

# Computer-Aided Sensitivity Analysis for Optimal Systems

M. Heidari, *Student Member, IEEE*, S. Filizadeh, *Member, IEEE*,  
and A. M. Gole, *Senior Member, IEEE*

**Abstract**-- In this paper, a computer-aided sensitivity analysis tool is developed for analyzing nonlinear optimum systems. To enable sensitivity analysis around the optimum points, the concept of second-order sensitivity analysis is introduced. The developed tool not only provides the conventional sensitivity indices (which are useful for non-optimal solutions), but also provides the second-order sensitivity indices. The proposed method have been exemplified using Selective Harmonic Elimination (SHE) switching scheme for voltage sourced converters.

**Index Terms**—Sensitivity analysis, transient simulation, selective harmonic elimination, STATCOM.

## I. INTRODUCTION

**S**IMULATION-BASED optimization tools [1], [2], which have been developed recently, allow the designer to optimize the design parameters of complex nonlinear systems in a much shorter time and with less simulation intensity (i.e., number of simulations required to determine the optimum) than conventional random or sequential search methods. In these new tools, the search for the optimum is carried out by a nonlinear optimization algorithm; transient simulation is used to evaluate a corresponding objective function whose minimization yields the optimum system performance [1].

An important issue to consider once the optimal solution is obtained is to determine the sensitivity of the design to the variations of the optimized parameters. Such variations can be caused by manufacturing tolerance, aging, change of operating conditions (e.g., temperature change), and measurement errors (in case of closed loop systems). Therefore, it is an indispensable step of the design procedure to determine how the performance of the optimized systems is affected when parameters deviate from their optimized values.

Conventional sensitivity analysis methods have been developed and used for such studies [3]. However, those methods have two main drawbacks. First, there is often no analytical solution for complex nonlinear systems; therefore analytical sensitivity assessment methods cannot be readily applied. Second, the majority of sensitivity analysis methods use first-order derivatives to estimate sensitivity to parameter

changes [3], [4]. However, such derivatives vanish around an optimum (unless optimized under constraints), thus they fail to provide useful information.

In this paper, a computer-aided sensitivity analysis tool is developed for nonlinear systems for which analytical formulation of the performance index is not attainable. To enable sensitivity analysis even around optimum points, the concept of second-order sensitivity analysis is introduced and implemented. The developed sensitivity analysis tool not only provides the conventional sensitivity indices (which may be useful for non-optimal solutions), but also provides the second-order sensitivity indices.

The usefulness of the proposed methods, which have been implemented in the PSCAD/EMTDC [5] electromagnetic transient simulation program, is demonstrated by studying Selective Harmonic Elimination (SHE) switching scheme for voltage sourced converters..

## II. SENSITIVITY ANALYSIS OF NONLINEAR SYSTEMS

The majority of conventional sensitivity analysis methods are based on first-order derivatives of the system performance. The well-known Bode logarithmic sensitivity index is a widely-used index defined as follows [3]:

$$S_{x_i}^f = \frac{\partial f(x_1, \dots, x_n)}{\partial x_i} \frac{x_i}{f(x_1, \dots, x_n)} \quad (1)$$

where  $f$  is the performance function (e.g., THD of a converter current, or a bus voltage),  $x_i$  is the  $i$ -th adjustable parameter of the system (e.g., firing angle of a converter, or size of a capacitor), and  $S_{x_i}^f$  is the sensitivity index of  $f$  with respect to  $x_i$ . Having found all sensitivity indices, one can study the effect of parameter variations ( $\Delta x_i$ ) on the behavior of a system performance (represented by  $f$ ) using the following equation:

$$f(x_1 + \Delta x_1, \dots, x_n + \Delta x_n) \approx f(x_1, \dots, x_n) \left( 1 + S_{x_1}^f \frac{\Delta x_1}{x_1} + \dots + S_{x_n}^f \frac{\Delta x_n}{x_n} \right) \quad (2)$$

However, conventional sensitivity indices are not effective when the sensitivity of an optimum solution is considered. By definition, an optimum is a point at which all first order derivatives are zero (unless the optimum lies on the boundary of the optimization space). Under such circumstances, the sensitivity indices given by (1) become zero (or very small in

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M. Heidari, S. Filizadeh, and A. M. Gole are with the Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, MB R3T 5V6, Canada (e-mail: mheidari@ee.umanitoba.ca; sfilizad@ee.umanitoba.ca; gole@ee.umanitoba.ca)

case of an approximate solution) and hence fail to provide useful information.

