Clarke Extended Methodology for Two Three-phase Double-circuit Lines

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***Abstract*— A single real transformation matrix is applied to typical symmetrical systems with three-phase double-circuit transmission lines or two parallel three-phase double-circuit transmission lines. The objective is to determine the characteristic matrices (Z and Y) in the mode domain. The proposed analyses are based on eigenvector and eigenvalue studies, using linear combinations of Clarke matrix elements. Using the proposed extended techniques, Z and Y diagonal matrices are obtained for transposed cases of three-phase double-circuit line systems (a six order transformation matrix). In this case, the error comparisons between the exact eigenvalues and the results of a single real transformation matrix are presented when the non-transposed line is considered.**

Index Terms-Clarke matrix, eigenvector, eigenvalue, frequency, mode domain transformation, transmission lines.

I. Introduction

TRADITIONALLY, when the line impedance matrix (Z) and the line admittance matrix (Y) are considered frequency dependent, the diagonal YZ product determination in mode domain is a problem that requires working with frequency dependent transformation matrices.The use of exact eigenvectors (T_V and T_I matrices) leads to slow digital transient simulations. In phase domain line models, the transformation matrices are not needed, but the simulations can also be slow because of the phase domain numerical manipulations. [1]

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Therefore, an alternative is a single real transformation matrix application [1, 2]. The purpose of this paper is to replace the matrices T_V and T_I, and the eigenvectors of ZY and YZ, by a single real transformation matrix. For ideally transposed cases, considering a three-phase double-circuit line, the Z, Y, YZ product and ZY product matrices are transformed into diagonal matrices with this single real transformation matrix application. The proposed development uses a single homopolar mode reference for all phase conductors of the system, leading to exact eigenvalues for the three-phase double-circuit line cases. For two parallel three-phase double-circuit line cases, the most non-diagonal elements are zero. Considering a real three-phase double-circuit line case, the comparison between the exact eigenvalues and the results of the single real transformation matrix is represented. The errors are smaller than 2% (in modulus) and for some modes, the error is close to zero.

II. Mathematical Equations

After the proper determination of the electrical parameters (longitudinal impedance and transversal admittance values) in phase domain, the proposed methodology can be performed to mode analysis and electromagnetic transient simulations. In this paper, only mode analyses are performed. So, the relationships between transversal voltages u_F and the longitudinal currents i_F can be expressed by the following equations:

$$-\frac{du_F}{dx} = Z.i_F \quad (1)$$

$$-\frac{di_F}{dx} = Y.u_F \quad (2)$$

Applying the eigenvector and eigenvalue analyses for YZ and ZY product matrices, the λ diagonal eigenvalue matrix and the eigenvector matrices are determined. The eigenvector matrices, T_V and T_I, correspond to Z and Y matrices, respectively. The T_V and T_I matrices are related to equations 1 and 2, based on the following equation:

$$\lambda = T_V.Z.Y.T_V^{-1} = T_I.Y.Z.T_I^{-1} \tag{3}$$

If the T_V and T_I transformation matrices are used, equations (1) and (2) can be obtained in mode domain. The per unit length longitudinal impedance matrix (Z_{MD}) and transversal admittance matrix (Y_{MD}) are:

$$Z_{MD} = T_V.Z.T_I^{-1} \tag{4}$$

$$Y_{MD} = T_I.Y.T_V^{-1} \tag{5}$$

In general, these frequency dependent transformation matrices are different and have complex elements. Using the proposed methodology, the transformation matrices are changed into a single real transformation matrix (T_{SR}). The (T_{SR}) matrix is determined from linear combinations of Clarke matrix elements[3,4,5,6]. So, equation (3) is changed into the following:

$$\lambda_{SR} = T_{SR}.Y.Z.T_{SR}^{-1} = T_{SR}.Z.Y.T_{SR}^{-1} \tag{6}$$

Considering a single homopolar mode reference, the λ_{SR} matrix is equal to an exact eigenvalue matrix (λ)[7] as well as T_V and T_I being eigenvector matrices[8,9,10] for a transposed three-phase double-circuit line. This single homopolar mode reference is the link between the two three-phase circuits of the system. With this technique , a transformation matrix (T_{SR}) is obtained which has interesting characteristics: single, real, frequency independent, line parameter independent and identical to voltages and currents. In this paper, the proposed single homopolar mode reference is extended to a system with two parallel three-phase double-circuit lines and its results are analyzed.

III. THREE-PHASE DOUBLE-CIRCUIT LINE SYSTEMS

For this line type, an ideal transposition is assumed. Each three-phase circuit is ideally transposed, generating only one coupling impedance within a circuit and only one coupling impedance among the circuits. The generic structure and the average coupling impedance values of this line transposition type are shown in Figure 1. The average self phase impedance value is represented by A. The average coupling impedances are represented by R, within a circuit, and P, among the circuits. This transposition type can be considered an idealization of common three-phase double-circuit lines.

