low-frequency and high-frequency components at the dividing point of halfway between zero hertz and half the data sampling frequency. The output of the low-pass filter is the smoothed version of the input signal and used as the next QMF pair's input. The output of the high-pass filter is the detailed version of the original signal. Thus cascaded QMF pairs realize the MSD technique. Detail description about QMF can be found in [6]. Fig. 3 shows the MSD technique and QMF pair.

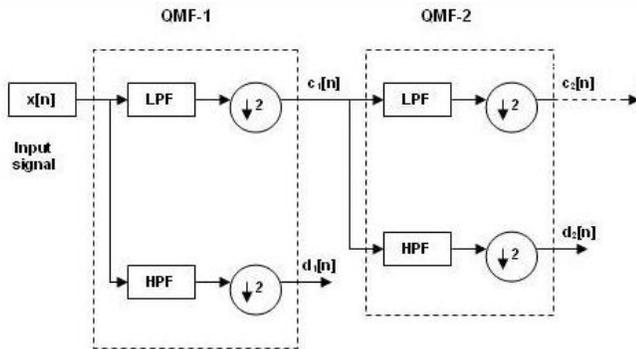


Fig. 3. Multiresolution Signal Decomposition

### III. POWER SYSTEM FAULT ANALYSIS

We consider the power system fault analysis as our application domain in the scope of this paper. More specifically, we focus on the automated fault analysis for the transmission network in South Africa. Presently, 98% of the transmission lines are equipped with the digital fault recorders (DFRs) on the feeder bays, with an additional few installed on the Static Var Compensators (SVCs) and 95% of these are remotely accessible via a X.25 communication system [7]. The DFRs and the associated settings are applied for the purpose of protection performance and disturbance analysis [8]. The DFRs trigger due to reasons like, power network fault conditions; protection operations; breaker operation and the like. Following IEEE COMTRADE standard [9], the DFR recordings are provided as input to the analysis software which uses Discrete Fourier Analysis and Superimposed current quantities [7].

The purpose of this study is to augment the existing fault analysis system with more robust and accurate algorithms and techniques to make it fully automated. So, we would first apply the abrupt changes detection algorithms to segment the fault recordings into different segments, viz., pre-fault segment, after initiation of fault, after circuit-breaker opening, after auto-reclosure of the circuit-breakers. Then on the each different segments specific signal processing and analysis would be performed to accomplish the fault recognition and analysis tasks. In the scope of this paper, we focus on the first task i.e., segmentation of the fault recordings by detecting the abrupt changes in the characteristics of the fault recordings using the WT. As power system fault and disturbance signals consist of abrupt changes, sharp edges, transitions and the

like, so WT analysis as proposed in this paper is quite suitable and accurate for the purpose.

### IV. SIGNAL DECOMPOSITION

In this section, we apply the multiresolution signal decomposition technique and quadrature mirror filter banks to decompose the fault signals from the DFRs into localized and detailed representation in the form of wavelet coefficients. Daubechies 1 and 4 wavelets are used as mother wavelets, i.e., the filters  $h[n]$  and  $g[n]$  as in (4) & (5) are chosen with one and four coefficients respectively and calculated as in [3]. Daubechies 1 wavelet can also be referred as ‘Haar’ wavelet [4].

Among many other choices of the mother wavelets, e.g., Coiflets, Meyer wavelet, Gaussian wavelet, Mexican hat wavelet, Morlet wavelet etc [3], Daubechies 1 and 4 wavelets have been chosen because Daubechies wavelets are compactly supported [3] wavelets with extremal phase and highest number of vanishing moments for a given support width [3], also the associated scaling filters are minimum-phase filters [3]. So, from the point of views of fast implementation and varying patterns of the fault signals, Daubechies wavelets appear to be the optimal choice for the mother wavelet for this specific application.

Daubechies 1 wavelet has the following mathematical description.

The scaling function  $\phi(x)$  is defined as

$$\phi(x) = 1, \quad \text{if } x \in [0,1],$$

$$\phi(x) = 0, \quad \text{if } x \notin [0,1]. \quad (6)$$

The wavelet function  $\psi(x)$  for this scaling function is defined as

$$\psi(x) = 1, \quad \text{if } x \in [0,0.5],$$

$$\psi(x) = -1, \quad \text{if } x \in [0.5,1],$$

$$\psi(x) = 0, \quad \text{if } x \notin [0,1]. \quad (7)$$

Fig. 4 shows the wavelet function for the Daubechies 1 mother wavelet.

