Passivity Enforcement via Residues Perturbation for Admittance Representation of Power System Components

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Abstract-- Employing an admittance representation in form of a black box model approximated by rational functions for linear power system components or network equivalents to be included in electromagnetic transient studies is a well-known method which improves the calculation efficiency. All of the methods that have been proposed to solve the rational approximation problem, have made efforts to overcome the problem of preserving passivity of the final model. Passivity is a vital property, since a nonpassive model may lead to an unstable transient simulation in time domain. The passivity violation regions are detected via a purely algebraic approach based on the existence of purely imaginary eigenvalues in the Hamiltonian matrix obtained from the state-space representation of the reduced-order model. Also a fast test method is presented to check passivity without direct calculation of eigenvalues of Hamiltonian matrix. Then a post-processing technique based on Quadratic Programming (QP) for passivity enforcement is presented. To increase calculation efficiency and faster convergence only one inequality constraint, in where eigenvalue of Hamiltonian matrix is in minimum, is considered in solving process of QP problem. The mentioned minimum point is detected by the bisection method instead frequency sweep method and therefore the efficiency of computation increases significantly.

Keywords: Quadratic Programming, Routh array test, bisection method, Hamiltonian matrix, passivity enforcement.

I. Introduction

Application of network equivalents for external systems in the well-known time domain programs such as EMTPs (Electro_Magnetic Transient Programs) has valuable merits in saving of the CPU memory and the run time. A large number of approaches have been appeared for construction of network equivalent which can be categorized into two main groups, time domain equivalent and frequency domain equivalent. The solution in frequency domain is essentially aimed on identification of rational functions that approximate the admittance matrix of external system seen from boundary bus(es). Among the various methods of fitting, the Vector Fitting (VF) has proved its efficiency and accuracy in the different applications [1-6]. Although VF can lead to a stable and precise approximation however the model may not be passive. A stable, however nonpassive network besides some passive networks or loads may lead to an unstable general system. Therefore passivity is an important property for a model, although its enforcement is a difficult task. In [7], the regions of passivity violation are searched by a frequency sweep method. The drawbacks of this detection method are:

1) The passivity violation regions may be outside of the considered frequency spectrum when examined by a frequency sweep. On the other hand the method needs a frequency sweep from 0 to ∞, to detect the nonpassive regions. Although transients in power systems will never be generated with an infinity frequency, the frequency sweep is needed to ∞ to guarantee the passivity of the approximated equivalent network or component.

2) The accurate detection of regions of passivity violation depends on the fineness of frequency sweep, where undoubtedly the higher fineness is more time-consuming.

Considering the above drawbacks, [8, 9] proposed a purely algebraic method based on the existence the purely imaginary eigenvalues of associated Hamiltonian matrices obtained from the state-space representation of the reduced-order model. This method is used in this paper too. The use of Hamiltonian matrix is also mentioned in [10]. In [8], a compensation procedure based on a small perturbation of the Hamiltonian matrix is also presented. In [7], a post-processing algorithm based on Quadratic Programming (QP) is employed to ensure the local passivity. However all frequency points found by means frequency sweep in nonpassivie regions are considered as constraints in QP algorithm. It causes a large-scale solution of QP that decreases the efficiency of calculation. Therefore in this paper to increases the efficiency of calculation of QP method only one constraint, where the worst passivity violation (where the eigenvalue is minimum measure) occurs, is selected to enforce the passivity and entered in QP algorithm. To detect magnitude of maximum violation in any nonpassive region, a frequency sweep method can be used.

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However by using the frequency sweep, there would be no guarantee to detect the maximum violation. This difficulty is overcomed by employing a bisection method based on Glover-Enns bounds [11, 12]. Also in some cases the network equivalent is inherently passive and it doesn't need any enforcement for passivity and it is sufficient just to check the passivity. Hence in this paper also a fast test method is proposed to check the existence of the purely imaginary eigenvalues of associated Hamiltonian matrices (without direct calculation of eigenvalues) that enhances the efficiency of calculation.