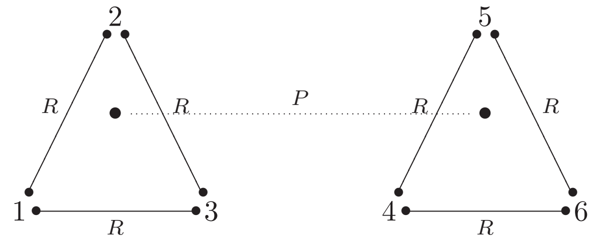

Fig. 1. Coupling impedances for operational transposition

For this case, the structure of the impedance matrix is shown in equation 6. The Y matrix has a similar structure to that of Z.

$$\mathbf{Z_{DL}} = \begin{bmatrix} A & R & R & P & P & P \\ R & A & R & P & P & P \\ R & R & A & P & P & P \\ P & P & P & A & R & R \\ P & P & P & R & A & R \\ P & P & P & R & R & A \end{bmatrix} \tag{7}$$

The result determined through equation (6) is a diagonal matrix and the matrix elements are the exact eigenvalues, if the T_{SR} transformation matrix is described by:

$$\mathbf{T_{SR}} = \begin{bmatrix} -\frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & -\frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \end{bmatrix} \tag{8}$$

The Z_{MDDL} is calculated as:

$$Z_{MDDL} = T_{SR}.Z_{DL}.T_{SR}^{-1} \tag{9}$$

The λ_{SR} matrix can be described by:

$$\lambda_{\mathbf{SR}} = \begin{bmatrix} K_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_1 \end{bmatrix} \tag{10}$$

where the values defined for K_1, K_2 and K_3 below are represented as: $K_1 = a - r$; $K_2 = a + 2r - 3p$; $K_3 = a + 2r + 3p$.

In the previous matrix, there are four equal modes and two different modes. Therefore, it is possible to obtain exact modes for three-phase double-circuit transposed lines applying a single homopolar mode reference. This reference is identified as the fourth row of the T_{SR} matrix where the value elements are equal. The single homopolar mode reference connects the two three-phase circuits of the considered system, creating a diagonal matrix for transposed cases.

IV. EIGENVALUE ANALYSES FOR THREE-PHASE DOUBLE-CIRCUIT LINE

The next figures show a new approach to the calculation of transients on transmission lines with frequency-dependent parameters. In Figure 2 it is observed that the relative differences between the values in the exact modes and quasi-modes are not greater than 2%.

For the calculations of these values the following relation is used:

$$\varepsilon = \frac{\lambda - \lambda_{SR}}{\lambda}.100 \qquad (11)$$

Where λ_{SR} is quasi-mode matrix elements and λ are exact eigenvalue matrix elements.

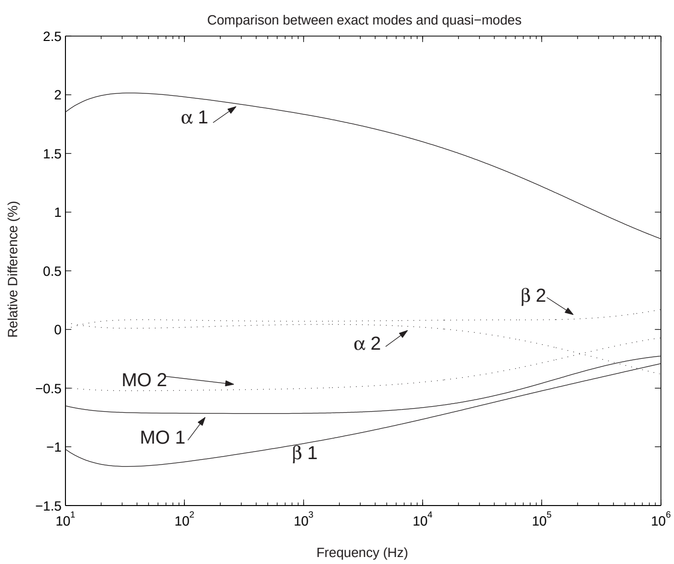

Fig. 2. Relative differences between the values in the exact modes and quasi-modes