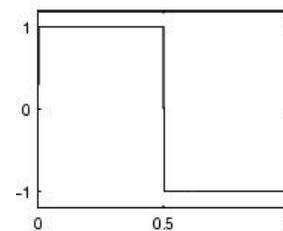


Fig. 4. Daubechies 1 Wavelet Function

For the Daubechies 4 wavelet, the scaling function  $\phi(x)$  has the form

$$\phi(x) = c_0\phi(2x) + c_1\phi(2x-1) + c_2\phi(2x-2) + c_3\phi(2x-3), \quad (8)$$

where

$$c_0 = (1 + \sqrt{3})/4, \quad (9)$$

$$c_1 = (3 + \sqrt{3})/4, \quad (10)$$

$$c_2 = (3 - \sqrt{3})/4, \quad (11)$$

$$c_4 = (1 - \sqrt{3})/4. \quad (12)$$

It is not possible in general to solve directly for  $\phi(x)$ ; the approach is to solve for  $\phi(x)$  iteratively until  $\phi_j(x)$  is very nearly equal to  $\phi_{j-1}(x)$ , where

$$\begin{aligned} \phi_j(x) = & c_0\phi_{j-1}(2x) + c_1\phi_{j-1}(2x-1) + c_2\phi_{j-1}(2x-2) \\ & + c_3\phi_{j-1}(2x-3). \end{aligned} \quad (13)$$

The Daubechies 4 wavelet function  $\psi(x)$  for the four-coefficient scaling function is given by

$$\psi(x) = -c_3\phi(2x) + c_2\phi(2x-1) - c_1\phi(2x-2) + c_0\phi(2x-3). \quad (14)$$

The Daubechies 4 scaling function and wavelet function are shown in Fig. 5 (a) & (b) respectively.

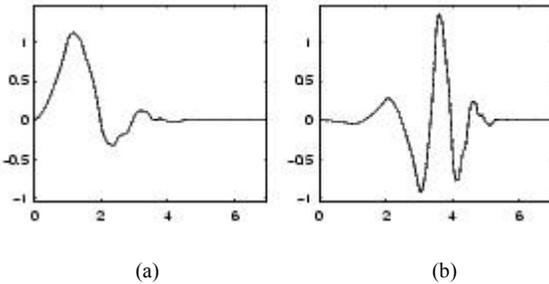


Fig. 5. Daubechies 4: (a) Scaling Function, (b) Wavelet Function

After transforming the original fault signal using the mother wavelets described above, we obtain the smoothed and detailed versions, viz.,  $c_1[n]$  and  $d_1[n]$  [see (4), (5)]. Signal  $d_1[n]$  can be considered to be the difference between the original signal  $x[n]$  and  $c_1[n]$ , and called the wavelet transform coefficient at scale one. We will use  $d_1[n]$  for threshold checking to estimate the change time-instants described in the following section.

#### V. APPLICATION OF THRESHOLD METHOD

We will use the threshold method on the wavelet transform coefficients of the original fault signal to detect the jumps and sharp cusps [10] in order to estimate the time-instants of the abrupt changes. Mathematically we say that signal  $x(t)$  has a sharp  $\alpha$ -cusp at  $t$  if for  $0 \leq \alpha < 1$ ,

$$|x(t + \Delta t) - x(t)| \geq K |\Delta t|^\alpha, \quad (15)$$

as  $\Delta t \rightarrow 0$  for some constant  $K \geq 0$ . We can consider it to be a jump if  $\alpha = 0$ . In practice, we can consider a cusp to be an abrupt change of the level of the trend over a small time period.

As discussed in the previous section, after transforming the original fault signal using the wavelet transform, we will search progressively across the finer wavelet scales for the largest wavelet coefficients on that scale [1]. As wavelet coefficients are the changes of the averages, so a coefficient of large magnitude implies a large change in the original signal. Large wavelet coefficients that are co-located in time across different scales provide estimates of the cusp points [1] hence time-instants of the abrupt changes. The change time-instants can be estimated by the instants when the wavelet coefficients exceed a given threshold which is equal to the ‘universal threshold’ of Donoho and Johnstone [2] to a first order of approximation.

