Interfacing Convolution Based Linear Models to an Electromagnetic Transients Program

B. Gustavsen, and O. Mo

Abstract—**This paper describes a convolution based terminal equivalent for the PSCAD/EMTDC simulation environment, implemented as a user-defined component. This frequency dependent terminal equivalent (FDTE) reproduces the terminal behavior of a multi-port linear system given as a phase domain state-space model. The model can be identified using available software. A user-written PSCAD component reads the state space parameters from file and includes the model in the simulation via a companion model and trapezoidal based convolution. In addition is implemented a frequency dependent transfer function model (FDTF) that calculates the transfer between a set of inputs and outputs. The accuracy of the FDTE/FDTF components is validated by numerical examples. Also is shown an example from network equivalencing where large savings in computation time is achieved, compared to a traditional modeling procedure.**

Index Terms—**state space model, convolution, electromagnetic transients, PSCAD, vector fitting.**

I. INTRODUCTION

E MTP type programs are widely applied for the analysis of power system transient behavior. Such programs have available a wide range of models for representing the various devices in the system. Although most situations can be adequately handled with existing models, situations frequently arise where additional modeling capability is required.

Black-box equivalents of frequency dependent linear devices and systems can be readily established via rational fitting of terminal responses, leading to pole-residue or state space models. This feature is particularly useful for wide band modeling of transformers [1],[2] and for the calculation of frequency dependent network equivalents (FDNEs) [3],[4],[5].

As the current version of PSCAD [6] lacks this modeling capability, the authors have created a so-called user-defined model that permits to include state space models via convolutions. This allows to model 1) terminal equivalents that interact with the system and 2) pure transfer function models. This paper describes the user-defined models and their implementation in the PSCAD environment. In addition is demonstrated its usage for FDNE modeling.

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II. INCLUSION OF STATE SPACE MODEL BY CONVOLUTIONS

A. Terminal Equivalent

The linear device/system to be modeled is characterized by its terminal admittance matrix $\mathbf{Y}$ that relates terminal voltages $\mathbf{v}$ and terminal currents $\mathbf{i}$.

$$\mathbf{i}(s) = \mathbf{Y}(s)\,\mathbf{v}(s) \tag{1}$$

It is assumed that $\mathbf{Y}$ has been approximated over the frequency band of interest by a common pole set, leading to a residue model on the form

$$\mathbf{Y}(s) \cong \sum_{m=1}^{N} \frac{\mathbf{R}_m}{s - a_m} + \mathbf{D} + s\mathbf{E} \tag{2}$$

where $\mathbf{D}$ and $\mathbf{E}$ are possibly zero. The pole-residue model is seen to be characterized by a common pole set for all elements of $\mathbf{Y}$, which is a requirement for having $\mathbf{Y}$ symmetrical. In order to ensure a stable simulation we must also require that $\mathbf{Y}$ has all its poles stable and that the model is passive. The common pole set requirement can be enforced by fitting all matrix elements simultaneously using the pole relocating vector fitting (VF) [7],[8],[9], which also enforces the stable poles requirement. Passivity can be enforced by subjecting the model to a post processing step which attempts to remove any passivity violations by perturbation [10].

The pole-residue model (2) is converted into a state-space model (3) by appropriate rearrangement of the coefficients. $\mathbf{A}$ becomes diagonal where the poles in (2) are repeated as many times as there are terminals, while $\mathbf{B}$ is a selector matrix having one's and zeros only. The elements of $\mathbf{C}$ and $\mathbf{A}$ are real or come in complex conjugate pairs while the elements of $\mathbf{D}$ and $\mathbf{E}$ are real only.

$$\mathbf{Y}(s) \cong \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} + s\mathbf{E} \tag{3}$$

As shown in the Appendix, application of trapezoidal integration to (1) gives at the nth time step

$$i(n) = i_1(n) + i_2(n) \tag{4}$$

where

$$\mathbf{x}(n) = \boldsymbol{\alpha} \cdot \mathbf{x}(n-1) + \mathbf{b}\,v(n-1) \tag{5a}$$

$$i_1(n) = \mathbf{c}^T\,\mathbf{x}(n) + g\,v(n) \tag{5b}$$

and

$$i_2(n) = -i_2(n-1) + \frac{2e}{\Delta t}(v(n) - v(n-1)) \qquad (6)$$

Equations (4)-(6) assume a single input and single output. The extension to multi-port systems is straightforward and will not be shown here.

