Quadratic Integration Method

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***Abstract*— This paper presents a new approach for time domain transient simulation of electric power systems with or without power electronic (switching) subsystems. The new methodology has been named quadratic integration method. The method is based on the following two innovations: (a) the nonlinear system model equations (differential or differential-algebraic) are reformulated to a fully equivalent system of quadratic equations, by introducing additional state variables, and (b) the system model equations are integrated assuming that the system states vary quadratically within a time step (quadratic integration).**

The proposed method yields an implicit integration scheme which demonstrates improved convergence characteristics and most importantly improved solution precision. The approach also demonstrates superior behavior compared to traditionally used methods (such as the trapezoidal integration rule) in terms of accuracy and numerical stability properties, especially for switching systems. The method is free of artificial numerical oscillations. Details about the numerical properties of the method are discussed in the paper.

The proposed methodology and its performance is demonstrated on several test systems including (a) linear R-L-C electric circuit, (b) system with nonlinear inductance, and (c) power electronic circuit (switching system). The methodology is expected to be very useful for systems with power electronics and nonlinear devices such as saturable transformers/reactors and surge arresters.

***Index Terms*— Nonlinear systems, numerical integration, power electronics, power system transient simulation, switching systems, time-domain simulation.**

I. Introduction

THIS paper presents a new approach for time domain transient simulation of electric power systems with or without power electronic (switching) subsystems. The new methodology has been named quadratic integration method. The method is based on the following two innovations: (a) the nonlinear system-model equations (nonlinear differential or differential-algebraic equations) are reformulated to a fully equivalent system of quadratic equations, by introducing additional state variables and algebraic equations, and (b) the system model equations are integrated using an implicit numerical scheme assuming that the system states vary quadratically within a time step (quadratic integration).

Dynamic simulation is a very important tool in power system transient analysis. A great number of numerical integration methods have been proposed and used for power system time-domain simulation, to transform the ordinary differential equations to algebraic equations at each time step [1-9]. Such methods include backward Euler, Trapezoidal, Simpson's rule, explicit Runge-Kutta methods, Gear's method, or other linear multi-step methods, mainly of the backward differentiation formula (BDF) family. In many situations the equations describing the operation of a power system are stiff and thus implicit methods are preferred, though more expensive in terms of computation time. Among these methods the trapezoidal integration is one of the most popular ones in network transient analysis, due to its merits of low distortion and absolute stability (A-stability). For example, the trapezoidal rule is used in EMTP [10-12], Spice [10], and Virtual Test Bed [10].

However, the trapezoidal rule has several drawbacks that limit its applicability and indicate that some improvements in dynamic simulation methods are needed. Two major disadvantages of the trapezoidal integration scheme are its low accuracy compared to other existing methods (trapezoidal rule is order two accurate) and the artificial numerical oscillations that are often encountered, especially in the simulation of power electronic circuits, where switching events, and therefore discontinuities, occur. Specifically, the numerical values of certain variables oscillate around the true values. The magnitude and frequency of such numerical oscillations are directly related to the parameters of the energy storage elements and the simulation time step. In several cases this problem is so severe that the simulation results are erroneous.

The problem has been studied in the literature and several solutions have been proposed [10-20]. The numerical oscillations associated with the trapezoidal integration have been identified to result from two different reasons. One type of numerical oscillations is caused by an overly large simulation time step as compared to the smallest time constant in the system [10]. This problem may occur when simulating stiff system such as a power system with electric machines and power electronic devices. Another type of numerical oscillations is caused by step changes in certain state variables, i.e. when the trapezoidal rule is used as a pure

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differentiator [10]. This kind of numerical oscillations are often observed in power electronic circuits when inductive or capacitive elements are present. Note that since a step change in a state variable can be seen as an infinitely small time constant, the two types of numerical oscillation are not completely unrelated.

Several approaches have been proposed to suppress these numerical oscillations. One popular approach is to use the trapezoidal rule with damping [13]. However, this method introduces artificial elements in the system that may affect the true solution to some extend. Another interesting approach is to apply the critical damping adjustment (CDA) scheme, as proposed in [11] and [12]. This approach suggest to switch from the trapezoidal integration rule to another integration method that does not have an oscillation problem, like backward Euler, for one time step after the discontinuity and then switch back to the trapezoidal rule again and continue the simulation normally. This idea has been extensively studied and several similar approaches of combination of trapezoidal and backward Euler rules have been proposed [14-17]. The choice of the implicit Euler method presents several implementation advantages and is therefore preferred. However, backward Euler method is order one accurate and trapezoidal method order two, so the accuracy remains quite low. In addition the use of one integration method instead of two would be preferable. The Gear's second order method has been proposed as an alternative. The method does eliminate such numerical oscillations; however, it is as accurate as the trapezoidal method, so it does not provide any advantage in terms of accuracy. Furthermore it is not A-stable, which is a desired property. Filter interpolation was used in [19] and a method based on wave digital filters has been also suggested and studied [20].

This paper introduces a new numerical integration method for power system simulation. The method is order four accurate and therefore much more precise compared to all the traditionally used methods in power system applications. Furthermore, the proposed method does not suffer from the numerical oscillation problem, contrary to the trapezoidal rule. The method is referred to as quadratic integration method.