Consider Fig. 1, in which a single-variable functions  $f$  is shown. The function attains its minimum at  $x_0=1$ , where the first-order derivative of the function becomes zero. A local estimation of the function using its Bode sensitivity index (1) yields  $f_1(x) = f(x_0) \left(1 + S_x^f \frac{\Delta x}{x}\right) = f(x_0)$ , as  $S_x^f = 0$  at the optimum.

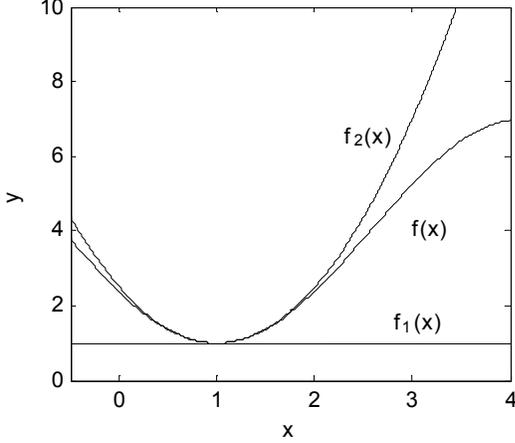


Fig. 1. First-order ( $f_1$ ) and second-order ( $f_2$ ) estimation around an optimum.

To allow assessment of sensitivity around optimal points of a given performance index, second-order sensitivity analysis is proposed. The second-order sensitivity indices are defined as follows.

$$S_{x_i}^f = \frac{1}{2} \frac{x_i^2}{f} \frac{\partial^2 f}{\partial x_i^2} \quad S_{x_i x_j}^f = \frac{x_i x_j}{f} \frac{\partial^2 f}{\partial x_i \partial x_j} \quad (3)$$

Using the second-order derivatives one can obtain an estimation of a given multi-variable function by considering both first and second-order terms as follows.

$$\begin{aligned} f(x_1, \dots, x_n) &\approx f(x_1, \dots, x_n) \left(1 + S_{x_1}^f \frac{\Delta x_1}{x_1} + \dots + S_{x_n}^f \frac{\Delta x_n}{x_n} + \dots\right. \\ &+ S_{x_1 x_1}^f \left(\frac{\Delta x_1}{x_1}\right)^2 + S_{x_n x_n}^f \left(\frac{\Delta x_n}{x_n}\right)^2 + S_{x_1 x_2}^f \frac{\Delta x_1}{x_1} \frac{\Delta x_2}{x_2} + \dots \\ &\left. + S_{x_1 x_n}^f \frac{\Delta x_1}{x_1} \frac{\Delta x_n}{x_n} + S_{x_1 x_2}^f \frac{\Delta x_2}{x_2} \frac{\Delta x_3}{x_3} \dots + S_{x_{n-1} x_n}^f \frac{\Delta x_{n-1}}{x_{n-1}} \frac{\Delta x_n}{x_n}\right) \end{aligned} \quad (4)$$

For the single-variable function  $f$ , an improved estimation of the function using its second-order sensitivity indices is as follows.

$$f_2(x) = f(x_0) \left(1 + S_x^f \frac{\Delta x}{x} + S_{xx}^f \left(\frac{\Delta x}{x}\right)^2\right) \quad (5)$$

As it can be seen in Fig. 1,  $f_2$  is a more accurate representation of  $f$  than  $f_1$ . Note that if the estimations are obtained around a non-optimal point (where the first-order derivatives are non-zero), the first-order sensitivity analysis

will still provide acceptable estimation of the function; however, for further accuracy and in order to estimate functions around their local optima, second order indices are necessary.

### III. EVALUATION OF SENSITIVITY INDICES

#### A. Evaluation of the Performance Function

If an analytical expression of the performance function of a given system is readily available, one can use (1) and (3) to calculate first and second-order sensitivity indices. Such formulation however, is usually not attainable for complex systems, and therefore, numerical methods need to be employed for their calculation. Evaluation of the performance function for such systems is possible either through direct measurement or through simulation.