Finally, the realization (4)-(6) is cast in the form of an companion network as shown in Fig. 1, where

$$i_{his}^1(n) = -\mathbf{c}^T \mathbf{x}(n) \qquad (7)$$

$$i_{his}^2(n) = i_2(n-1) + \frac{2e}{\Delta t}v(n-1) \qquad (8)$$
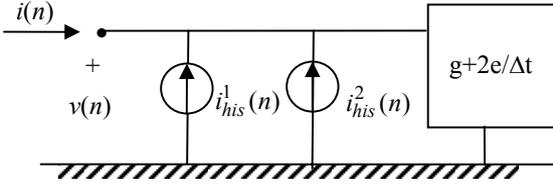
Fig. 1 Companion network

From the circuit in Fig. 1 is seen that the expression for calculating $i^2_{his}(n)$ in (8) can be simplified into

$$i_{his}^2(n) = -i_{his}^2(n-1) + \frac{4e}{\Delta t}v(n-1) \qquad (9)$$

In the multi-terminal case, there is one history current source ($i_{his}=i^1_{his}+i^2_{his}$) from ground to each terminal, and the conductance becomes a matrix. Thus, the electrical relationship becomes

$$\mathbf{i}(n) = \mathbf{G}\,\mathbf{v}(n) - \mathbf{i}_{his}(n) \qquad (10)$$

Note that in (10), the conductance is a matrix and the voltage and currents are vectors.

B. Transfer Equivalent

A transfer equivalent **H** has been developed that defines the relation between a set of inputs **u** and outputs **y**.

$$\mathbf{y}(s) = \mathbf{H}(s)\mathbf{u}(s) \qquad (11)$$

As the number of inputs is in general different from the number of outputs, the resulting $\mathbf{H}(s)$ is non-square and the common pole set requirement no longer applies. One may therefore fit the columns of **H** independently, resulting in that the poles (diagonal elements of **A**) are non-repeated.

$$\mathbf{H}(s) \cong \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} + s\mathbf{E} \qquad (12)$$

The convolution for **H** is in principle identical to that for **Y**. However, as the model does not directly interact with the electrical solution, the convolution is realized directly using (10) without usage of a companion network. This means that the voltage vector **v** in (10) is interpreted as inputs while the current vector **i** is interpreted as outputs. That way, the code used for realizing the FDTE component can be reused with only minor modifications.

III. FDTE IMPLEMENTATION

A. Interface With Electric Circuit

The FDTE model is electrically interfaced to the rest of the electric circuit using a companion model consisting of a conductance network and a number of current sources. The complete companion model of a five terminal model is shown in Fig. 2. The model is here configured with a separate model reference ground as an additional terminal. The user can also specify that the reference ground be common to the PSCAD global ground (not shown).

The conductance branch values g are computed directly from the conductance matrix **G** in (10) as follows

$$g_{ii} = \sum_j G_{ij} \qquad (13)$$

$$g_{ij} = -G_{ij} \qquad (14)$$

Conductance g_{ii} connects terminal i with the model reference terminal while g_{ij} connects terminals i and j. The current sources in Fig. 2 are the history current sources of the companion network, given as the sum of (7) and (8).

The actual implementation of the companion network is done as shown in Fig. 3 since PSCAD does not allow to connect current sources directly between nodes.
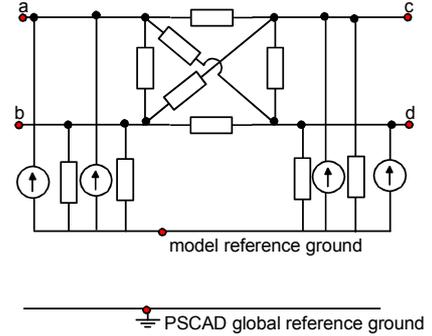
Fig. 2 Companion model with isolated model reference ground terminal.
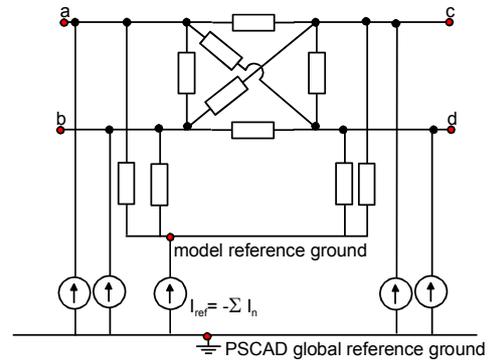
Fig. 3 Implementation in PSCAD

B. User Interface

The graphical symbol for the FDTE is shown in Fig. 4. The left component is configured as a two terminal component with grounded model reference ground, while the right

component is configured as a 10 terminal device with insulated model reference ground.