The universal threshold  $T$  is given by

$$T = \sigma \sqrt{2 \log_e n}, \quad (16)$$

where  $\sigma$  is the median absolute deviation of the wavelet coefficients, divided by 0.6725 [2] and  $n$  is the number of samples of the wavelet coefficients. Instead of standard deviation median absolute deviation is used because median is hardly influenced by a small fraction of extreme values [10].

After determining the time-instants when the wavelet coefficients of the fault signal exceed the threshold, we mark them using unit impulses, indicating the abrupt change time-instants.

#### VI. APPLICATION RESULTS

In this section, we present the practical application results of the power system fault analysis method developed according to the above discussed signal decomposition and representation using the WT and then applying the threshold method on the detailed version of the fault signal, followed by heuristic smoothing filtering operation. MATLAB<sup>®</sup> with Wavelet toolbox [11] has been used for implementing the application. The whole procedure detects the change time-instants thus segments the fault signal. We are interested in the change time-instants to be indicated as unit impulses.

After normalizing the original fault signal using its mean value, it is transformed into the smoothed and detailed version using the WT and then the threshold method is applied on the detailed version to determine the change time-instants. Then smoothing filter operations are applied on this segmented model to perform sequentially the following smoothing operations:

- It removes confusing multiple close-spikes and combines them into single unit impulse.

- It removes any unwanted glitches which can otherwise result in false positives for the abrupt changes.
- The segments in the power system fault analysis signals are during the pre-fault condition and following events like fault initiation, circuit-breaker opening and reclosing. These events are predefined and so are the number of segments. So, any bigger number of segmentation possibly indicates transients, power swings and the like. Estimation of the number of segment(s) is also performed and checked.
- Based on the modeling of the segments, analysis is done for estimating the event-critical change instants, rejecting others.

Fig. 6 shows the result for the fault signal, sampled at a sampling frequency of 2.5 kHz [7], obtained from the Eskom DFRs during a phase to ground fault. The WT uses ‘Daubechies 1’ [3] mother wavelet.

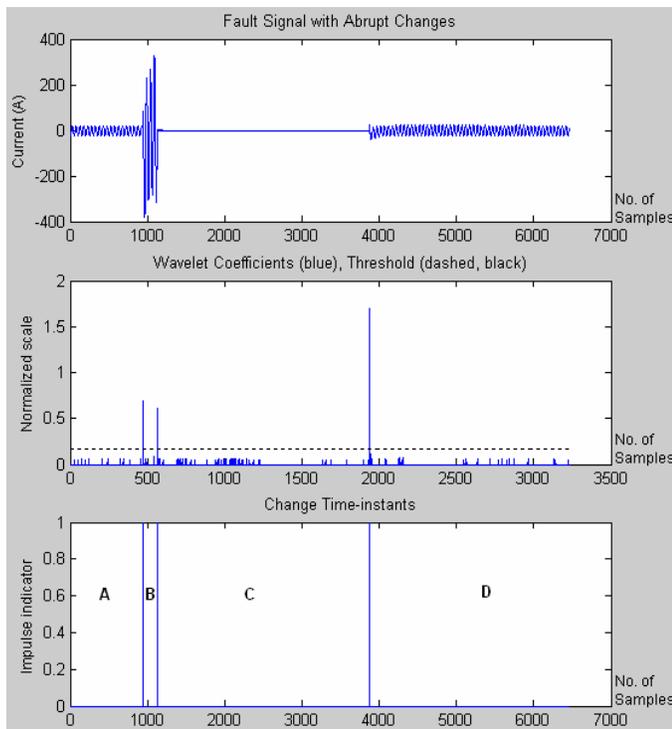


Fig. 6. Segmentation of the RED-Phase Current signal

In Fig. 6, the original DFR recording for the current during the fault in the RED-Phase is shown in the top section, wavelet coefficients for this fault signal (in blue) and the universal threshold (in black, dashed) are shown in the middle section and the change time-instants computed using the threshold checking (middle section) followed by smoothing filtering is shown in the bottom section. It is to be noted that only the high-pass filter output of the QMF pair is shown, so

the wavelet coefficients in the middle section indicate half of the total samples of the original signal. The time-instants of the changes in the signal characteristics, in the lower plot in Fig. 6, indicate the different signal segments owing to different events during the fault, e.g., segment A indicates the pre-fault section and the fault inception, segment B indicates the fault, segment C indicates opening of the circuit-breaker, segment D indicates auto-reclosing of the circuit-breaker and system restore.