Electromagnetic transient simulation programs are able to accurately model and simulate power systems, which include nonlinear and switching devices [6]. In this paper, PSCAD/EMTDC program has been chosen as the simulation platform to simulate and evaluate the performance index of a complex network.

Although PSCAD/EMTDC is able to simulate the system behavior accurately, it is not able to calculate the sensitivity indices directly. However, it can be used as an evaluator of the performance function [1], and it is interfaced with an external algorithm that uses such function evaluations for estimation of respective sensitivity indices.

#### B. Numerical Estimation of the Sensitivity Indices

For a positive incremental change in one of the variables of a multi-variable function, the following expression can be used to estimate the function variation using the Taylor series expansion (third and higher order terms are neglected).

$$\begin{aligned} f(x_1, \dots, x_i + \Delta x_i, \dots, x_n) - f(x_1, \dots, x_n) &\approx \\ \frac{\partial f}{\partial x_i} \Delta x_i + \frac{1}{2} \frac{\partial^2 f}{\partial x_i^2} \Delta x_i^2 \end{aligned} \quad (6)$$

Similarly for a negative increment, we obtain:

$$\begin{aligned} f(x_1, \dots, x_i - \Delta x_i, \dots, x_n) - f(x_1, \dots, x_n) &\approx \\ -\frac{\partial f}{\partial x_i} \Delta x_i + \frac{1}{2} \frac{\partial^2 f}{\partial x_i^2} \Delta x_i^2 \end{aligned} \quad (7)$$

Combining (6) and (7) yields the following expressions for estimated values for both first and second order derivatives of the function with respect to the incremented variable.

$$\begin{aligned} \frac{\partial f}{\partial x_i} &\approx \frac{f(x_1, \dots, x_i + \Delta x_i, \dots, x_n) - f(x_1, \dots, x_i - \Delta x_i, \dots, x_n)}{2\Delta x_i} \\ \frac{\partial^2 f}{\partial x_i^2} &\approx \frac{f(x_1, \dots, x_i + \Delta x_i, \dots, x_n) + f(x_1, \dots, x_i - \Delta x_i, \dots, x_n)}{\Delta x_i^2} \\ &- \frac{2f(x_1, \dots, x_n)}{\Delta x_i^2} \end{aligned} \quad (8)$$

Note that estimation of partial derivatives requires two function evaluations, i.e.,  $f(x_1, \dots, x_i + \Delta x_i, \dots, x_n)$  and  $f(x_1, \dots, x_i - \Delta x_i, \dots, x_n)$ , which in the case of simulation-based approach leads to two transient simulations of the system.

Having found estimations of the above partial derivatives, one can use the same approach to determine mixed partial derivatives as follows. Note that the right-hand side terms in (8) and (9) depend only on function evaluations for various parameter values, which can be obtained using simulation of the network for the corresponding parameters. Also note that calculation of mixed partial derivatives (9) requires the derivatives obtained in (8).

$$\begin{aligned} \frac{\partial^2 f}{\partial x_i \partial x_j} &\approx \frac{f(x_1, \dots, x_i + \Delta x_i, \dots, x_j + \Delta x_j, \dots, x_n) - f(x_1, \dots, x_i - \Delta x_i, \dots, x_j + \Delta x_j, \dots, x_n)}{2\Delta x_i \Delta x_j} \\ &+ \frac{f(x_1, \dots, x_i + \Delta x_i, \dots, x_j - \Delta x_j, \dots, x_n) - f(x_1, \dots, x_i - \Delta x_i, \dots, x_j - \Delta x_j, \dots, x_n)}{2\Delta x_i \Delta x_j} - \frac{f(x_1, \dots, x_i, \dots, x_n)}{\Delta x_i \Delta x_j} \quad (9) \\ &- \frac{1}{2} \frac{\Delta x_i}{\Delta x_j} \frac{\partial^2 f}{\partial x_i^2} - \frac{1}{2} \frac{\Delta x_j}{\Delta x_i} \frac{\partial^2 f}{\partial x_j^2} \end{aligned}$$

Fig. 2 shows a schematic diagram of the interface between the simulation program and the external sensitivity index calculator.