Clicking on the symbol allows to modify the number of terminals and whether the reference node is to be treated as separate or connected to the global ground. In addition is specified the number of poles (dimension of **A**), which is used for user memory allocation as explained in Section III-C. Also must be specified the name of a text file that holds the model parameters (**A**,**B**,**C**,**D**,**E**). The current version does not allow to connect terminals directly to ground; instead a small resistor must be used if grounding is desired.
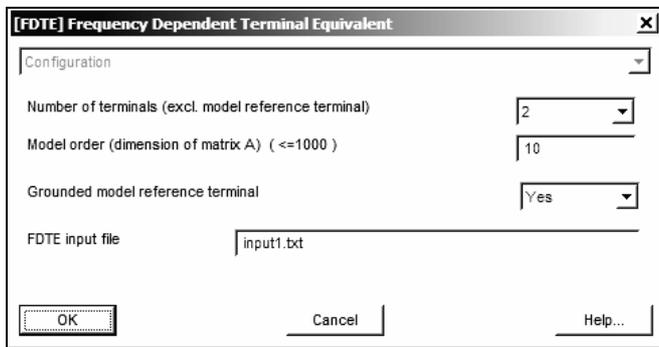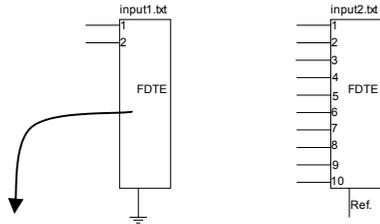
Fig. 4 FDTE component and user dialogue.

C. Interface Between Model and PSCAD

The conductance elements used for realizing the conductance matrix of the companion model are entered into the PSCAD system conductance matrix via RLC-branches within the model Branch Definition segment in PSCAD. In the first simulation step, the required memory is allocated via the model FORTRAN Definition segment while taking into account the model size. In subsequent time steps, the FORTRAN definition segment extracts the terminal voltages with respect to the model reference ground and passes them on to a user-written FORTRAN subroutine which returns the history current sources to be used for the next time step. In the first time step, the subroutine initializes the resistance/conductance matrix for the companion network as well as the coefficients used for updating the history current sources. All data to be used at later time-steps (coefficients and state variables) are stored in internal single dimension arrays that are predefined by PSCAD. That way, an arbitrary number of user defined models can be specified, without waste of memory.

IV. FDTF Implementation

Implementation of the FDTF component is easier than that of the FDTE as the PSCAD interfacing does not require usage

of a companion network. In each time step the values at the input terminals (signal) are transferred to the component subroutine which returns values to be placed at the output terminals. As with FDTE, the model parameters are read from a text file.

The graphical symbol for the FDTF is shown in Fig. 5. The component is here configured with 4 inputs and two outputs. The input/out terminals are "signal" nodes in PSCAD. One is therefore free to connect to the input terminals any signal quantities such as measured voltages/currents and control signals. The signals on output terminals are available for plotting and for use with other devices that take signal quantities as input.

Fig. 5 FDTF component and user dialogue.

V. Example: FDNE Simulation

One of the possible applications of the FDTE/FDTF capability is the FDNE modeling of a part of a power system. To demonstrate this we consider the three-phase distribution system in Fig. 6. As external terminals is selected buses A and B.
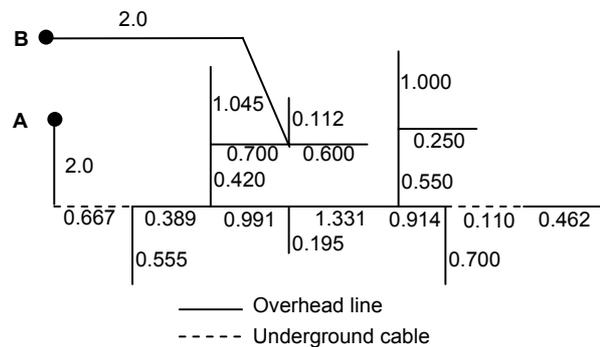
Fig. 6 Distribution system (lengths in km)