II. DEFINITION OF PASSIVITY

Passivity may be defined in a loose sense as the inability of a given structure to generate energy. The linear time-invariant multiport system can be converted into state-space form as follows:

$$\begin{aligned} \dot{x}(t) &= \boldsymbol{A}x(t) + \boldsymbol{B}u(t) \\ y(t) &= \boldsymbol{C}x(t) + \boldsymbol{D}u(t) \end{aligned} \tag{2}$$

where the dot denotes time differentiation. The number of ports and the dynamic order of the function in the approximation are p and n respectively. Then the state vector $x(t) \in \Re^n$,the input and out-put vectors $x, y \in \Re^p$, $A \in \Re^{n\times n}$, $B \in \Re^{n\times p}$, $C \in \Re^{p\times n}$ and $D \in \Re^{p\times p}$.The poles and residues of system are included in the matrices A and C respectively. The input-output transfer function matrix of the system can be obtained from (2) as follows:

$$Y(s) = C(sI - A)^{-1} B + D \tag{3}$$

where s is the Laplace operator. The precise definition of passivity requires that the transfer matrix under investigation be positive real. This condition requires that the Hermitian part of the transfer matrix must be nonnegative definite on the imaginary axis, i.e.,

$$G(j\omega) = \frac{1}{2}(Y(j\omega) + Y^*(j\omega)) \geq 0 \qquad \forall\omega \tag{4}$$

where * denotes complex conjugate transpose.

The condition (4) can be checked by ensuring that all its eigenvalues are nonnegative at any frequency

$$\lambda_i(j\omega) \geq 0 \qquad \forall\lambda_i(j\omega) \in \lambda(G(j\omega))) \wedge \forall\omega \tag{5}$$

The direct application of the definition (5) for testing passivity, however, requires a frequency sweep since this condition needs to be checked at any frequency. The results of such tests, therefore, depend on accurate sampling of frequency axis, which is not a trivial task. For this reason, purely algebraic passivity tests are high desirable that will be described in section III.

III. CHARACTERIZATION OF PASSIVITY VIOLATIONS

Regarding to drawbacks of frequency sweep to detect non-passive regions, in this section an effective tool for deriving the passivity conditions in a purely algebraic form based on Hamiltonian matrices associated to the state-space realization will be used. Let first recall the definitions of the Hamiltonian matrices for the hybrid cases (with emphasis on $\boldsymbol{Y}$ form for network equivalent applications) [13],

$$\begin{aligned} \boldsymbol{N}_\delta &= \begin{pmatrix} \boldsymbol{A} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{A}^T \end{pmatrix} + \begin{pmatrix} \boldsymbol{B} \\ -\boldsymbol{C}^T \end{pmatrix} (2\delta\boldsymbol{I} - (\boldsymbol{D}+\boldsymbol{D}^T))^{-1} \begin{pmatrix} \boldsymbol{C} & \boldsymbol{B}^T \end{pmatrix} \\ &= \begin{pmatrix} \boldsymbol{A}+\boldsymbol{B}\boldsymbol{Q}^{-1}\boldsymbol{C} & \boldsymbol{B}\boldsymbol{Q}^{-1}\boldsymbol{B}^T \\ -\boldsymbol{C}^T\boldsymbol{Q}^{-1}\boldsymbol{C} & -\boldsymbol{A}^T - \boldsymbol{C}^T\boldsymbol{Q}^{-1}\boldsymbol{B}^T \end{pmatrix} \end{aligned} \tag{6}$$

where $\boldsymbol{Q} = (2\delta\boldsymbol{I} - \boldsymbol{D} - \boldsymbol{D}^T)$

$\boldsymbol{N}_\delta$ is a Hamiltonian matrix, meaning

$$\boldsymbol{J}^{-1}\boldsymbol{N}_\delta\boldsymbol{J} = -\boldsymbol{N}_\delta^T \quad \text{where} \quad \boldsymbol{J} = \begin{pmatrix} 0 & \boldsymbol{I} \\ -\boldsymbol{I} & 0 \end{pmatrix}$$

where ^{T} denote transpose. $\boldsymbol{N}_\delta$ matrix depends on a scalar parameter, δ, which is related to the spectrum of frequency-dependent eigenvalues of the Y matrix that is expressed by following theorem 1 [13].