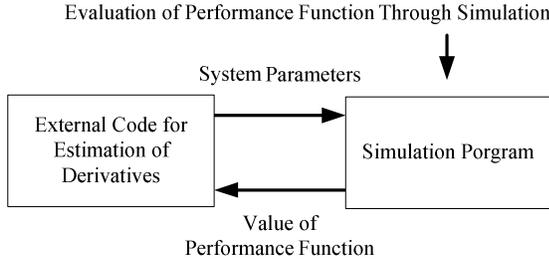


Fig. 2. Multiple simulation technique used for estimation of derivatives.

#### IV. CASE STUDY: SELECTIVE HARMONIC ELIMINATION

In this section the proposed sensitivity analysis tool is used to assess the performance of a Selective Harmonic Elimination (SHE) scheme for generation of firing pulses for high power converters. These techniques gained popularity in high power applications [7], [8] due to their lower switching losses and better harmonic spectrum than conventional pulse-width modulation (PWM) methods. In these methods, by choosing proper switching angles, a certain number of harmonics are eliminated from the output voltage of the converter.

Fig. 4 shows a typical output voltage of a two-level converter (shown in Fig. 3) with SHE switching pattern. As shown, the waveform has five chops in each quarter-cycle, which provides five degrees of freedom to improve the output voltage of the converter. By proper selection of those angles, the fundamental voltage can be controlled, and four harmonics can be eliminated. Increasing the number of the chops enables elimination of more harmonic components.

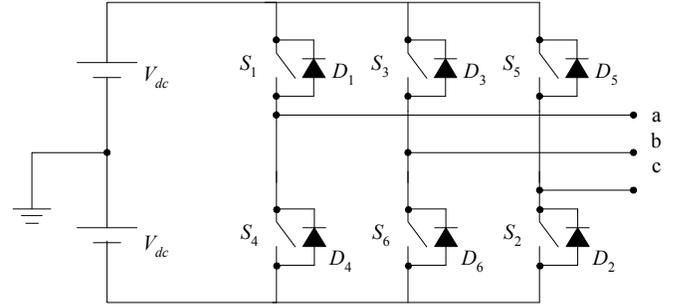


Fig. 3. Two-level inverter

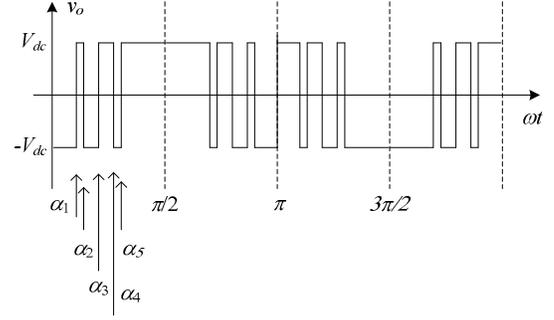


Fig. 4. Harmonic elimination switching pattern

Analytical solution for the switching angles is available for idealized converters, in which the dc-side voltage is constant. However, switching at exact instants of time specified by the idealized solutions may not be possible due to digital implementation of SHE. Such firing mismatch will introduce remnant harmonics in the synthesized output voltage. Sensitivity analysis can be used to quantify the deviations in the harmonic spectrum from the idealized situation.

In this section, an ideal voltage sourced converter (VSC) with a constant dc voltage is first considered. Simulation-based sensitivity analysis is used to assess the harmonic spectrum of the voltage waveform in presence of firing mismatch. Since an analytical solution is available for this idealized case, the results obtained using simulation can be verified against those obtained analytically. In the second example, simulation-based sensitivity is done for a static VAR compensator (STATCOM) with a capacitive dc-link, which introduces fluctuations in the dc voltage. Complexity of the network and its interactions with the VSC in the latter example prohibits an analytical formulation of the problem; this necessitates the use of the proposed simulation-based tool.

