Accurate Transmission Line Modeling Through Optimal Time Delay Identification

Luciano De Tommasi and Bjørn Gustavsen

Abstract--Frequency dependent transmission line modeling by the Method of Characteristics requires to calculate a rational approximation of the characteristic admittance $\mathbf{Y}_c$ and the propagation function, H. Most models rely on fitting the modal components of H in order to handle the delay terms of the line. This paper compares various techniques for delay extraction and fitting with emphasis on accuracy and computational efficiency. The comparison includes asymptotic magnitude fitting, phase reconstruction from magnitude data followed by vector fitting (VF), and VF with time delay included in the optimization. The latter approach is shown to be the most accurate, at the cost of longer computation times. The alternative fitting strategies are applied to one example of transient overvoltage calculation on an underground cable system.

Keywords: Magnitude fitting, minimum phase shift function, transmission line model, rational approximation, method of characteristics, time delay identification, vector fitting.

I. Introduction

Frequency dependent transmission line models are commonly used in the simulation of electromagnetic transients in power systems and verification of electronics high-speed interconnects. Such line models are usually formulated via the method of characteristics (MoC) due to its efficient handling of the delay effects. The procedure requires to fit with rational functions the characteristic admittance $\mathbf{Y}_C$ and the propagation function **H**, thereby achieving a highly efficient time domain implementation by recursive convolutions [1]. The fitting is done within a modal framework [1]-[4] or in the phase domain [5]-[7].

The rational approximation of the characteristic admittance $\mathbf{Y}_c$ is straightforward due the smoothness of the responses, both in the modal domain and in the phase domain. The fitting of **H** is a lot more challenging due to the need for taking into account the associated time delays. In the modal formulation, each mode is fitted with a rational function plus a single time delay. The poles and delays obtained from modes are used in some phase domain models (e.g. [6],[7]) as known quantities for the final phase domain fitting.

The fitting of the modes of **H** has traditionally been based on asymptotic magnitude fitting using real poles and real zeros with subsequent refinement and delay extraction [2]. This procedure offers robust high order fittings and is currently used in several EMTP-type simulators. The accuracy was improved in [4] by fitting the modes using the pole relocating algorithm known as vector fitting (VF) [8]. The modal delay was precalculated from the magnitude shape of the propagation function around a single frequency point. However, in some situations the accuracy was found to be unsatisfactory due to inaccurate delay extraction. Therefore, delay optimization within the fitting procedure was introduced [7],[9], thereby greatly improving the accuracy of the mode fittings. On the other hand, when fitting **H** using the Universal Line Model (ULM) scheme [6], there is no direct relation between the accuracy of the modal fitting and the final result provided by the phase domain fitting, since only poles and delays determined by the modal fitting are used for the final fitting of residues in the phase domain.

In this paper we compare alternative approaches for fitting the modes of **H** when applied to the modeling of a single core underground cable by the ULM. The identification of poles and delays is done by three alternative procedures:

1. asymptotic Bode-type fitting of magnitude function with subsequent refinement and delay identification;
2. delay extraction through phase reconstruction from magnitude data, followed by VF;
3. VF with simultaneous identification of delay and rational function.

The different approaches are compared in terms of accuracy for the fitting of modes and the final phase domain fitting in ULM, and when simulating transient voltages on an underground cable. Approaches 2 and 3 are based on the relaxed VF version [10].

II. The Universal Line Model

The main advantage of ULM [6] is the procedure used for fitting the propagation function:

$$\mathbf{H}(s) = e^{-\sqrt{\mathbf{Z}(s)\cdot\mathbf{Y}(s)}\,l}, \qquad (1)$$

where **Z** and **Y** denote the series impedance and shunt

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admittance matrix.

The first step in ULM is to fit the modes of **H**, which can be obtained either using a real, constant transformation matrix **T** evaluated at a high frequency point [6], or by a complex, frequency dependent **T**(ω) [7].