Fig. 7 shows another result for the RED-phase voltage recording, sampled at a sampling frequency of 2.5 kHz [7], from the Eskom DFRs during a phase to ground fault. The WT uses ‘Daubechies 4’ [3] mother wavelet.

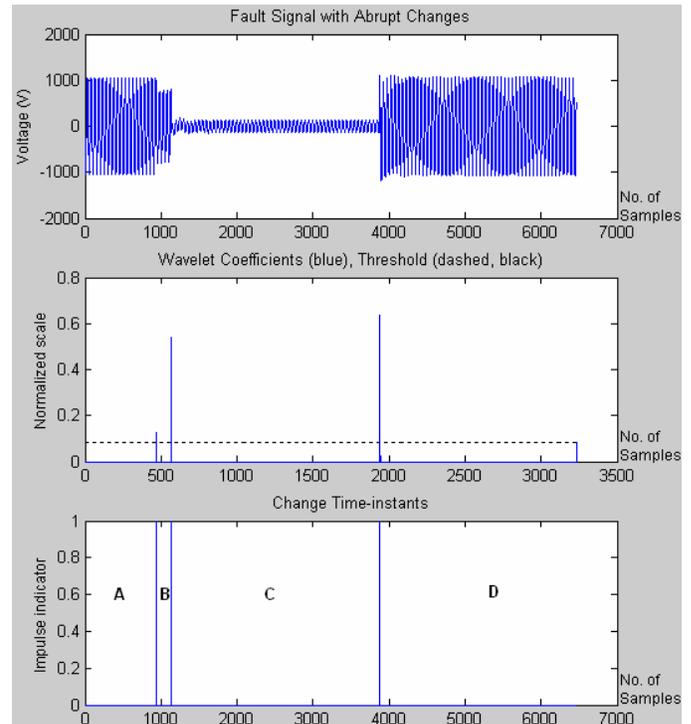


Fig. 7. Segmentation of the RED-Phase Voltage signal

In Fig. 7, the original DFR recording for the voltage during the fault in the RED-Phase is shown in the top section, wavelet coefficients for this fault signal (in blue) and the universal threshold (in black, dashed) are shown in the middle section and the change time-instants computed using the threshold checking (middle section) followed by smoothing filtering is shown in the bottom section. It is to be noted that only the high-pass filter output of the QMF pair is shown, so the wavelet coefficients in the middle section indicate half of the total samples of the original signal. The time-instants of the changes in the signal characteristics in the lower plot in Fig. 7 indicate the different signal segments owing to different events during the fault, e.g., segment A indicates the pre-fault section and the fault inception, segment B indicates the fault, segment C indicates opening of the circuit-breaker, segment D indicates auto-reclosing of the circuit-breaker and system restore.

### A. Comments on Application Results

Following the discussion of the applied algorithms and the application results, the following comments can be cited.

- The intended application is not meant for real-time analysis, so computation time is not a critical factor. However, the proposed algorithm for the abrupt change detection and signal segmentation took an average computation time of 0.431 seconds. An Intel® Celeron® 1.9 GHz computer was used for all the application tests using MATLAB® [11]. It is to be noted that the complete automatic disturbance recognition and analysis tasks have to be performed within five minutes of the acquiring of the fault signals, abrupt change detection and segmentation being the first step.
- The proposed algorithm using the wavelet transform is considerably faster and more robust compared to the traditional peak value detection and superimposed current quantities algorithms [7]. Also, this algorithm based on the wavelet transform and threshold method facilitates further signal processing and analysis in the subsequent stages of automatic disturbance recognition and analysis, focusing on the different segments and helping to determine quickly parameters like duration of the fault etc directly from the abrupt change detection based segmentation itself. This cannot be done using the traditional peak value detection and superimposed current quantities algorithms [7].
- Instead of Fourier transform, Wavelet transform is particularly suitable for the power system disturbance and fault signals which may not be periodic and may contain both sinusoidal and impulse components.
- Wavelet coefficients are greatly adaptive to the fault signal pattern variations.
- Wavelet transform provides a local representation (both in time and frequency) of a given signal, thus the necessary time-frequency resolution for the power system fault analysis can be achieved. This is not possible with the traditional Fourier transform which provides a global representation of a signal.