Theorem 1. Assume A has no imaginary eigenvalues, δ is not a singular value of $(\boldsymbol{D} + \boldsymbol{D}^T)/2$, and $\omega_0 \in \boldsymbol{R}$. Then, δ is an eigenvalue of $\boldsymbol{G}(j\omega)$ if and only if $(\boldsymbol{N}_\delta - j\omega_0 I)$ is singular.

A passivity test can be readily designed by using the critical level $\delta = 0$ and hence

$$\boldsymbol{N}_{\delta=0} = \begin{pmatrix} \boldsymbol{A} - \boldsymbol{B}\left(\boldsymbol{D}+\boldsymbol{D}^T\right)^{-1}\boldsymbol{C} & \boldsymbol{B}\left(\boldsymbol{D}+\boldsymbol{D}^T\right)^{-1}\boldsymbol{B} \\ -\boldsymbol{C}^T\left(\boldsymbol{D}+\boldsymbol{D}^T\right)^{-1}\boldsymbol{C} & -\boldsymbol{A}^T + \boldsymbol{C}^T\left(\boldsymbol{D}+\boldsymbol{D}^T\right)^{-1}\boldsymbol{B}^T \end{pmatrix} \tag{7}$$

On the other hand, pure imaginary eigenvalues of matrix N correspond to the exact locations where the real part of the symmetric admittance matrix becomes singular. The powerful merit of above technique based on Hamiltonian matrix is being independent of the frequency. This pure algebraic method overcomes the mentioned drawbacks of frequency sweep method. Although due to the numerical noise, the detection of imaginary eigenvalues of matrix N is difficult, however employing the special properties of the eigenvalues of Hamiltonian matrix N, this problem can be solved, namely the eigenvalues of N are symmetrical with respect to the real and imaginary axes. Although based on the properties of the eigenvalues as mentioned above the precise locations of singular values of $G(j\omega)$ are known, however this does not contain any information about passivity violation regions. For this purpose a method based on the slope of eigenvalues of $G(j\omega)$ at its singular locations is employed. The slope of eigenvalues of $G(j\omega)$ at singular locations can be determined by following equation [14].

$$\frac{d\lambda}{d\omega}=\frac{v^t\left(CA\left(\omega^2 I+A^2\right)^{-2}2\omega B\right)u}{v^t u} \tag{8}$$

where u and v^t are the corresponding right and left eigen-vector respectively.

The singular frequencies in a vector $S_a=[\omega_1,\omega_1,\ldots,\omega_T]$ where $\omega_1<\omega_2<\ldots<\omega_T$ are found. Starting from the highest frequency, ω_T, and counting the positive and the negative slopes in the specified frequencies, then in any frequency where the number of positive and negative slopes equates, there is an indication of a local nonpassive region.

IV. A FAST TEST METHOD TO CHECK PASSIVITY

In some cases the network equivalent is inherently passive and it doesn't need any enforcement for passivity and it is sufficient just to check the passivity. Hence a fast test method to check the existence of the purely imaginary eigenvalues of associated Hamiltonian matrices (without direct calculation of eigenvalues) enhances the efficiency of calculation.