Each element of the modal propagation matrix $\mathbf{H}^m$ (diagonal) may be written as:

$$h_i^m(\omega) = e^{-\left(\alpha_i(\omega) + j\frac{\omega}{v_i(\omega)}\right)l}, \qquad (2)$$

where α is the attenuation, v is the velocity and l is the line length. Such modes are approximated with a rational function plus a single time delay:

$$h_i^m(s) \cong \left(\sum_{m=1}^{N} \frac{r_{im}}{s - a_{im}}\right) e^{-s\tau_i}. \qquad (3)$$

The assumption of a rational function plus a delay term on the right side of (3) enables an accurate fitting of h_i^m with relatively low orders.

Finally, **H**(s) is fitted in the phase domain taking the poles $\{a_{im}\}$ and delays $\{\tau_i\}$ as known quantities:

$$\mathbf{H}(s) \cong \sum_{i=1}^{n} \sum_{m=1}^{N} \frac{\mathbf{R}_{im}}{s - a_{im}} e^{-s\tau_i}. \qquad (4)$$

III. Asymptotic Fitting of Magnitude Function and Time Delay Identification

Asymptotic fitting of magnitude functions was introduced by Bode [14] and later used in transmission line modeling by J. Marti [2]. Since only the magnitude function is considered in the fitting process, the time delay must be calculated by comparing the phase angle of the propagation function with the one of the rational approximation.

Basically, asymptotic fitting allocates poles and zeros by tracking the magnitude of the original function as function of frequency. A new pole/zero is allocated whenever the asymptote of the fitting function deviates from the original function by more than a predefined tolerance. That way, the fitting function freely adapts itself to the shape of the original function. This implies that the order of the approximation is not established “a-priori”, but results from the required accuracy specified as input for the fitting routine. The procedure was designed assuming real poles and zeros located in the left half-plane. Thus, the rational function belongs to the class of minimum phase shift functions.

The obtained poles and zeros are further refined using an optimization procedure [2]. In this work, we consider a zeros optimization scheme based on the Gauss-Newton nonlinear least squares algorithm, as implemented in the MATLAB function `lsqcurvefit`. It takes as initial values the poles and zeros determined by asymptotic fitting and optimizes simultaneously the whole set of zeros to minimize the *rms*-error. Poles are taken as known quantities in this process. We also found that similar results in accuracy are achieved when considering zeros as known and optimizing poles.

Finally, delay identification is performed by using a suitable algorithm to minimize the *rms*-error. Our implementation is based on the *Brent’s Method*. More about this issue is given in section V.

IV. Time Delay Extraction Through Phase Reconstruction from Magnitude Data

When using the VF algorithm, both the real and imaginary part of $h = h_i^m$ are used in the fitting process. This makes it necessary to remove a suitable time delay before the fitting is carried out:

$$h_\tau(\omega) = h(\omega)e^{j\omega\tau} \cong \left(\sum_{m=1}^{N} \frac{r_m}{j\omega - a_m}\right). \qquad (5)$$

Equations (2) and (5) lead to the expression of the phase angle of h_τ:

$$\varphi_\tau(\omega) = \angle h_\tau(\omega) = \omega\left(\tau - \frac{l}{v(\omega)}\right). \qquad (6)$$

Note that if $\tau \neq \tau_0$, where τ_0 is the lossless time delay:

$$\tau_0 = \frac{l}{v(\omega \to +\infty)}, \qquad (7)$$

the phase angle φ_τ diverges as $\omega \to +\infty$, resulting $\varphi_{\tau<\tau_0}(\omega \to +\infty) \to -\infty$ and $\varphi_{\tau>\tau_0}(\omega \to +\infty) \to +\infty$, whereas $\varphi_{\tau_0}(\omega \to +\infty) \to 0$.