##### A. Case I: Inverter with Ideal DC Voltage

For a two-level converter with SHE firing, the harmonic content of the output voltage is calculated as follows [9]:

$$V_n = \left| \frac{2\sqrt{2}V_{dc}}{n\pi} \left( 1 + 2 \sum_{i=1}^5 (-1)^i \cos(n\alpha_i) \right) \right| \quad (10)$$

where,  $\alpha_i$  is the  $i$ -th switching angle,  $V_n$  is the rms value of the  $n$ -th harmonic of the voltage, and  $2V_{dc}$  is the total dc bus voltage. To regulate the fundamental and eliminate  $N-1$

harmonic components, a system of  $N$  nonlinear equations of the form (10) needs to be solved simultaneously for the  $N$  switching angles. [9]. For a waveform with 5 chopping angles, the system of equations will be as follows.

$$\begin{cases} \left| 1 + 2 \sum_{i=1}^5 (-1)^i \cos(\alpha_i) \right| = m \\ \left| 1 + 2 \sum_{i=1}^5 (-1)^i \cos(k\alpha_i) \right| = 0, k = 5, 7, 11, 13 \end{cases} \quad (11)$$

where  $m$  is the modulation index defined as follows:

$$m = \frac{\pi\sqrt{2}}{4V_{dc}} V_1 \quad (12)$$

Table I shows the 5 chopping angles for a two-level VSC with constant dc-bus voltage of 24 kV (total), when the fundamental phase voltage is regulated to 8 kV rms, i.e.,  $m=0.74$  in the system of equations in (11).

TABLE I  
SWITCHING ANGLES TO ELIMINATE 5<sup>TH</sup>, 7<sup>TH</sup>, 11<sup>TH</sup>, AND 13<sup>TH</sup> HARMONICS

$V_{dc} = \pm 12$ kV, $V_1 = 8$ kV, $m=0.74$				
SHE Switching Angles				
$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$
11.0°	23.3°	29.9°	46.3°	50.8°
Harmonic Spectrum of the Output Voltage				
$V_1$	$V_5$	$V_7$	$V_{11}$	$V_{13}$
8.0 kV	0.0 kV	0.0 kV	0.0 kV	0.0 kV

As an example, the impact of firing angle mismatch on the fifth order harmonic (which is the lowest order targeted harmonic) is considered. The developed sensitivity analysis tool has been used to calculate the sensitivity indices (derivatives) of  $|V_5|$  with respect to the switching angles  $\alpha_i$ . The results are shown in the Table II.

TABLE II  
SENSITIVITY ANALYSIS RESULTS OF THE FIFTH ORDER HARMONIC

$ V_5 $	$\frac{\partial}{\partial \alpha_1}$	$\frac{\partial}{\partial \alpha_2}$	$\frac{\partial}{\partial \alpha_3}$	$\frac{\partial}{\partial \alpha_4}$	$\frac{\partial}{\partial \alpha_5}$
1	-0.033	0.011	-0.036	-0.038	0.031
$\frac{\partial}{\partial \alpha_1}$	2.445	-1.233	0.130	0.168	-2.263
$\frac{\partial}{\partial \alpha_2}$	-1.233	1.357	-0.752	-1.179	0.072
$\frac{\partial}{\partial \alpha_3}$	0.130	-0.752	1.076	0.123	-1.380
$\frac{\partial}{\partial \alpha_4}$	0.168	-1.179	0.123	2.212	-2.165
$\frac{\partial}{\partial \alpha_5}$	-2.263	0.072	-1.380	-2.165	2.608

The table shows both first and second order derivatives, which are estimated with the developed tool (for example

$$\frac{\partial |V_5|}{\partial \alpha_3} = -0.036 \text{ [kV/deg]} \quad \text{and} \quad \frac{\partial^2 |V_5|}{\partial \alpha_2 \partial \alpha_3} = -0.752 \text{ [kV/deg}^2\text{)].}$$

Suppose that there is a  $0.1^\circ$  switching error for each of the switching angles (note that for a 60 Hz system,  $0.1^\circ$  corresponds to  $4.6 \mu\text{s}$  mismatch in switching time). Using (4), an explicit first-order or quadratic function for  $|V_5|$  (depending on whether first- or second-order derivatives are used) with respect to switching angles  $\alpha_1 \dots \alpha_5$  can be obtained, which can then be used to estimate the worst-case scenario. Table III shows the results for the worst-case combination of the switching angles using the estimated derivatives in Table II.

Note that the estimated remnant 5<sup>th</sup> harmonic using first-order derivatives is 0.3%, which is far less than the actual value of 1.9% obtained using (10) for the deviated angles. Second-order derivatives however, provide a closer estimation of 2.1%.