Since *N* is Hamiltonian, the characteristics polynomial of *N*, in $a(s)=\det(sI-M)$, is a polynomial of s^2; $a(s)=p(-s^2)$. Therefore *N* has imaginary eigenvalues if and only if *p* has real nonnegative roots. The coefficients of polynomial *p* could be computed from *N* by the Leverrier-Feddeva algorithm [15]. Also a *Sturm* method can be used to test whether *p* has real nonnegative roots [16]. Consider two polynomials with real coefficients as below:

$$\begin{aligned}\alpha(x)&=\alpha_0x^m+\alpha_1x^{m-1}+\cdots+\alpha_m, \quad \alpha_0\neq 0\\ \beta(x)&=\beta_0x^m+\beta_1x^{m-1}+\cdots+\beta_m, \quad \beta_0\neq 0\end{aligned} \tag{9}$$

The *Strum sequence* associated with $\alpha(x)$ and $\beta(x)$ is a set of polynomials $\{k_if_i(x)\}$ where k_i are arbitrary positive constants and:

$$\begin{aligned}f_0(x)&=\alpha(x)\\ f_1(x)&=\beta(x)\\ f_i(x)&=f_{i+1}(x)q_i(x)-f_{i+2}(x) \qquad i=0,1,2,\ldots\end{aligned} \tag{10}$$

Equations (10) represents *Euclid's algorithm* applied to $\alpha(x)$ and $\beta(x)$ while the signs of the reminders reversed. A second fundamental tool in the qualitative study of polynomial is the Cauchy index $I_a^b\gamma(x)$, where $\gamma(x)=\beta(x)/\alpha(x)$, and it is known that:

$$I_a^b\gamma(x)=V(a)-V(b) \tag{11}$$

where $V(x_0)$ denotes the number of variations in sign of the sequence of $f_0(x), f_1(x), f_2(x),\ldots$.

The algorithm of Routh, presented first by British mathematician, E. J. Routh, enables the Strum sequence to be constructed without explicitly carrying out the polynomial divisions in (10).

Specifically, the *Routh array* (r_{ij}) is a set of rows as follows:

$$\begin{array}{llllll} r_{01} & r_{02} & r_{03} & \cdots & r_{0m} & r_{0,m+1}\\ r_{11} & r_{12} & r_{13} & \cdots & r_{1m} & \\ r_{21} & r_{22} & r_{23} & \cdots & & \\ \multicolumn{6}{l}{\cdots\cdots\cdots\cdots\cdots} \end{array} \tag{12}$$

where:

$$r_{0j}=\alpha_{j-1},\quad r_{1j}=\beta_{j-1}, \qquad j=1,2,3,\ldots \tag{13}$$

and:

$$r_{ij}=\frac{-1}{r_{i-1,1}}\begin{vmatrix} r_{i-2,1} & r_{i-2,j+1}\\ r_{i-1,1} & r_{i-1,j+1}\end{vmatrix}, \qquad i=2,3,4,\ldots \tag{14}$$

It is assumed that the array is in a regular form, i.e., all $r_{i1}\neq 0$. The problem which was resolved by Routh [17, 18] is to determine when a given polynomial with real coefficients as:

$$f(x)=\alpha_0x^n+\alpha_1x^{n-1}+\cdots+\alpha_n, \quad \alpha_0>0 \tag{15}$$

is the characteristic polynomial of an asymptotically stable linear system. This requires that all the zeros of (15) have negative real parts. In this case, the required and sufficient condition is:

$$v(r_{01},r_{11},r_{21},\ldots)=0 \tag{16}$$

In other words, all the first column elements in the Routh array generated by (14) should be positive. The Routh algorithm can readily be modified to determine the number of real zeroes of a real polynomial *f(x)*. The modified Routh array ($\tilde{r}_{ij}$) is formed starting with the first two rows as:

$$\begin{aligned}(\tilde{r}_{0j})&=\left[(-1)^n a_0,(-1)^{n-1}a_1,\ldots,-a_{n-1},a_n\right]\\ (\tilde{r}_{1j})&=\left[(-1)^n na_0,(-1)^{n-1}(n-1)a_1,\ldots,-a_{n-1}\right]\end{aligned} \tag{17}$$

The rows are formed by the coefficients of the polynomials $f(-x)$ and $f'(-x) = df(-x)/dx$.