In the following example the system is energized at bus A from a three-phase voltage source with bus B being open circuited. A ground fault occurs at A3, 20 ms after energization.
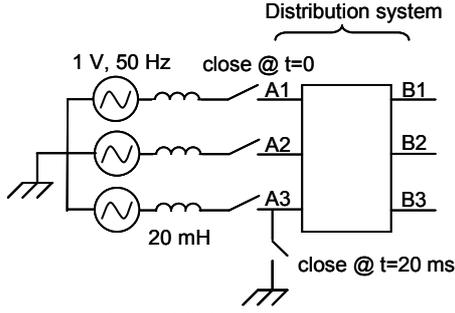
Fig. 7 Energization and ground fault initiation

The transient voltage at A3 and B3 is to be calculated in three different ways:

1. Using a detailed model of the system, with explicit modeling of each cable/line segment by the phase domain line model in PSCAD.
2. Using a six-terminal FDTE of the distribution system with respect to buses A and B.
3. Using a three-terminal FDTE with respect to bus A. The voltage at B3 is obtained by an FDTF that has the voltage at A1, A2, A3 as inputs and the voltage at B3 as output.

With the FDTE/FDTF approaches, the terminal admittance matrix $\mathbf{Y}$ is calculated in the frequency range 10 Hz – 100 kHz with respect to the two three-phase buses A and B. Each cable/line segment is represented by an exact PI equivalent in phase domain co-ordinates that takes into account skin effect in conductors and ground. For that purpose, a standalone program was used for calculating the line series impedance and shunt capacitance matrices.

With the six-terminal FDTE approach (item #2 in list), the obtained $\mathbf{Y}$ is fitted directly using a common pole set of 60 poles. The rational fitting of the 36 elements is shown in Fig. 8.

With the combined FDTE/FDTF approach (item #3 in list), the 6×6 $\mathbf{Y}$ is partitioned into four 3×3 parts, according to buses A and B

$$\begin{bmatrix} \mathbf{i}_A \\ \mathbf{i}_B \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{AA} & \mathbf{Y}_{AB} \\ \mathbf{Y}_{BA} & \mathbf{Y}_{BB} \end{bmatrix} \begin{bmatrix} \mathbf{v}_A \\ \mathbf{v}_B \end{bmatrix} \qquad (15)$$

Since bus B is open circuited we have $\mathbf{i}_B$=0. From (15) we now get

$$\mathbf{i}_A = (\mathbf{Y}_{AA} - \mathbf{Y}_{AB}\mathbf{Y}_{BB}^{-1}\mathbf{Y}_{BA})\mathbf{v}_A = \mathbf{Y}_{AA}^{red}\,\mathbf{v}_A \qquad (16)$$

$$\mathbf{v}_B = (-\mathbf{Y}_{BB}^{-1}\mathbf{Y}_{BA})\mathbf{v}_A = \mathbf{H}\,\mathbf{v}_A \qquad (17)$$

As we are only interested in the voltage on terminal B3, we select the last (third) row of $\mathbf{H}$ in (17). Thus, $\mathbf{H}$ becomes of size 1×3.