In cable systems, the lossless delay may be difficult to precalculate due to the various dielectric materials. In [4] was therefore introduced the concept of calculating τ_0 directly from the shape of the magnitude function. As already noted in [9], we have verified that h_{τ_0} approaches a minimum phase shift function for both underground cables and overhead lines. This has been done by checking the very good agreement between $\angle h_{\tau_0}$ and the minimum-phase shift phase angle extracted from the magnitude function $|h(\omega)|$ by the Bode phase-integral theorem. The theorem gives an analytical expression of the phase angle φ for a minimum phase shift function [18]:

$$h_\varphi(\omega) = |h(\omega)|e^{j\varphi(\omega)}, \qquad (8)$$

as a function of logarithm of its magnitude $|h|$:

$$\varphi(\omega) = \frac{\pi}{2} \left.\frac{d(\ln|h(\omega_1)|)}{d(\ln\omega_1)}\right|_{\omega_1=\omega} + \Delta(\omega), \qquad (9)$$

where:

$$\Delta(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\left(\left|\frac{d(\ln|h|)}{du}\right| - \left|\frac{d(\ln|h|)}{du}\right|_{u=0}\right)\ln\left(\coth\frac{|u|}{2}\right)du, \quad (10)$$

and

$$u = \ln \frac{\omega_1}{\omega}. \quad (11)$$

The phase angle $\varphi(\omega)$ is evaluated through (9-10) using the known magnitude $|h(\omega)| = |h_{\tau_0}(\omega)|$. We have found the following highly accurate agreement:

$$\angle h_{\tau_0}(\omega) \cong \varphi(\omega), \quad \text{for each } \omega. \quad (12)$$

Solving (12) for τ_0 gives:

$$\tau_0 = \frac{l}{v(\omega)} + \frac{\varphi(\omega)}{\omega}, \quad \text{for each } \omega. \quad (13)$$

The last equation clearly indicates that a single frequency point ω is enough in order to evaluate τ_0, and that τ_0 is practically independent from the selected frequency point Ω.

Note that the approach described in [4], used the formula (13) with a single high frequency point Ω.

V. SIMULTANEOUS IDENTIFICATION OF DELAY AND RATIONAL FUNCTION (COMBINED PROCEDURE)

It was shown that by extracting a time delay slightly larger than the lossless one, a more accurate approximation on the form (3) can be achieved [9]. An example of this, here is given in Fig. 1, where the propagation function $h(s)$ of a 50 km long overhead line has been subjected to a rational approximation (N=8 poles) over the frequency interval [1 Hz, 10 MHz], after backwinding (5) using either a lossless time delay τ_0 or a larger delay $\tau_1 > \tau_0$. The fitting is done using the VF algorithm with enforcement of stable poles. It is seen that compensation with the time delay τ_1 gives a less negative phase angle over the range of frequencies where the magnitude of the original function is not negligible yet. This enables a quite more accurate fitting result (*rms*-error: 2.55*E*–4 versus 7.76*E*–4).

The delay which gives the smallest *rms*-error is dependent on the order of the approximation, see fig. 2. It is observed that the lower is the order, the greater is the time delay that minimizes the *rms*-error. In [7] was introduced the idea of optimizing the delay such that the *rms*-error of the fitting is minimized. The basic idea is to search for the time delay which gives the smallest possible *rms*-error of:

$$h e^{s\tau_i} \cong \left(\sum_{m=1}^{N} \frac{r_m}{s - a_m} \right). \quad (14)$$

An efficient implementation was presented in [9], by using the Brent's Method which combines the Golden Section Search with Parabolic Interpolation. This algorithm is conveniently available in the MATLAB environment (routine `fminbnd`). The practical use requires writing a routine which calculates the rational approximation and its *rms*-error for a given time delay. The optimization procedure is robust and accurate. The only drawback is its increased computational cost due to the additional calls to the VF routine.