Computing the modified Routh array $(\widetilde{r}_{ij})$, (14), the parameter k_p, the number of positive real zeros can be computed as follows:

$$k_p = n - v(\widetilde{r}_{01}, \widetilde{r}_{11}, \widetilde{r}_{21}, \ldots) \quad (18)$$

V. PASSIVITY ENFORCEMENT

To enforce the passivity in the nonpassive regions detected in section III, we follow the Quadratic Programming (QP) method proposed in [7] with some modifications to increase efficiency calculations.

The quadratic programming problem is as following form

$$\min_{x} \ \frac{1}{2} x^T H x + f^T x \quad (19)$$

Such that

$$A \cdot x \le b$$
$$A_{eq} \cdot x = b_{eq}$$
$$l_b \le x \le u_b$$

where H, A, and A_{eq} are matrices, and f, b, b_{eq}, l_b, u_b, and $\boldsymbol{x}$ are vectors. The QP problem can be computed using the MATLAB command, *quadprog*.

In [7], all frequency points in a nonpassive region, detected using frequency sweep, are used as inequality constraints included in QP problem. This causes a large-scale problem and decrease efficiency computation. Instead of it, we propose only one inequality constraint in the point where the eigenvalue of $G(j\omega)$ is minimum is considered.

This decreases the problem to a medium-scale and with a less number of iterations QP problem will be solved. To detect the mentioned point, instead of frequency scan in nonpassive region, an efficient method based on H_∞-norm approximation is presented. Let first consider the minimum dissipation, *diss(H)*, of a transfer matrix defined by:

$$diss(H) = \inf_{\mathrm{Re}\, s > 0} \ \lambda_{\min}((H(s) + H(s)^*)/2) \quad (20)$$

Then consider the following theorem [13].

Theorem: Let A be stable and $\delta < \lambda_{\min}((D + D^T)/2)$. Then $diss(H) \le \delta$ if and only if N_δ has imaginary eigenvalues.

Above theorem suggests a bisection algorithm for computation of *diss(H)*. Let γ_{lb} and γ_{ub} be the lower and upper bounds of *diss(H)*, respectively. In a nonpasssive region the upper bound will be zero. For the lower bound, one can use modification of Enns and Glover bounds [11, 12] used for H_∞-norm approximation:

$$\gamma_{lb} = \min\{\lambda_{\min}((L + L^*)/2), \lambda_{\min}((D + D^T)/2)\} \quad (21)$$

where; $L = L_o L_c$

L_o and L_c can be computed by solving the observability and controllability Grammian Lyapunov equations as follows:

$$\begin{aligned} A L_c + L_c A^T + B B^T &= 0 \\ A^T L_o + L_o A + C^T C &= 0 \end{aligned} \quad (22)$$

Then a bisection algorithm [13] can be used to detect the maximum violation in any nonpassive region.

VI. COMPUTED RESULTS

In this section two examples will be presented to demonstrate the accuracy and efficiency of passivity check and compensation algorithm.

Example 1: This is a fourth order rational approximation of a single port system to explain more the described techniques in this paper. Fig. 1 shows frequency spectrum eigenvalues of real part of Y. The Routh array test described in section IV and then the calculation of eigenvalues of the Hamiltonian matrix of state-space realization identifies a nonpassive region, [0.4856, 2.8060] kHz.

The lower bound computed by (21) is -0.1191. Then we use the bisection algorithm to detect the minimum eigenvalue of *Real(Y)* in the mentioned nonpassive region. The minimum of eigenvalues of *Real(Y)* will be -0.0242 in frequency 1.2028 kHz. Then we solve QP problem subject to only one inequality constraint in the point minimum eigenvalue of *Real(Y).* Table 1 summarizes the residues and constant term before and after perturbation by using the proposed QP in this paper and [7]. The errors in two cases are 0.2235 and 0.6553 respectively that shows the more accuracy for our proposed method. Also in 1st iteration, the passivity will be enforced at all frequency spectrums.