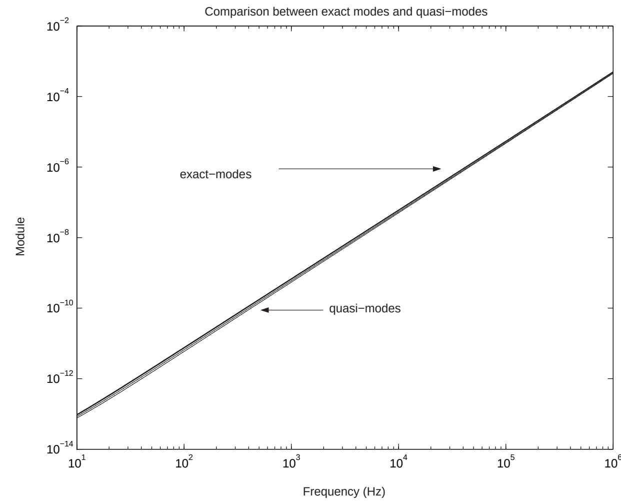

Fig. 3. Comparison between the modules in the exact modes and quasi-modes

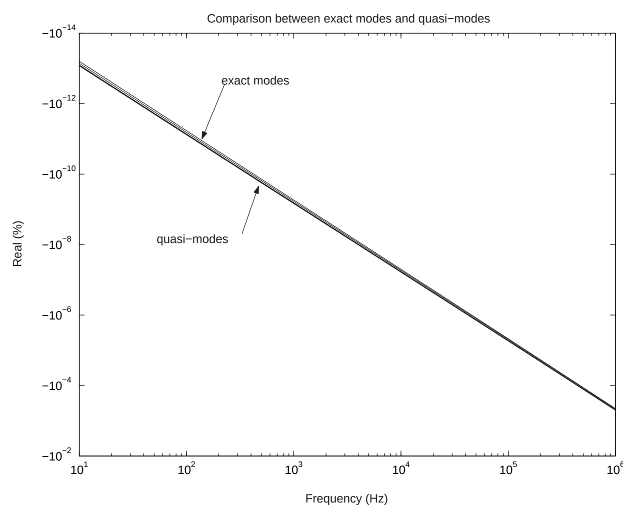

Fig. 4. Comparison between the real part in the exact modes and quasi-modes

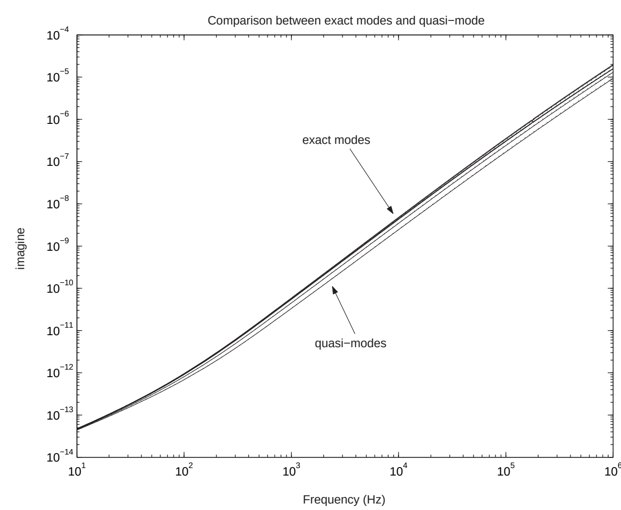

Fig. 5. Comparison between the imaginary part in the exact modes and quasi-modes

V. TWO PARALLEL THREE-PHASE DOUBLE-CIRCUIT LINES

A system with two parallel three-phase circuit-double lines is shown in Figure below.

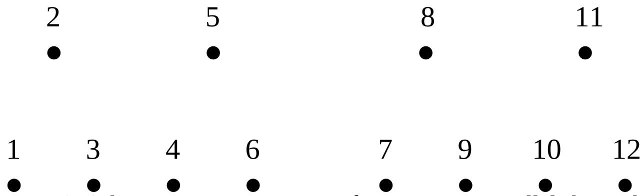

Fig. 6. Schematic representation of system two parallel three-phase

The system shown in Figure is performed with some ideal conditions. The assumptions are: the distances between the lines are equal and each three-phase circuit is ideally transposed. Because of this, the average self phase impedance value is represented by A. Within a circuit, the coupling impedances are represented by D. Considering the coupling impedances among the adjacent circuits, the H symbol is used to represent these values. The W and J symbols represent the coupling impedances among the circuits in non-adjacent or in extreme system positions, respectively. This system is identified through the Z_{Tp} matrix, shown in equation 12.

$$\begin{bmatrix} A & D & D & H & H & H & W & W & W & J & J & J \\ D & A & D & H & H & H & W & W & W & J & J & J \\ D & D & A & H & H & H & W & W & W & J & J & J \\ H & H & H & A & D & D & H & H & H & W & W & W \\ H & H & H & D & A & D & H & H & H & W & W & W \\ H & H & H & D & D & D & H & H & H & W & W & W \\ W & W & W & H & H & H & A & D & D & H & H & H \\ W & W & W & H & H & H & D & A & D & H & H & H \\ W & W & W & H & H & H & D & D & A & H & H & H \\ J & J & J & W & W & W & H & H & H & A & D & D \\ J & J & J & W & W & W & H & H & H & D & A & D \\ J & J & J & W & W & W & H & H & H & D & D & A \end{bmatrix} \quad (12)$$