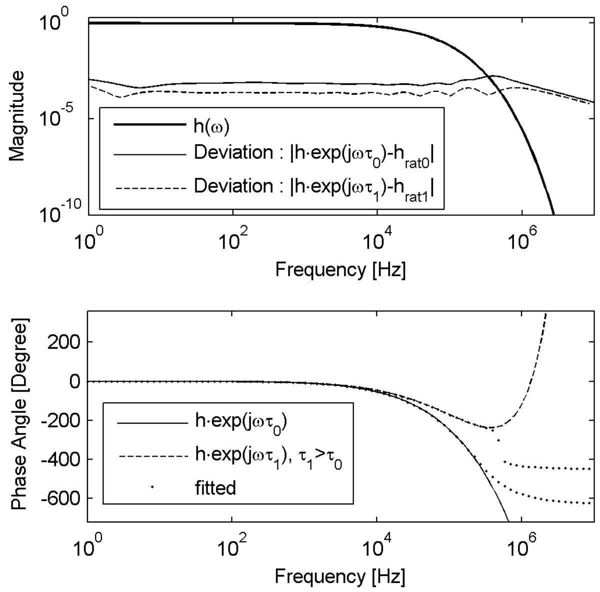

Fig. 1. Rational Approximation of a propagation function, backwinded with lossless time delay τ_0 and $\tau_1 > \tau_0$.

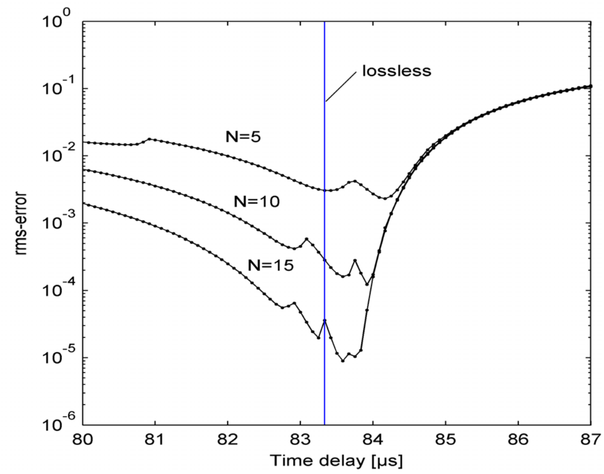

Fig. 2. Effect of order (N) on *rms*-error. Stable poles only.

VI. TEST CASE AND COMPARISON

A. Case

As test case we consider a 10 km single core underground cable, see Fig. 3 and table I.

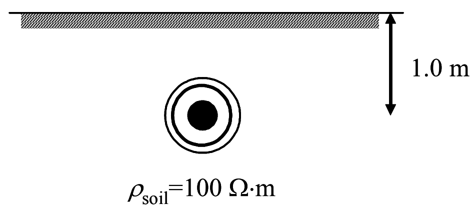

Fig. 3 Underground cable.

TABLE I CABLE DATA

Item	Property
Core	OD=39 mm , ρ= 3.365E–8 Ω·m
Insulation	t=18.25 mm, ε_r=2.85
Sheath	t=0.22 mm , ρ= 1.718E–8 Ω·m
Jacket	t=4.53 mm, ε_r=2.51

B. Fitting Modal Responses

Figs 4 and 5 respectively show the fitting of the coaxial mode and the ground mode, using the alternative fitting approaches described in Sections III and IV. With both modes, the combined procedure and the phase reconstruction procedure are seen to give a substantially more accurate result than usage of asymptotic fitting. The combined procedure is particularly more accurate when fitting the ground mode.

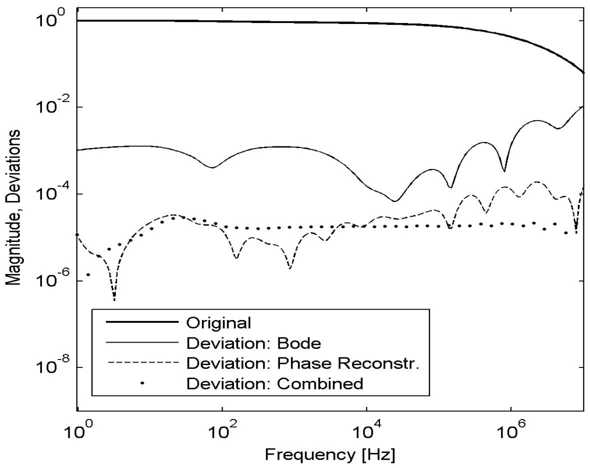

Fig. 4 Fitting the coaxial mode, N=16.