The proposed methodology is presented in section II of the paper. Section III contains a brief discussion on the numerical stability properties of the method. Section IV demonstrates the method in some simple examples and presents some preliminary results. Finally, section V concludes the paper and describes the future research steps on the study of the proposed methodology.

II. Description of Quadratic Integration Method

This section presents the key features of the quadratic integration method. The method is based on two innovations: First, the nonlinear system-model equations (nonlinear differential or differential-algebraic equations) are reformulated to a fully equivalent system of quadratic equations, by introducing additional state variables and additional algebraic equations. This step aims in reducing the nonlinearity of the system to at most quadratic in an attempt to improve the efficiency of the solution algorithm. It is independent of the integration method and thus can be applied in combination with any numerical integration rule. Second, the system model equations are integrated using an implicit numerical scheme assuming that the system states vary quadratically within a time step (quadratic integration).

The basic concept in the derivation of the quadratic integration method is illustrated in Fig. 1. In the trapezoidal rule it is assumed that the system functions/states vary linearly throughout a time step. In this approach it is assumed that they vary quadratically within an integration step. Note that within an integration time step of length h, defined by the interval $[t-h,t]$, the two end points, $x(t-h)$, $x(t)$, and the midpoint x_m $\left(x_m = x(t-h/2)\right)$ fully define the quadratic function in the interval $[t-h,t]$. This quadratic function is integrated in the time interval $[t-h,t]$ resulting in a set of algebraic equations for this integration step. The solution of the equations is obtained via Newton's method. Note that by virtue of the first step the resulting algebraic equations are either linear or quadratic. The proposed method demonstrates improved convergence characteristics of the iterative solution algorithm.

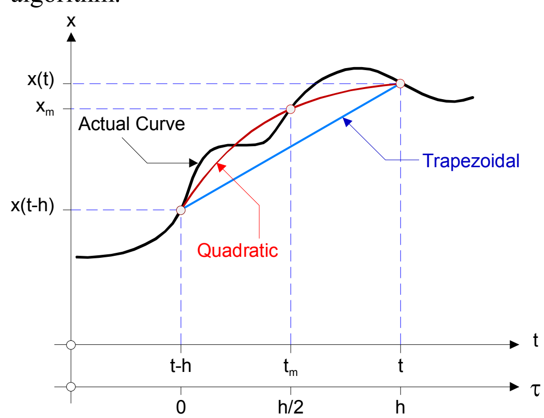

Fig. 1. Illustration of quadratic integration method.

The quadratic integration method belongs to the category of implicit, one-step, Runge-Kutta methods. More specifically it is an implicit Runge-Kutta method based on collocation and it can be alternative derived based on the collocation theory. The basic idea is to choose a function from a simple space, like the polynomial space, and a set of collocation points, and require that the function satisfy the given problem equations at the collocation points [21-23]. The method has three collocation points, at $x(t-h)$, x_m, and $x(t)$. It uses the Lobatto quadrature rules and is a member of the Lobatto-methods family. Any Lobatto method with s collocation points has an order of accuracy of $2s-2$, and therefore the method is order four accurate [21-23].

Assuming the general nonlinear, non-autonomous dynamical system:

$$\dot{x} = f(t,x), \tag{1}$$

the algebraic equations at each integration step of length h, resulting from the quadratic integration method, are:

$$x_m - \frac{h}{3} f(t_m, x_m) + \frac{h}{24} f(t, x(t)) = x(t-h) + \frac{5h}{24} f(t-h, x(t-h)) \tag{2}$$
$$x(t) - \frac{2h}{3} f(t_m, x_m) - \frac{h}{6} f(t, x(t)) = x(t-h) + \frac{h}{6} f(t-h, x(t-h))$$

Solution of the system via Newton's method yields the value of the state vector $x(t)$. Note that the value at the midpoint, x_m, is simply an intermediate result and it is discarded at the end of the calculations at each step.

For the special case of a linear system

$$\dot{x} = Ax + Bu \tag{3}$$

the algebraic equations at each time step become:

$$\begin{bmatrix} \frac{h}{24}A & I - \frac{h}{3}A \\ I - \frac{h}{6}A & -\frac{2h}{3}A \end{bmatrix} \cdot \begin{bmatrix} x(t) \\ x_m \end{bmatrix} = \begin{bmatrix} \left(I + \frac{5h}{24}A \right) \\ \left(I + \frac{h}{6}A \right) \end{bmatrix} \cdot x(t-h) + \begin{bmatrix} -\frac{h}{24}B & \frac{5h}{24}B & \frac{h}{3}B \\ \frac{h}{6}B & \frac{h}{6}B & \frac{2h}{3}B \end{bmatrix} \cdot \begin{bmatrix} u(t) \\ u(t-h) \\ u_m \end{bmatrix} \tag{4}$$

I is the identity matrix of proper dimension and h the length of the integration step.