TABLE III  
THE WORST CASE FOR 5<sup>TH</sup> HARMONIC

$\Delta \alpha_1$	$\Delta \alpha_2$	5 <sup>th</sup> Order Harmonic		
-0.1°	+0.1°	First-Order Estimation	Second-Order Estimation	Analytical Result
+0.1°	-0.1°			
-0.1°	-0.1°	0.023 kV (0.3% of $V_1$ )	0.167 kV (2.1% of $V_1$ )	0.149 kV (1.9% of $V_1$ )
+0.1°	+0.1°			

Note that the switching angles given in Table II are an optimum solution for the selective harmonic elimination problem laid out in (11). However, they are an optimal solution for elimination of the 5<sup>th</sup> harmonic as well. Since first-order derivatives tend to vanish around an optimum, they fail to provide an acceptable estimation of the remnant harmonics. On the other hand, the proposed second-order technique can estimate the system performance with considerably higher accuracy.

### B. Case II: SHE for a Static Compensator

In practice, voltage-sourced converters are often supplied on the dc side through a capacitor. Unlike an idealized dc source, a capacitor will experience voltage fluctuations during normal and transient operation of the converter. Although provisions for minimizing such fluctuations are incorporated into the design of the capacitive dc buses, small voltage ripple will still be present.

Fig. 5 shows the schematic diagram of a static VAR compensator (STATCOM) with a capacitive dc bus. The VSC is connected through a transformer to an ac network. Other system specifications are listed in Table IV. SHE switching pattern with five chops is used in order to shape the output voltage. The corresponding switching angles for harmonic elimination (obtained using idealized equations in (10)) are shown in Table V. Under normal operating conditions the STATCOM injects 4.7 MVAR to the network (when the angles listed in Table V are used for switching). Note that due to the variations in the dc voltage, the harmonics are not completely eliminated although the residual values are quite small. It has been shown in [8] that the idealized solution of

(10) is quite close to the solution obtained using optimization-based simulation of the non-ideal network.

The complexity of this nonlinear network makes formulation of the sensitivity of harmonics to switching angle variations excessively difficult when dc voltage fluctuations and firing mismatch are considered simultaneously. Note that although in this case the idealized formulation in (10) becomes invalid, the method proposed here is still capable of estimating sensitivity indices through detailed simulation of the network.

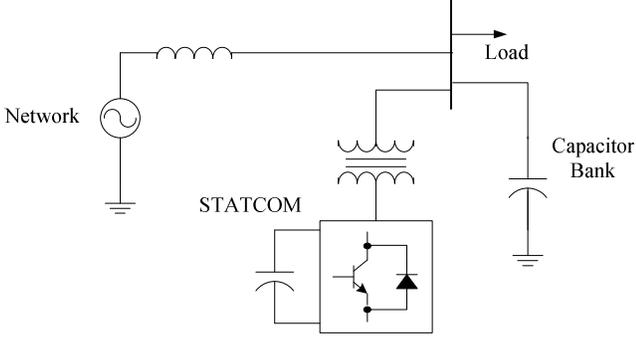


Fig. 5. STATCOM case

TABLE IV  
SYSTEM PARAMETERS

Network	20 kV, SCR = 3.5
STATCOM Transformer	4.8 kV/20 kV, 8.0 MVA, $X_l = 15\%$
Capacitor Bank	4.0 MVAR
Load	18.0 MVA, pf = 0.9
STATCOM Converter	4.8 kV, C = 0.27 pu

TABLE V  
STATCOM SWITCHING ANGLES AT THE NOMINAL OPERATING POINT

$V_{dc} = \pm 8.5$ kV				
SHE Switching Angles				
$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$
11.0°	23.3°	29.9°	46.3°	50.8°
Harmonic Spectrum of Output Voltage				
$V_1$	$V_5$	$V_7$	$V_{11}$	$V_{13}$
11.33 kV	0.03 kV	0.01 kV	0.00 kV	0.00 kV

The dc-bus voltage and the resulting ac voltage and at the VSC and network terminals are shown in Fig. 6, 7 and 8, respectively.