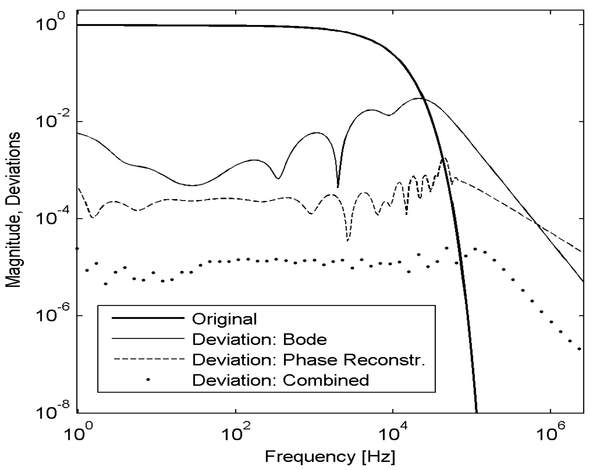

Fig. 5 Fitting the ground mode, N=16.

C. Phase Domain Fitting

Poles and delays obtained by mode fitting are used as known quantities for the final fitting of **H** in the phase domain by (4).

Fig. 6 shows the fitting of element H_{11}. It is seen that all approaches give a satisfactory result, despite the poor accuracy that Bode fitting gave for the modes. Fig. 7 shows the corresponding result for element H_{22}. Bode fitting now gives a somewhat poor result while the combined approach gives a highly accurate result.

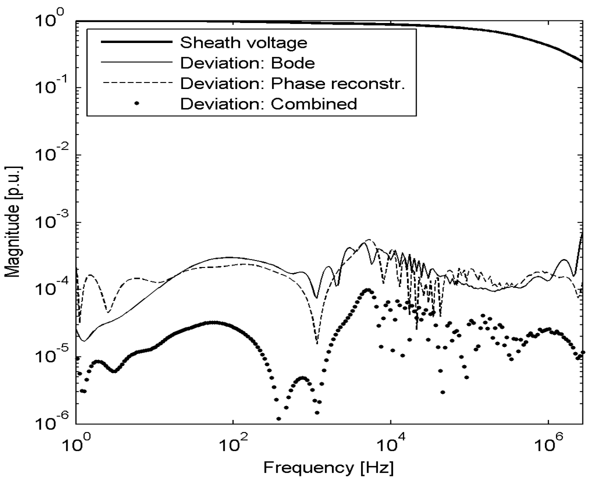

Fig. 6 Element H_{11}.

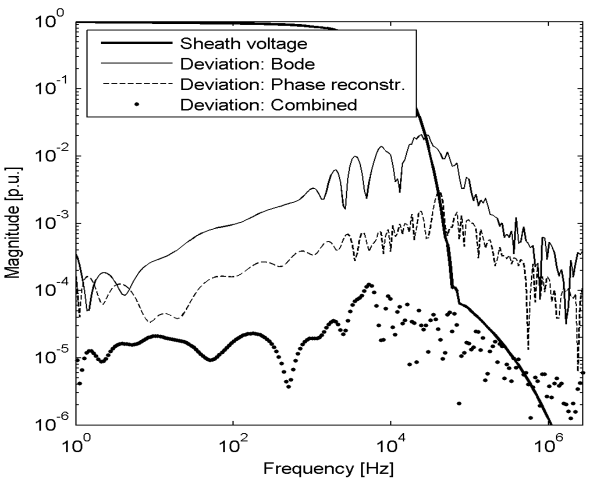

Fig. 7 Element H_{22}.

D. Time Domain Simulation

The rational models were exported to the EMTP-RV [16] time domain simulation environment. As a first test, we calculated the induced sheath voltage at the cable far end (V_4) when a unit step voltage is applied to the core conductor, see Fig. 8.

Fig. 9 shows the resulting sheath voltage as calculated by a numerical inverse Fourier transform [15]. The deviations from the Fourier solution are shown for the alternative fitting techniques, indicating a quite accurate result with all approaches.

The same result is shown in Fig. 10 when reducing the fitting order to 12 poles per mode. The deviation with Bode fitting now becomes quite large. This result can be explained by the lower pole redundancy in the phase domain fitting process (4), thus increasing the importance of extracting accurate poles and delays.