The proposed integration approach has the following advantages: (a) improved accuracy and numerical stability, and (b) free of fictitious numerical oscillations. Details about the numerical properties of the method are discussed next

III. Numerical Properties

The numerical stability properties of a numerical integration method can be studied using the first order test equation:

$$\dot{x} = ax. \tag{5}$$

Applying the quadratic integration method yields at each time step:

$$\begin{bmatrix} x(t) \\ x_m \end{bmatrix} = \begin{bmatrix} \left(\frac{12 + 6ah + a^2h^2}{12 - 6ah + a^2h^2} \right) \\ \left(\frac{12 - 0.5a^2h^2}{12 - 6ah + a^2h^2} \right) \end{bmatrix} x(t-h) \tag{6}$$

and therefore:

$$x(t) = \frac{12 + 6ah + a^2h^2}{12 - 6ah + a^2h^2} \cdot x(t-h), \tag{7}$$

where h is the integration step. Setting $z = ah$ yields the characteristic polynomial for the method:

$$R(z) = \frac{z^2 + 6z + 12}{z^2 - 6z + 12} \tag{8}$$

Note that the eigenvalue a of the system can be complex, so z is in general a complex number.

The region of absolute stability is given by the set of values z such that $|R(z)| \le 1$. A method is called A-stable if the region of absolute stability in the complex z-plane contains the entire left half plane. This means that independently of the step size $h > 0$, a stable eigenvalue a of the original continuous time system, with $\mathrm{Re}(a) < 0$, will be still represented as a stable mode in the discrete time system, and thus the discrete system mimics accurately the behavior of the original system, in terms of stability. Note that for $\mathrm{Re}(z) < 0$ it follows that $|R(z)| \le 1$. Therefore, the proposed method is A-stable.

Furthermore, the absolute stability region is exactly the left-hand half complex plane. This property is called strict A-stability. If the dynamical system under study includes an unstable mode, then, irrespectively of the integration step-size, this mode will remain unstable in the descretized system. This is not the case for other methods, for example, the backward Euler, or the BDF linear, multi-step methods, where the numerical stability domain extends in the right-hand plane, where $\mathrm{Re}(z) > 0$. In this case, if the real dynamical system includes an unstable mode, this mode could appear as stable for some step size, in the discrete system.

Comparing the quadratic and the trapezoidal integration methods the following hold:

1. Both the trapezoidal method and the quadratic integration method are strictly A-stable. The characteristic polynomial for the trapezoidal method is $R(z) = \frac{2+z}{2-z}$, and it holds that $R(z) \le 1$ in the whole left-hand complex plane, i.e., $\mathrm{Re}(z) < 0$.
2. The trapezoidal method is order two accurate. The quadratic integration is order four. Therefore, in terms of accuracy, quadratic integration is much preferable.
3. It has been observed in applications that the trapezoidal method can provide an oscillatory solution even for systems that have exponential solutions as the simple test equation above. This is apparent if one considers the term $R(z) = \frac{2+z}{2-z}$ for a physically stable system. Note that it is possible to select the integration time step ($z = ah$), so that this term is negative (for example any real value for z, with $z < -2$). This can occur when larger integration steps are selected. In this case the solution will be oscillatory, oscillating around the true solution of the problem. In the case of the quadratic integration, the corresponding term $R(z) = \frac{z^2 + 6z + 12}{z^2 - 6z + 12}$ can never be negative as long as $\mathrm{Re}(z)$ is negative, i.e. as long as the physical system is stable. This can be a very nice characteristic in many applications.

IV. Preliminary Results

This section discusses the application of the method to some preliminary test cases. The examples are fairly simple and their goal is mainly to demonstrate clearly the application of the methodology.

A. Linear RLC circuit

The first example is a simple series RLC circuit, as illustrated in Fig. 2. An AC voltage source of 10Vrms value and of 60Hz frequency is the input of the circuit. Using the capacitor voltage and the circuit current as the two system states the system equations in the standard state space representation are:

$$\frac{d}{dt}\begin{bmatrix} v_C \\ i \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix} \cdot \begin{bmatrix} v_C \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} \cdot u = A \cdot \begin{bmatrix} v_C \\ i \end{bmatrix} + B \cdot u, \quad (9)$$

where v_C is the capacitor voltage and i the circuit current. The input u is $u = 10 \cdot \sqrt{3} \cdot \sin(120 \cdot \pi \cdot t)$, in V.

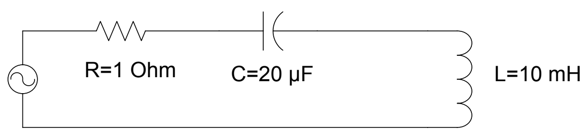

Fig. 2. Series RLC circuit.