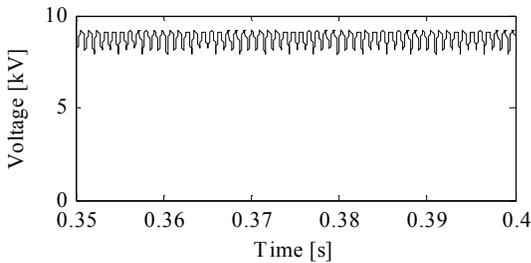


Fig. 6. STATCOM dc-bus voltage

As shown the dc voltage has high frequency ripple around its average value of 8.5 kV. Such ripple is also reflected on the ac side in the form of fluctuations on the pulses comprising the

output voltage. The impact of firing mismatch on the fifth order harmonic in the presence of ripple is quantified through the sensitivity indices given in Table VI, which are obtained using the proposed simulation-based sensitivity analysis.

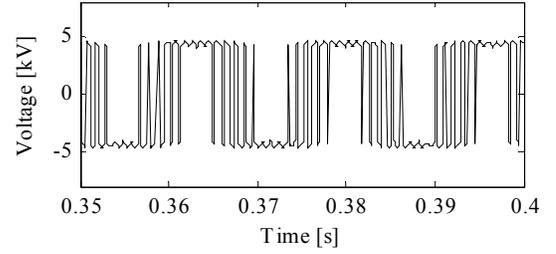


Fig. 7. Phase-a voltage of the converter.

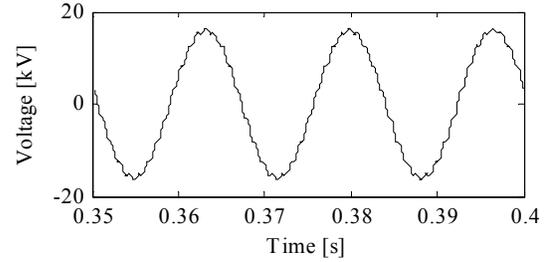


Fig. 8. Phase-a network voltage.

TABLE VI  
SENSITIVITY ANALYSIS RESULTS FOR THE STATCOM VOLTAGE

$ V_5 $	$\frac{\partial}{\partial \alpha_1}$	$\frac{\partial}{\partial \alpha_2}$	$\frac{\partial}{\partial \alpha_3}$	$\frac{\partial}{\partial \alpha_4}$	$\frac{\partial}{\partial \alpha_5}$
1	-0.095	0.045	-0.083	-0.107	0.112
$\frac{\partial}{\partial \alpha_1}$	2.677	-1.533	0.365	0.559	-2.745
$\frac{\partial}{\partial \alpha_2}$	-1.533	2.123	-1.310	-1.530	0.201
$\frac{\partial}{\partial \alpha_3}$	0.365	-1.310	1.666	0.375	-2.365
$\frac{\partial}{\partial \alpha_4}$	0.559	-1.530	0.375	2.577	-2.842
$\frac{\partial}{\partial \alpha_5}$	-2.745	0.201	-2.365	-2.842	3.749

Similar to the previous example, the above indices are used to obtain first and second-order approximations of the magnitude of the fifth harmonic around the optimized set of switching angles. Using such approximations, one can perform a study of the worst-case scenario of the reemergence of the fifth harmonic when firing angles mismatch their optimized values within a given tolerance. Table VII shows the worst-case combination of the switching angles and the estimated remnant fifth harmonic using both first and second-order approximations. As shown, the first order method gives an estimated value of 0.4%, while the value predicted by the second order method is 2.2%. Note that a direct simulation of the network for the worst case combination of switching angles (an analytical expression is unavailable for this case) also yielded a 2.2% of fifth harmonic, which matches the second

order estimation.

TABLE VII  
THE WORST CASE FOR 5<sup>TH</sup> HARMONIC FOR THE STATCOM

$\Delta\alpha_1$	-0.1°	5 <sup>th</sup> Order Harmonic		
$\Delta\alpha_2$	+0.1°	First-Order Estimation	Second-Order Estimation	Simulation Result
$\Delta\alpha_3$	-0.1°			
$\Delta\alpha_4$	-0.1°	0.044 kV	0.246 kV	0.251 kV
$\Delta\alpha_5$	+0.1°	(0.4% of $V_1$ )	(2.2% of $V_1$ )	(2.2% of $V_1$ )

## V. CONCLUSION

The paper introduced simulation-based methods for evaluation of the sensitivity of the performance of a circuit to perturbations in its design parameters. The second-order sensitivity indices were proposed to enable sensitivity analysis around optimal solutions, where conventional first-order indices vanish. The approach proposed in the paper uses transient simulation of the network for numerical estimation of first and second-order derivatives, which will be used subsequently to obtain a quadratic approximation of the performance function, which is used to perform such studies as worst-case analysis.