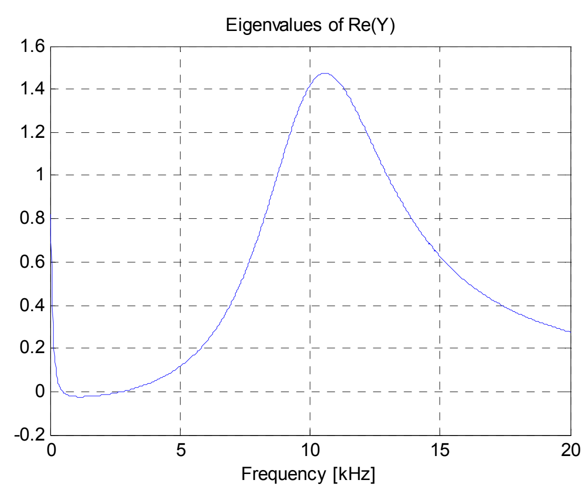

Fig. 1: Frequency spectrum of eigenvalues.

Fig. 2 and 3 show frequency spectrum of the eigenvalues before (blue line) and after (red line) application of the perturbation; through employing the proposed QP method, versus employing the method in [7] respectively.

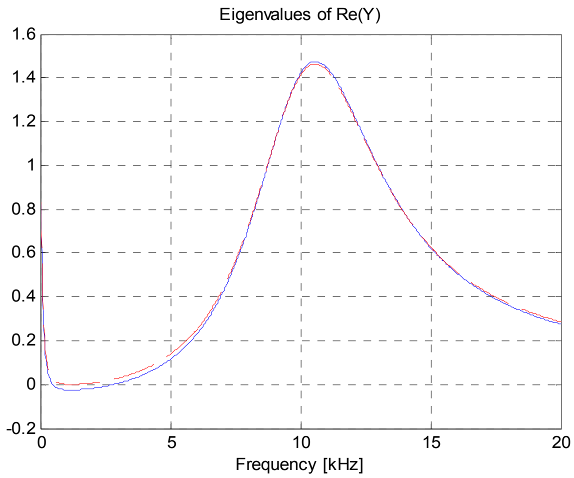

Fig. 2: Frequency spectrum of the eigenvalues before and after application of the perturbation; through employing the proposed QP method.

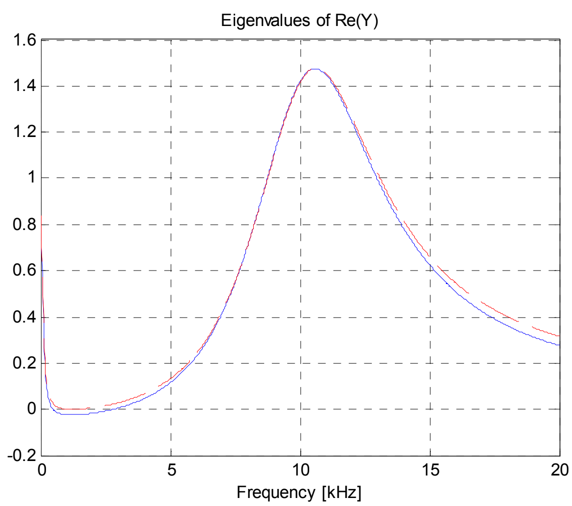

Fig. 3: Frequency spectrum of the eigenvalues before and after application of the perturbation; through employing the method in [7].