The system is simulated using the trapezoidal rule and the quadratic integration method. The results are also compared to the analytical solution of the system, so that the improved accuracy of the quadratic integration compared to the trapezoidal method is demonstrated. The trapezoidal rule yields the algorithm of equations (10):

$$\left(I - \frac{h}{2}A\right) \cdot \begin{bmatrix} v_C(t) \\ i(t) \end{bmatrix} = \left(I + \frac{h}{2}A\right) \cdot \begin{bmatrix} v_C(t-h) \\ i(t-h) \end{bmatrix} + \frac{h}{2}B \cdot \left(u(t) + u(t-h)\right) \quad (10)$$

The quadratic integration yields equations (11).

$$\begin{bmatrix} \frac{h}{24}A & I - \frac{h}{3}A \\ I - \frac{h}{6}A & -\frac{2h}{3}A \end{bmatrix} \cdot \begin{bmatrix} v_C(t) \\ i(t) \\ v_{Cm} \\ i_m \end{bmatrix} = \begin{bmatrix} \left(I + \frac{5h}{24}A\right) \\ \left(I + \frac{h}{6}A\right) \end{bmatrix} \cdot \begin{bmatrix} v_C(t-h) \\ i(t-h) \end{bmatrix} + \begin{bmatrix} -\frac{h}{24}B & \frac{5h}{24}B & \frac{h}{3}B \\ \frac{h}{6}B & \frac{h}{6}B & \frac{2h}{3}B \end{bmatrix} \cdot \begin{bmatrix} u(t) \\ u(t-h) \\ u_m \end{bmatrix} \quad (11)$$

Fig. 3 presents a graph of the capacitor voltage for the first 60Hz-period. Fig. 4 shows one period of the circuit current, while Fig. 5 shows the inductor voltage for the duration of the simulation, until steady-state is reached. A time step of 0.1 ms was used. Note that the results of both integration method and the analytical solution are very close and therefore cannot be distinguished in the graphs.

Fig. 6 presents the absolute error of the circuit current for the two methods, compared to the analytical solution, for the duration of the simulation. The current axis is logarithmic. Note that the quadratic integration method is almost three orders of magnitude more accurate compared to the trapezoidal.

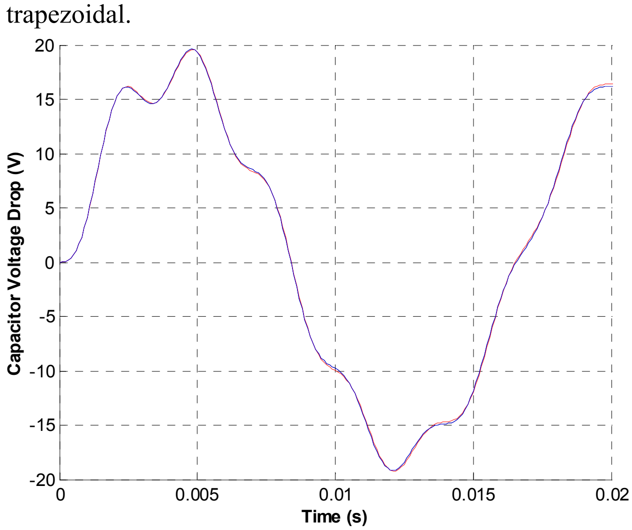

Fig. 3. Capacitor voltage computed using trapezoidal (red) and quadratic (blue) integration (waveforms are too close to be distinguished in the graph).

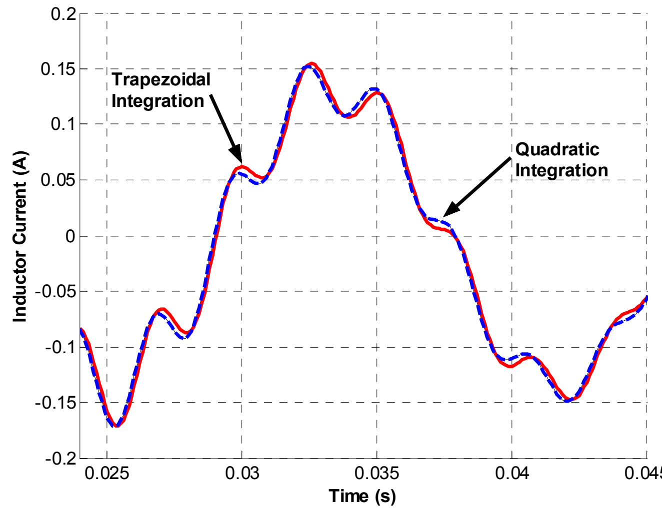

Fig. 4. Circuit current computed using trapezoidal (red) and quadratic (blue) integration.

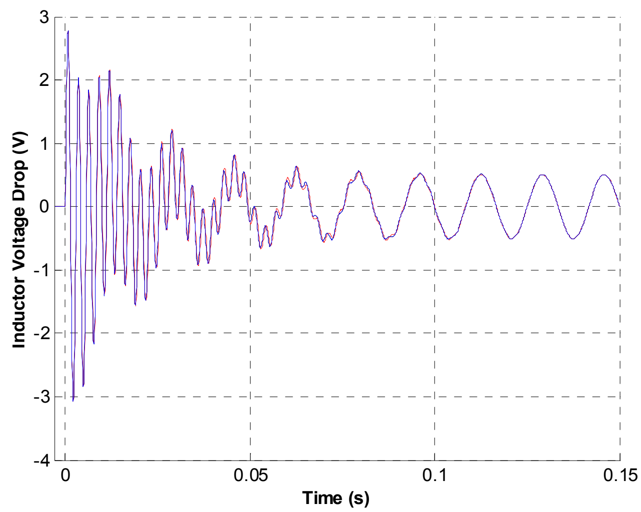

Fig. 5. Inductor voltage using trapezoidal (red) and quadratic (blue) integration.