The paper presented an implementation of the method in the PSCAD/EMTDC and used the developed tool to assess the sensitivity of the harmonic content of the output voltage of a VSC to variations of the switching angles under selective harmonic elimination. Since sensitivity around an optimal solution was considered, first order derivatives provided a poor estimation of the remnant harmonics while second-order indices yielded estimations of much higher accuracy. It was shown that the estimated results agreed well with the analytical solution (for the idealized VSC with constant dc voltage) and direct simulation for the STATCOM example with fluctuating dc voltage.

## VI. REFERENCES

- [1] A. M. Gole, S. Filizadeh, R. W. Menzies, and P. L. Wilson, "Optimization-Enabled Electromagnetic Transient Simulation", IEEE Trans. Power Delivery, Vol. 20, pp. 512 – 518, Jan 2005
- [2] S. Filizadeh, and A. M. Gole, "A Gradient-Based Approach for Power System Design Using Electromagnetic Transient Simulation Programs",

- in Proc. International Conference on Power System Transients, (IPST'05), 2005
- [3] Mansour Eslami and Richard S. Marleau, "Theory of Sensitivity of Network: A Tutorial", IEEE Trans. Education, Vol. 32, No. 3, pp. 319 – 334, 1989
- [4] J. A. Martinez, "Computational Aspects of Sensitivity Analysis", in Proc. IEEE Power Engineering Society Winter Meeting, pp.799 – 800, 2001
- [5] EMTDC User's Guide, Manitoba HVDC Research Center, Winnipeg, MB, Canada, 2003
- [6] H.W. Dommel, "Digital Computer Solution of Electromagnetic Transients in Single and Multiphase Networks", IEEE Trans. Power Apparatus and Systems, Vol. PAS-88, no. 4, pp. 388–399, Apr. 1969
- [7] Lie Xu and Vassilios G. Agelidis, "A VSC Transmission System Using Flying Capacitor Multilevel Converters and Selective Harmonic Elimination PWM Control", in Proc. International Power Engineering Conference (IPEC'05), 2005
- [8] S. Filizadeh, and A. M. Gole, "Harmonic Performance Analysis of an OPWM-Controlled STATCOM in Network Applications", IEEE Trans. Power Delivery, Vol. 20, No. 2, pp. 1001 – 1008, Apr 2005
- [9] John N. Chiasson, Leon M. Tolbert, Keith J. McKenzie, and Zhong Du, "A Complete Solution to the Harmonic Elimination Problem", IEEE Trans. Power Electronics, Vol. 19, No. 2, pp. 491 – 499, Mar 2004

## VII. BIOGRAPHIES

**Maziar Heidari** (S'06) received his B.Sc. and M.Sc. degrees in electrical engineering from Amirkabir University of Technology in 2002, and 2005 respectively. Currently he is working toward his Ph.D. degree in University of Manitoba. His research interests include simulation-based tools and techniques, power electronic applications in power systems, and drive systems.

**Shaahin Filizadeh** (M'05) received his B.Sc. and M.Sc. degrees in electrical engineering, from Sharif University of Technology in 1996 and 1998, respectively. He obtained his Ph.D. from the University of Manitoba in 2004. He is currently an assistant professor with the Department of Electrical and Computer Engineering, University of Manitoba. His areas of interest include electromagnetic transient simulation, nonlinear optimization, and power electronic applications in power systems and vehicle propulsion.

**Aniruddha M. Gole** (SM'04) obtained the B.Tech. (EE) degree from IIT Bombay, India in 1978 and the Ph.D. degree from the University of Manitoba, Canada in 1982. Professor Gole holds the NSERC Industrial Research Chair in Power Systems Simulation at the Department of Electrical and Computer Engineering at the University of Manitoba. As an original member of the design team, he has made important contributions to the PSCAD/EMTDC simulation program. Dr. Gole is active on several working groups of CIGRE and IEEE and is a Registered Professional Engineer in the Province of Manitoba.