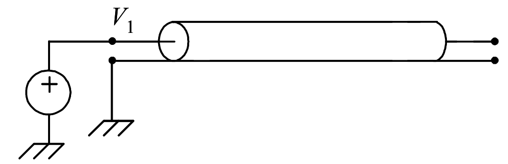

Fig. 8 Step voltage excitation.

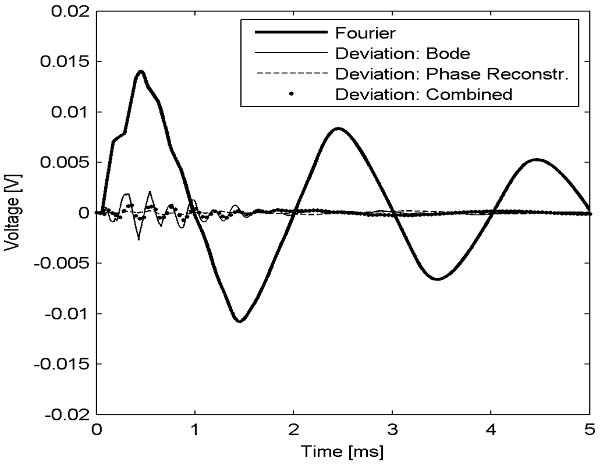

Fig. 9 Induced sheath voltage, N=16.

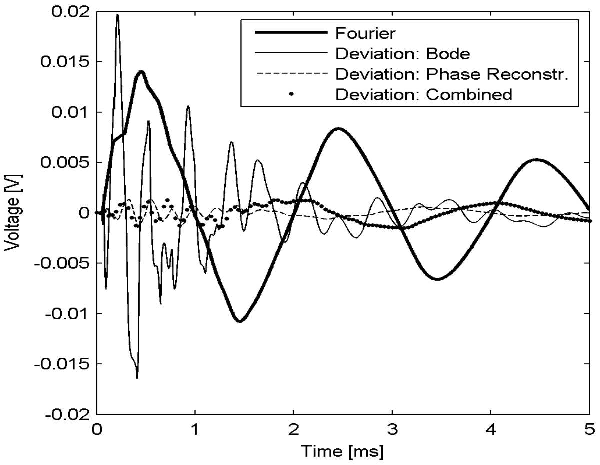

Fig. 10 Induced sheath voltage, N=12.

The fitting order was further reduced to 8 poles per mode, and the core voltage (V_3) was simulated when a unit step voltage is applied to the cable sheath (V_2), see Fig. 11. The simulation result (Fig. 12) again shows that phase reconstruction and the combined approach give a substantially more accurate result than magnitude fitting (Bode).

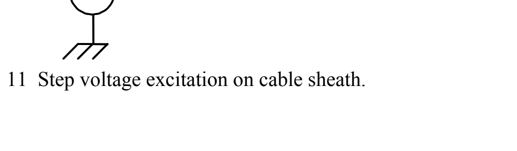

Fig. 11 Step voltage excitation on cable sheath.

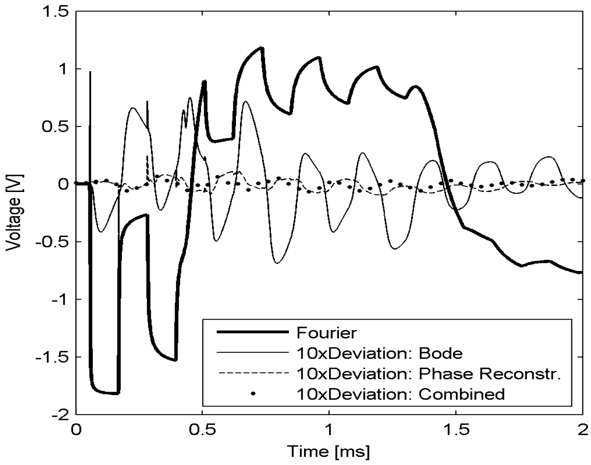

Fig. 12 Core voltage at far end, N=8.