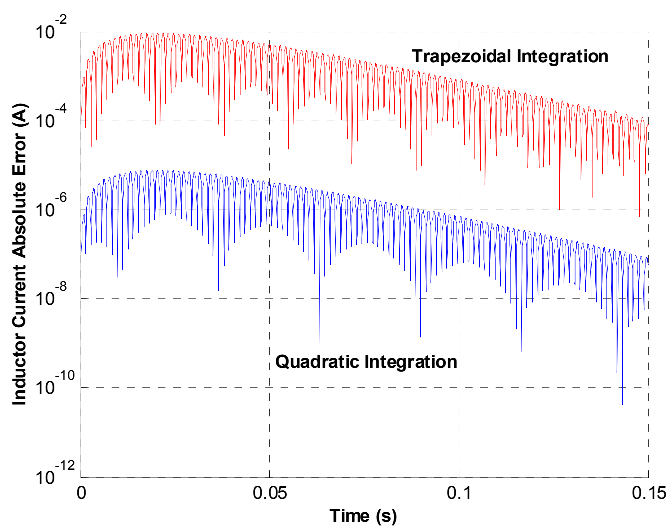

Fig. 6. Absolute error of circuit current of trapezoidal (red) and quadratic (blue) integration.

B. Nonlinear inductor

The second test-case is an RL circuit with a nonlinear inductor. Two cases of nonlinear inductor are studied: (a) an inductor which has a high order nonlinear flux-current characteristic, and (b) a piecewise linear inductor with two linear segments. The circuit in both cases is as in Fig. 7. The voltage source is a sinusoidal AC source of 60 Hz and of 10 V rms.

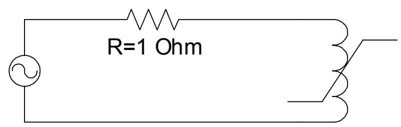

Fig. 7. RL circuit with nonlinear inductor.

a) High-order nonlinear inductor

The circuit equations are:

$$\frac{d\lambda}{dt} = -R \cdot i + V_{AC}, \tag{12}$$

where λ is the inductor flux and i the inductor current. The nonlinear characteristic of the inductor is

$$i = i_0 \cdot \left(\frac{\lambda}{\lambda_0}\right)^8 \cdot \mathrm{sgn}(\lambda). \tag{13}$$

The proposed method consists of quadratization of the equations first and then application of the quadratic integration. The equivalent quadratic system with linear differential and quadratic algebraic equations is (note the introduction of two additional variables, z_1 and z_2 and two additional equations):

$$\begin{aligned} \frac{d\lambda}{dt} &= -R \cdot i + V_{AC} \\ 0 &= i - i_0 \cdot z_2^2 \cdot \mathrm{sgn}(\lambda). \\ 0 &= \lambda_0^2 \cdot z_1 - \lambda^2 \\ 0 &= z_2 - z_1^2 \end{aligned} \tag{14}$$

Fig 8 illustrate the computed inductor voltage and current waveforms using the quadratic integration, with a time step of 10 μs. Fig 9 shows the inductor flux waveform. The system parameters are $i_0 = 10\,A$ and $\lambda_0 = 0.03\,Wb$.

b) Piecewise linear inductor

In this case the circuit equations are:

$$\frac{di}{dt} = \begin{cases} -\dfrac{R}{L_1} \cdot i + \dfrac{1}{L_1} \cdot V_{AC}, |i| \le i_0 \\ -\dfrac{R}{L_2} \cdot i + \dfrac{1}{L_2} \cdot V_{AC}, |i| \ge i_0 \end{cases} \tag{15}$$

The inductance values are $L_1 = 1\,mH$ and $L_2 = 0.1\,\mu H$. The switching occurs at $i_0 = 12A$. Figures 10 and 11 show the waveforms of the inductor voltage and current when computed using the trapezoidal rule with a time step of 10 μs. Note that the results contain numerical oscillations when switching from the first to the second model. Figures 12 and 13 show the same voltage and current waveforms when the system is simulated using quadratic integration with the same time step. Note that the oscillations are eliminated, when this method is used.

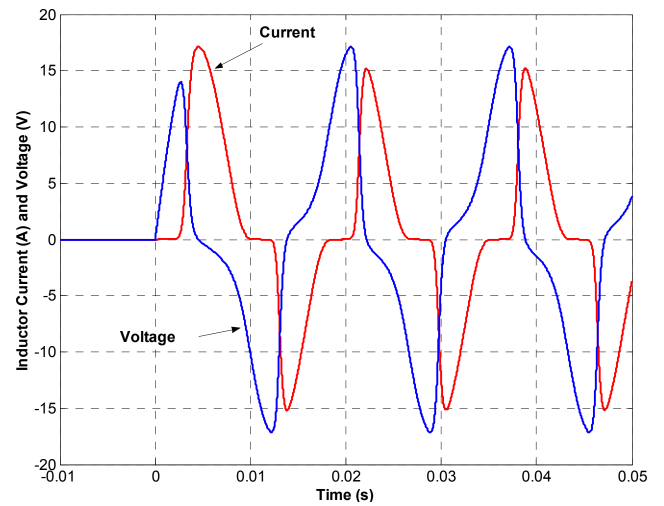

Fig. 8. Inductor voltage and current waveforms of a nonlinear inductor using quadratic integration.