The obtained $\mathbf{Y}_{AA}^{red}$ is fitted using a common pole set of 40 poles, while the columns of $\mathbf{H}$ are fitted independently using 40 poles per column, see Figs 9 and 10.
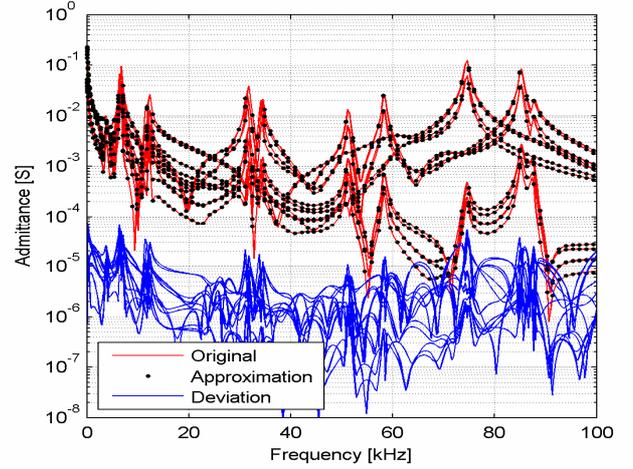
Fig. 8 Rational fitting of $\mathbf{Y}$ (6×6)
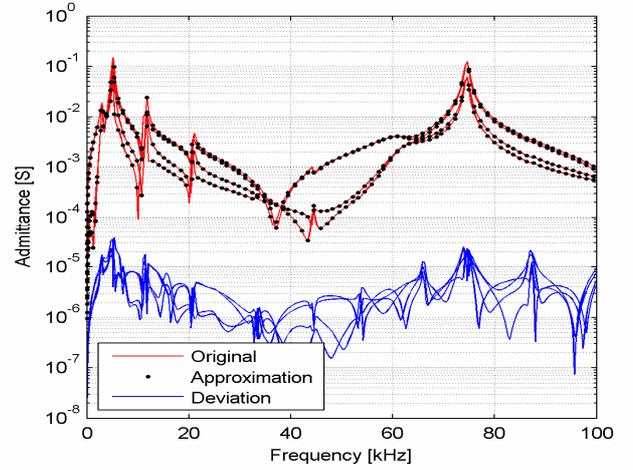
Fig. 9 Rational fitting of $\mathbf{Y}^{red}$ (3×3)
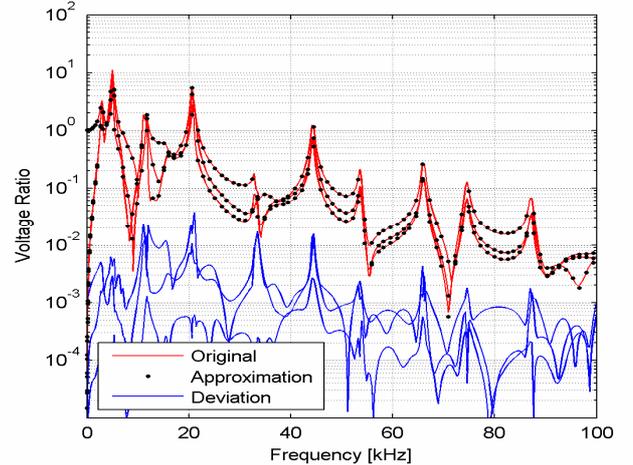
Fig. 10 Rational fitting of $\mathbf{H}$ (1×3)

Figs 11 and 12 compare the simulated voltage on B3 using the three approaches. It is seen that an excellent agreement has been achieved. The (small) discrepancies are possibly due to differences in the line/cable constants routine of PSCAD and the one used for establishing $\mathbf{Y}$, as well as the inaccuracies of the rational fittings observed in Figs 8–10.

Fig. 11 Transient voltage on B3

Fig. 12 Transient voltage on B3 (expanded view)

TABLE I CPU TIME

	ΔT=0.2 μs	ΔT=5 μs
Regular PSCAD	33.23	–
FDTE (6×6)	15.73	0.91
FDTE (3×3) + FDTF (1×3)	6.6	0.53

TABLE II MATRIX SIZES

	FDTE (6×6)	FDTE (3×3)	FDTF (1×3)
A	(360×360)	(120×120)	(120×102)
B	(360×6)	(120×3)	(120×3)
C	(6×360)	(3×120)	(1×120)
D	(6×6)	(3×3)	(1×3)
E	(6×6)	(3×3)	(1×3)

VI. DISCUSSION

In Section V, the new FDTE/FDTF component was used for realizing a network equivalent of a distribution system having many short lines and cables. Here it was found that the FDTE/FDTF approach could greatly reduce computation time as compared to explicit modeling of each line/cable, since a larger time step could be used. The increased time step was possible because the dominant transient was of fairly low frequency, while at the same time the constraint on time step length due to the transmission line travel time was removed. In harmonic studies, the FDTE/FDTF is expected to be highly efficient since only a limited frequency band needs to be considered, thereby permitting low order approximations and long time steps.

When modeling devices from measured responses, there is no alternative to the FDTE/FDTF approach, other than using an equivalent lumped network with many branches. Typical applications include wide band modeling of transformers and motors, and the calculation of transferred voltage to taps and internal points in windings.

Initialization from power frequency steady state can easily be achieved when interfacing FDTE/FDTF to general EMTP-type programs. This type of initialization is however not available in PSCAD.

The FDTE/FDTF models are fully compatible with the interpolation feature in PSCAD since all history current source currents for the next time step are stored in dedicated PSCAD internal storage arrays controlled by the simulation engine. Consequently, there are no memory effects in model that will interfere with the interpolation feature.

The state-space models used in this work were obtained using the relaxed version of vector fitting [9] as implemented in `vectfit2.m`. This software can be downloaded from [12].

VII. CONCLUSIONS

This paper has described the implementation of a frequency dependent terminal equivalent (FDTE) and a frequency dependent transfer function (FDTF) for the PSCAD simulation environment. The FDTE/FDTF components were coded as user-defined components in the FORTRAN language, thereby giving a highly efficient code. Usage of the FDTE/FDTF modeling approach was found to greatly reduce

We next compare the computational effort by the three approaches, excluding writing results to disk file.