### VII. CONCLUSIONS

We have presented in this paper the wavelet transform used for detecting the abrupt changes in the signals recorded during disturbances in the transmission network of South Africa. Power system disturbance and fault signals may not be periodic and may contain both sinusoidal and impulse components. So, we propose the use of wavelet transform, particularly the dyadic-orthonormal wavelet transform to decompose the original fault signal into the smoothed and detailed version in terms of the wavelet coefficients using the multiresolution signal decomposition technique. Then we make a progressive search on that wavelet scale for the largest wavelet coefficients. The change time-instants can be

estimated by the time-instants when the wavelet coefficients exceed a given threshold (which is equal to the 'universal threshold' of Donoho and Johnstone [2] to a first order of approximation). This is followed by smoothing operation. We have been mainly interested in estimating the change time-instants and the results obtained from the MATLAB® implementation are quite good. So, the use of the dyadic-orthonormal wavelet transform to transform the fault signals into the smoothed and detailed version, followed by the threshold checking is quite effective in detecting the abrupt changes in the signals originating from power system faults to segment them into the event-specific sections.

### VIII. REFERENCES

- [1] P.F. Craigmile and D.B. Percival, "Wavelet-Based Trend Detection and Estimation", Department of Statistics, Applied Physics Laboratory, University of Washington, Seattle, WA, 2000.
- [2] D.L. Donoho and I.M. Johnstone, "Ideal Spatial Adaptation by Wavelet Shrinkage", *Biometrika*, vol. 81, no. 3, pp. 425-455, 1994.
- [3] I. Daubechies, *Ten Lectures on Wavelets*, Philadelphia: Society for Industrial and Applied Mathematics, 1992.
- [4] S. Mallat, *A wavelet tour of signal processing*, Academic Press, 1998.
- [5] S. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 11, no. 7, pp. 674-693, July 1989.
- [6] G. Strang and T. Nguyen, *Wavelets and filter banks*, Wellesley-Cambridge Press, 1996.
- [7] E. Stokes-Waller, "Automated Digital Fault Recording Analysis on the Eskom Transmission System", Southern African Conference on Power System Protection, 1998.
- [8] E. Stokes-Waller and P. Keller, "Power Network and Transmission System based on Digital Fault Records", Southern African Conference on Power System Protection, 1998.
- [9] *IEEE Standard Common Format for Transient Data Exchange (COMTRADE) for Power Systems*, IEEE Standard C37.111-1991, Version 1.8, February 1991.
- [10] Y. Wang, "Change curve estimation via wavelets", *Journal of American Statistical Association*, vol. 93, pp. 163-172, 1998.
- [11] MATLAB® Documentation – Wavelet Toolbox, Version 6.5.0.180913a Release 13, The Mathworks Inc., Natick, MA.

### IX. BIOGRAPHIES



**Abhisek Ukil** received the B.E. degree in electrical engineering from the Jadavpur University, Calcutta, India, in 2000 and the M.Sc. degree in electronic systems and engineering management from the University of Applied Sciences, South Westphalia, Soest, Germany, and Bolton Institute, UK, in 2004.

He is currently pursuing the D.Tech. degree at Tshwane University of Technology, Pretoria, South Africa.



**Rastko Živanović** (M'97) received the Dipl.Ing. and M.Sc. degrees from the University of Belgrade, Belgrade, Serbia, in 1987 and 1991, and the Ph.D. degree from the University of Cape Town, Cape Town, South Africa, in 1997.

Currently, he is Professor with the Faculty of Engineering at Tshwane University of Technology, Pretoria, South Africa, where he has also been Lecturer and Senior Lecturer since 1992. His research interests include power system protection and control.