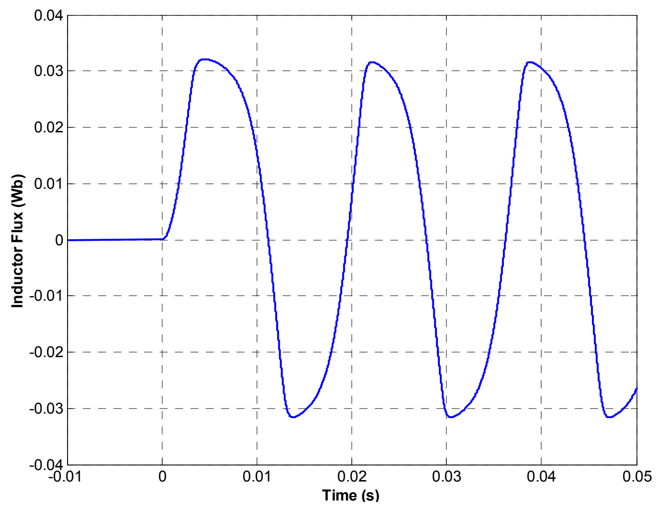

Fig. 9. Inductor flux waveform of a nonlinear inductor using quadratic integration.

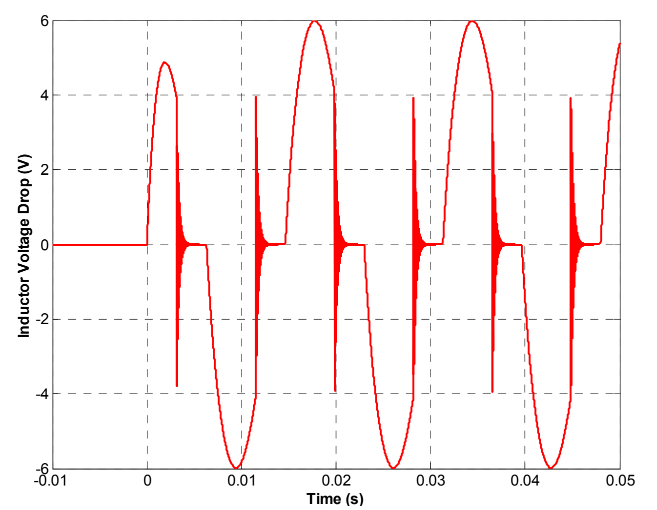

Fig. 10. Voltage of piecewise linear inductor using trapezoidal integration.

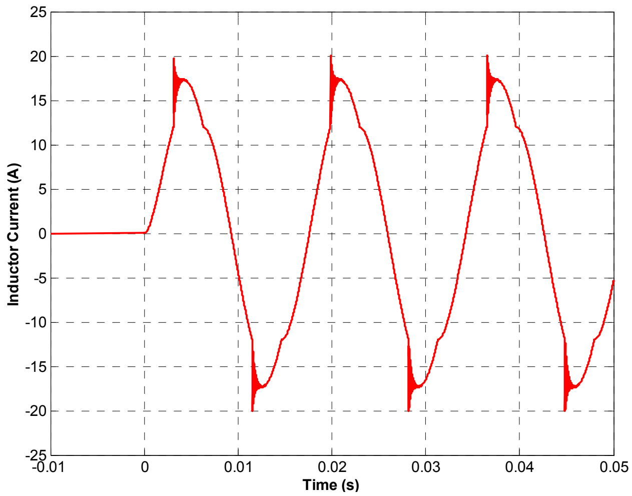

Fig. 11. Current of piecewise linear inductor using trapezoidal integration.

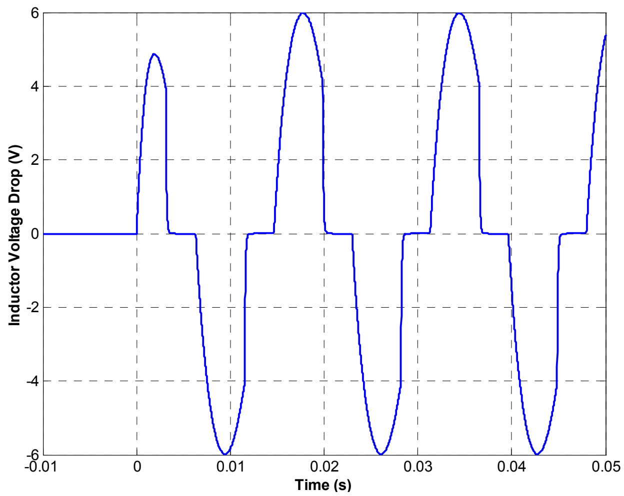

Fig. 12. Voltage of piecewise linear inductor using quadratic integration.