In the detailed PSCAD model, the shortest transmission line travel time is 0.38 μs, due to the 112 m line. Since the simulation time step length must be smaller than this value, a time step length of ΔT=0.2 μs was used in the simulation (Figs 11, 12), giving a total of 100,000 time steps. The second column in Table I shows the CPU time needed by the three calculation approaches when using this time step. It is seen that usage of FDTE + FDTF gives a reduction in computation time by about a factor of 5.

In this example, one is with the FDTE/FDTF approaches permitted to use a much larger time step than 0.2 μs since the presence of the short lines has been removed. The dominant frequency component in Fig. 12 is only about 4 kHz, corresponding to a period of 250 μs. The third column in Table I shows the CPU time when increasing the time step length to 5 μs. Clearly, the ability of using a bigger time step leads to large savings in computation time over the regular PSCAD approach. The change to the time domain responses was negligible. The size of the state-space matrices are listed in Table II. Note that **A** is diagonal and **B** is sparse.

computation time for a case of network equivalent modeling (FDNE) where the presence of short stub lines requires to use a very small time step with a traditional approach.

VIII. APPENDIX

Consider a first order pole-residue model

$$y = \frac{r}{s-a} u \qquad (18)$$

In the time domain we have

$$\dot{x} = ax + bu \qquad (19a)$$
$$y = rx \qquad (19b)$$

Trapezoidal integration gives at the nth time step

$$\frac{x(n)-x(n-1)}{\Delta t} = a\frac{x(n)+x(n-1)}{2} + \frac{u(n)+u(n-1)}{2} \qquad (20a)$$

$$y(n) = r\,x(n) \qquad (20b)$$

Collecting terms gives

$$x(n) = \alpha\,x(n-1) + \lambda u(n) + \mu u(n-1) \qquad (21a)$$

$$y(n) = r\,x(n) \qquad (21b)$$

where

$$\alpha = \frac{1+a\Delta t/2}{1-a\Delta t/2}, \quad \lambda = \mu = \frac{\Delta t/2}{1-a\Delta t/2} \qquad (22)$$

One of the terms in (21a) is removed by introducing the transformation

$$x'(n) = x(n) - \lambda u(n) \qquad (23)$$

This gives

$$x'(n) = \alpha\,x'(n-1) + (\alpha\lambda + \mu)u(n-1) \qquad (24a)$$

$$y(n) = r\,x'(n) + r\lambda u(n) \qquad (24b)$$

The coefficient in front of $u(n-1)$ in (24a) is scaled to unity,

$$x''(n) = \alpha\,x''(n-1) + u(n-1) \qquad (25a)$$

$$y(n) = r(\alpha\lambda + \mu)x''(n) + r\lambda u(n) \qquad (25b)$$

Simplifying the notation gives

$$x(n) = \alpha\,x(n-1) + u(n-1) \qquad (26a)$$

$$y(n) = c\,x(n) + g\,u(n) \qquad (26b)$$

For an Nth order approximation, (26a)-(26b) changes into

$$\mathbf{x}(n) = \boldsymbol{\alpha} \cdot \mathbf{x}(n-1) + \mathbf{b}\,u(n-1) \qquad (27a)$$

$$y(n) = \mathbf{c}^T \mathbf{x}(n) + g\,u(n) \qquad (27b)$$

where $\mathbf{x}$ and $\boldsymbol{\alpha}$ are column vectors of length N, $\mathbf{b}$ is a column vector of ones, and $\mathbf{c}^T$ is a row vector. In (27a), the "dot" denotes element-by-element multiplication. The multiplication with $\mathbf{b}$ does not need to be carried out since its elements are unity.

In the case of a complex conjugate pair in (18), the second term is deleted and the residue (r) of the first part is multiplied by a factor two [5]. This effectively reduces the computation time in half. Only the real part of $y(n)$ is retained.

Next, consider a capacitive term,

$$i = e\frac{dv}{dt} \qquad (28)$$

Trapezoidal integration gives at the nth time step

$$\frac{i(n)+i(n-1)}{2} = e\frac{v(n)-v(n-1)}{\Delta t} \qquad (29)$$

Rearranging terms gives

$$i(n) = -i(n-1) + \frac{2e}{\Delta t}(v(n)-v(n-1)) \qquad (30)$$

IX. REFERENCES

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