Using the single homopolar mode reference, the T_{SR} transformation matrix can be described by:

$$\begin{bmatrix} L1 & M & L1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ N & 0 & N1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ S & S & S & S1 & S1 & S1 & S & S & S & S1 & S1 & S1 \\ 0 & 0 & 0 & L1 & M & L1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L & 0 & L1 & 0 & 0 & 0 & 0 & 0 & 0 \\ S1 & S1 & S1 & D & S & S & S1 & S1 & S1 & S & S & S \\ 0 & 0 & 0 & 0 & 0 & 0 & L1 & M & L1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L & 0 & L1 & 0 & 0 & 0 \\ S1 & S1 & S1 & S1 & S1 & S1 & S & S & S & S & S & S \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L1 & M & L1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & N & 0 & N1 \\ S & S & S & S & S & S & S & S & S & S & S & S \end{bmatrix} \quad (13)$$

The values of the T_{SR} matrix elements are:

$$L = \frac{1}{\sqrt{6}}, L1 = \frac{-1}{\sqrt{6}}; M = \frac{2}{\sqrt{6}}; \quad (14)$$

$$N = \frac{1}{\sqrt{2}}, N1 = \frac{-1}{\sqrt{2}}; S = \frac{1}{\sqrt{12}}, S1 = \frac{-1}{\sqrt{12}}; \quad (15)$$

The single homopolar mode reference is determined by the twelfth row where all elements have the same value. This value leads to a unitary homopolar mode modulus. The homopolar mode connects all phase conductors in a single mode.

The Z_{MDTP} is calculated as:

$$Z_{MDTP} = T_{SR}.Z_{TP}.T_{SR}^{-1} \quad (16)$$

The λ_{SR} matrix can be described by:

$$\lambda_{\mathbf{SR}} = \begin{bmatrix} K6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K0 & 0 & 0 & K1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K1 & 0 & 0 & K2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & K6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K3 & 0 & 0 & K4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K4 & 0 & 0 & K5 \end{bmatrix} \quad (17)$$

The λ_{SR} matrix elements correspond to the T_{SR} transformation matrix applied to YZ_{TP} product. The Y_{TP} structure is similar to that of the Z_{TP} one. The values of the λ_{SR} matrix elements are:

$$K = a + 2d - \frac{9h}{2} + 3w - \frac{3j}{2} \quad (18)$$

$$K1 = \frac{3(h-j)}{2} \quad (19)$$

$$K2 = a + 2d + \frac{3h}{2} - 3w - \frac{3j}{2} \quad (20)$$

$$K3 = a + 2d + \frac{9h}{2} + 3w + \frac{3j}{2} \quad (21)$$

$$K4 = -\frac{3(h-j)}{2} \quad (22)$$

$$K5 = a + 2d - \frac{3h}{2} - 3w + \frac{3j}{2} \quad (23)$$

$$K6 = a - d \quad (24)$$

It is verified that the λ_{SR} matrix is symmetrical. Most of the non-diagonal elements are zero and there are eight equal modes. The other diagonal elements that are non-exact modes can be called quasi-modes.

The non-diagonal elements that are non-zero values present structures composed of the subtraction of two values(equations 18 and 24). So, these elements can become non-significant when compared with the diagonal elements, depending on several line characteristics. In future development, the error analyses for this case will be performed, investigating what are the main line characteristics which can minimize the non-zero non-diagonal elements. Probably, the investigation will be concentrated on the geometrical characteristics of the line.

VI. CONCLUSIONS

With linear combinations of Clarke matrix elements, the eigenvector and eigenvalue analyses are made, considering three-phase double-circuit and two parallel three-phase double-circuit lines. The proposed methodology applies a single homopolar mode reference, determining single real transformation matrices for analyzed lines in this paper. This homopolar mode reference connects all three-phase circuits of the analyzed systems. The proposed methodology has been tested on systems with three-phase double-circuit lines (a 6 order transformation matrix) and two parallel three-phase double-circuit lines (a 12 order transformation matrix). For transposed three-phase double-circuit lines, exact modes and diagonal matrices are obtained in mode domain. In transposed cases of two parallel three-phase double-circuit lines, exact modes are not obtained. Considering this case and the non-transposed three-phase double-circuit lines, in future development, error analyses must be performed, investigating if the errors between exact modes and quasi-modes are negligible. In this paper, it is used only two types of line structures. It can be considered special cases because of the symmetrical characteristics of the line examples. These examples are used to present the concept of a single homopolar mode reference that connects all phase conductors of the line. Basing on error analyses presented in this paper, the future objective is to obtain a model without convolution procedures where, probably, there are computer-time savings. These future objectives will be related to the single real transformation matrix and the single homopolar reference applications. So, for future development, other line structures will be checked.

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VII. BIOGRAPHIES

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