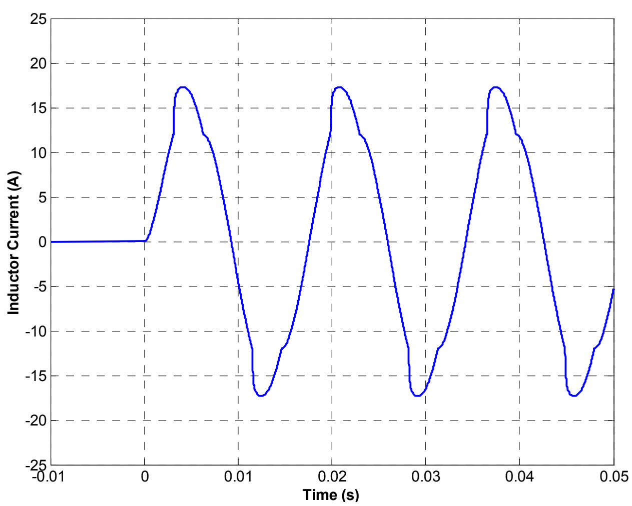

Fig. 13. Current of piecewise linear inductor using quadratic integration.

C. R-L circuit with diode

The last test case is a simple switching system, as illustrated in Fig 14. A sinusoidal voltage source of 10 V rms and 60 Hz frequency drives an inductive load through a diode. This scenario is often encountered in the simulation of power electronic systems. The diode is modeled using a piecewise linear model, as described by (16).

$$i_D = \begin{cases} \dfrac{1}{R_D} \cdot v_D, \; v_D \le V_{D0} \\ \dfrac{1}{r_D} \cdot v_D + V_{D0} \cdot \left(\dfrac{1}{R_D} - \dfrac{1}{r_D} \right), \; v_D \ge V_{D0} \end{cases} \tag{16}$$

where i_D and v_D are the diode current and voltage respectively. V_{D0} is the diode voltage at which the diode starts conducting and R_D and r_D are the diode resistances. The numerical values of the above constant are: $R_D = 10^6\ Ohm$, $r_D = 10^{-1}\ Ohm$, $V_{D0} = 0.7\ V$

The inductance value is $L = 1\ mH$. Using the diode voltage as state variable the system equations are:

$$\frac{dv_D}{dt} = \begin{cases} -\dfrac{R + R_D}{L} \cdot v_D + \dfrac{R_D}{L} \cdot V_{AC}, v_D \le V_{D0} \\ -\dfrac{R + r_d}{L} v_D + \dfrac{r_D}{L} \cdot V_{AC} + \dfrac{V_{D0}}{L} \left((R + r_d)\left(1 - \dfrac{r_D}{R_D}\right) + r_D\left(1 + \dfrac{r_D}{R_D}\right) \right) \end{cases} \tag{17}$$

A step size of 2 μs is used for the numerical integration, using the trapezoidal rule and the quadratic integration.

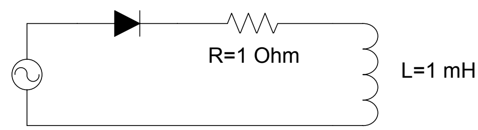

Fig. 14. RL circuit with diode.

Figures 15 through 18 show the diode voltage and the voltage across the inductor. When the trapezoidal integration is used severe numerical oscillations appear each time the diode is turned off. The quadratic integration successfully eliminates these oscillations.

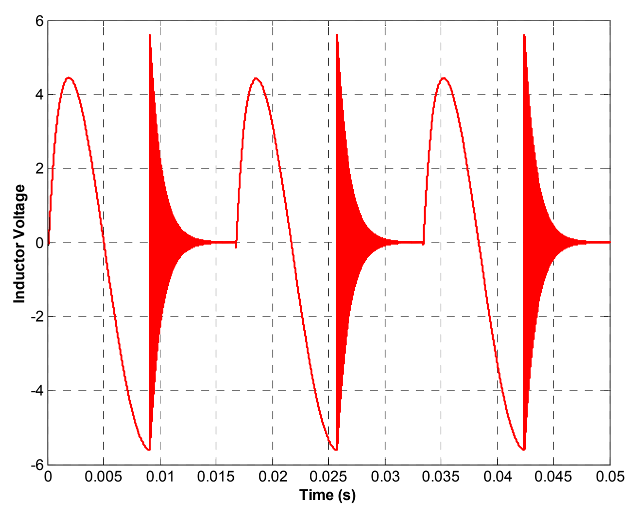

Fig. 15. Inductor voltage using trapezoidal integration.

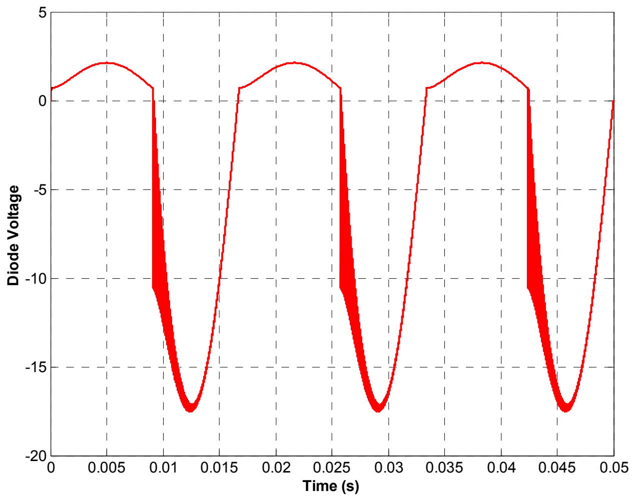

Fig. 16. Diode voltage using trapezoidal integration.