VII. Discussion

It was shown that including delay optimization in VF generally leads to substantially more accurate results for the fitted modes. This gave with the ULM approach a highly accurate result for the final fitting of **H** in the phase domain. It was however noted that some elements of **H** can be fitted with satisfactory result even with poorly fitted modes. This result is due to the high redundancy of the ULM approach as each element is fitted independently using all poles and delays.

The time domain simulation results showed that a model obtained via Bode fitting gave a substantially less accurate result than usage of VF. However, including delay optimization in VF (combined approach) did not lead to much further improvement when compared to a Fourier solution. The latter result could be caused by errors in the Fourier solution itself. Also, the simulation error generally depends on the fitting error of *all* elements of **H**, plus that of $\mathbf{Y}_c$.

In some situations is desirable to leave out the delay optimization due to the computational effort. This is particularly relevant with the Voltage Profile Component [17] where **H** needs to be fitted for a large number of alternative lengths.

The current implementation of ULM in PSCAD and EMTP-RV makes use of delay optimization, similarly to the combined procedure, but with a different optimization approach than Brent's Method.

VIII. Conclusions

Fitting the modes of the propagation function using the Vector Fitting (VF) algorithm leads to a better accuracy than asymptotic magnitude fitting. In general, the time delay to be used for "backwinding" should be chosen larger than the lossless delay. An optimal delay can be extracted by combining VF with Brent's Method in an iterative procedure, giving the best result in terms of *rms*-error and maximum error. With the Universal Line Model, the iterative approach generally results in a further improvement for the propagation function (phase domain), although satisfactory results can often be obtained without iterations.

APPENDIX

The phase angle of the minimum phase shift function h_φ is recovered from its magnitude $|h(\omega)|$ by means of (9-12).

Since the integrand of (12) is singular for u=0, the integration has to be intended in the sense of Cauchy principal value [12]:

$$\int_{-\infty}^{+\infty} = \lim_{\varepsilon \to 0^+} \left[\int_{-\infty}^{-\varepsilon} + \int_{\varepsilon}^{+\infty} \right]. \tag{15}$$

Note that the factor ln(coth(|u|/2)) peaks when u=0 (i.e. ω_l=ω), therefore the phase angle at a given ω mostly depends by the magnitude slope around ω [12].

The numerical implementation of (910) substitutes the limit $\varepsilon \to 0$ by a given value of the sampling interval and the limit to infinite by a given value of the upper/lower frequency.

We perform the numerical integration of (10) using an uniform sampling of the integral, i.e. $\omega_{lj}=\Delta j\omega$, where $j = -N, \ldots -1, 1, \ldots N$ and Δ is assumed as a constant, which corresponds to $u_j=j\Delta$. This gives a proper approximation of the integrand (10) in the sense (15), since the origin u=0 is positioned exactly at the center of the sampling interval. Such a goal is simply achieved generating the ω vector by means of the MATLAB command `logspace`. Note that the integral in (10) should be taken over a frequency range that spans at least two decades below and above ω, i.e. $\omega_{l\,low}=0.01\omega$, $\omega_{l\,high}=100\omega$.

The following shows a MATLAB code for calculating the phase angle of a minimum-phase shift function by (8)-(10). It is assumed that `absH` contains the magnitude function, given at N_s frequency samples. The code calculates the phase angle at the j-th frequency sample.

```
%First term in (9):
    phase1=(pi/2)*log((absH(j+1)/
           absH(j-1)))/(log(w(j+1)/w(j-1)));
%Second term in (9):
    phase2=0;
    term2=log((absH(j+1)/absH(j-1))) /(log(w(j+1)/w(j-1)));
    for k=2:Ns-1
      term1=log(absH(k+1)/absH(k-1)) /(log(w(k+1)/w(k-1)));
      if k~=j
        phase2=phase2+(abs(term1)-abs(term2))
        *log(coth(abs(log(w(k)/w(j)))/2))*log(w(k+1)/w(k));
      end
    end
    phase2=phase2/pi;
    phase_min(j)=(phase1-phase2); %Phase angle [rad]
```

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