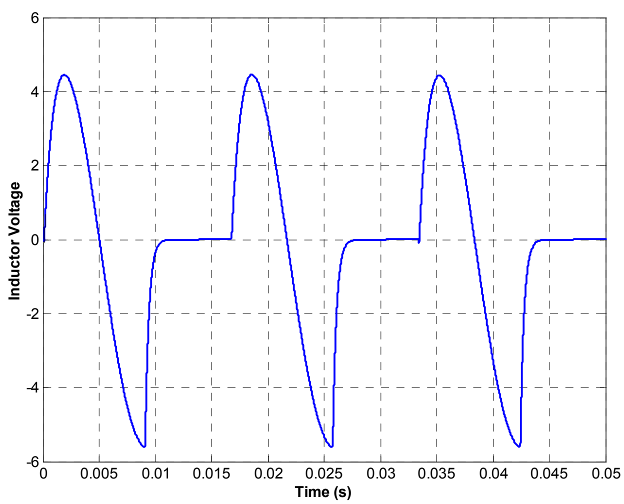

Fig. 17. Inductor voltage using quadratic integration.

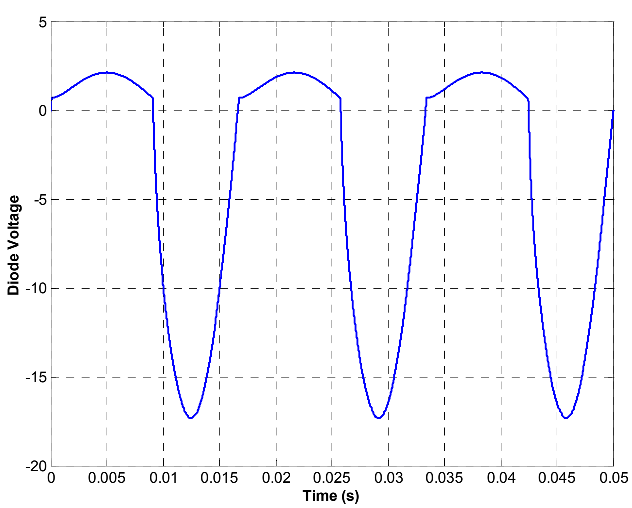

Fig. 18. Diode voltage using quadratic integration.

V. Comparison of Trapezoidal and Quadratic Integration Methods

As described, the proposed quadratic integration method is a one-step implicit Runge-Kutta method based on collocation. The trapezoidal integration method can be also viewed as a member of this category; however, trapezoidal rule uses two collocation points, while the proposed method uses three. This provides a great advantage in terms of accuracy. As every numerical integration method, the quadratic integration directly converts the system of differential equations to a set of algebraic equations, at each integration step. The formulation of these equations is straightforward and the procedure can also be automated. This can facilitate the process in more complicated models. However, the number of algebraic equations of the quadratic integration scheme is double compared to that of the trapezoidal rule, due to the additional collocation point. The end-result is increased computational effort compared to the trapezoidal method per iteration (approximately double when sparsity techniques are used). Nonetheless, the improved method accuracy (order four, compared to order two of the trapezoidal method) allows the use of larger time-steps, so that the total computational effort becomes less than that of trapezoidal integration, while the accuracy remains significantly higher. The trade-off between accuracy and computational speed applies also to higher order implicit Runge-Kutta methods. As the number of collocation points, and thus the order, increase, the computational effort also increases. It appears that the quadratic integration method achieves a good balance between accuracy and computational speed.

The proposed method also appears to possess better numerical properties and be more accurate when compared to linear, multi-step methods commonly used in power system transient analysis. The use of such methods is usually restricted to order two accurate methods. Detailed comparison of the quadratic integration and linear, multi-step methods used will be reported in future papers.

VI. Conclusions and Further Work

This paper introduced some preliminary concepts on a new numerical integration method for power system transient, time-domain simulation. The method is order four accurate and, therefore, much more precise compared to other traditionally used method. Furthermore, the method does not suffer from sustained numerical oscillations after discontinuities (switching events) as the popular trapezoidal integration method. Several simple test cases show that the methodology appears to be a good alternative for the simulation of power system transients.

The paper focuses on some basic concepts and some introductory work on the issue. The numerical properties of the method will be studied more thoroughly and the method will also be extensively compared to several other numerical methods that are commonly used in power system transient analysis. Furthermore, the method will be enhanced with an error estimation and control algorithm and therefore with variable step-size capabilities. The addition of variable step-size capabilities can be done in the same straightforward way as in any other integration method. Some study on singularity detecting codes will be also performed to allow the tracking of switching events. These features will allow a better handling of the switching discontinuities, which was not fully addressed in this paper. Finally more test cases including more complex systems will be simulated. The theoretical study and the additional results will be reported in future papers.

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