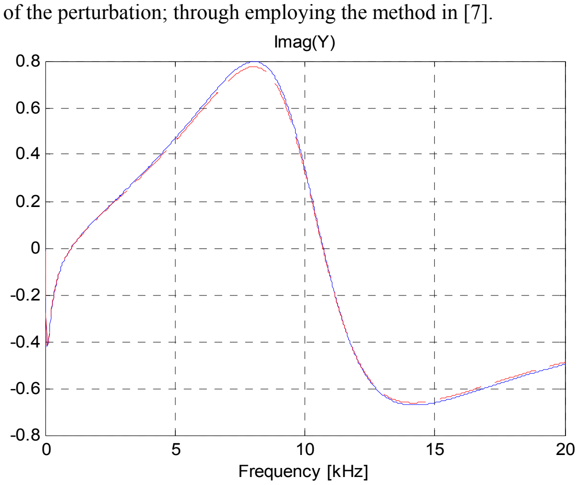

Fig. 4: Frequency spectrum of the imaginary eigenvalues before and after application of the perturbation; through employing the proposed QP method.

Also the imaginary part of **Y** without and with passivity enforcement is shown in Fig. 4 that demonstare the accuracy oh the proposed method. Table 1 shows the residues and the constant term before and after application of the perturbation; through employing the proposed QP method, versus employing the method in [7].

Table 1. The residues and constant term before and after application of the perturbation; through employing the proposed QP method, versus employing the method in [7].

The pole (× 1e4)	The residues/constant term before perturbation	The residues/constant term after perturbation; through employing the proposed QP method	The residues/constant term after perturbation ; through employing the method in [7].
-0.0006	7.3417e-14	0.0595	0.1982
-0.0628	538.5587	527.9080	522.7283
-1.8850 ± j6.2832	2.6928e4± j1.0771e4	2.6400e4± j1.0377e4	2.6135e4 ±j1.1107e4
Constant term (D)	0.0429	0.0604	0.0843

Example 3: In this example the proposed method is examined on a sample distribution network where the passivity enforcement of terminal admittance matrix of this system is looked for. The information of this network is provided by [19]. The distribution system has two 3-phase buses as terminals (A, B) shown in Fig. 5. The 6×6 admittance matrix Y is calculated for this system in a frequency range of 10 Hz–100 kHz. To increase the calculation efficiency all the elements of **Y** are fitted with a common pole set. To fit 50 complex pair poles are selected as initial poles.

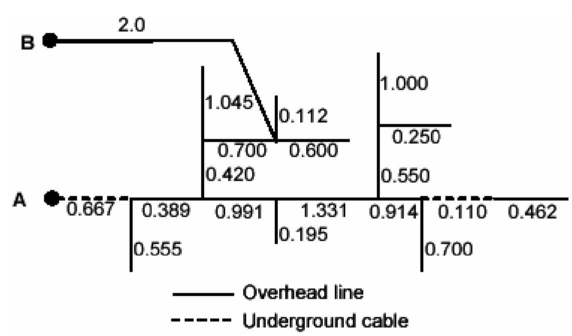

Fig. 5: Power system distribution system

Figs. 5 & 6 show fitting the magnitude and phase angle of 6×6 admittance matrix **Y** through an improved version of VF named vfit2 [19]. Fig. 7(a) shows the frequency spectrum of the six eigenvalues of the admittance matrix. As shown in Fig. 7(a), the passivity violations occur in frequencies of about 3kHz.

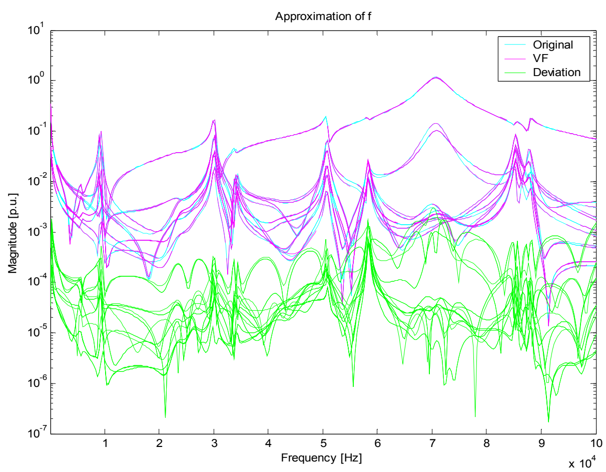

Fig. 5: Fitting of the magnitude by VF

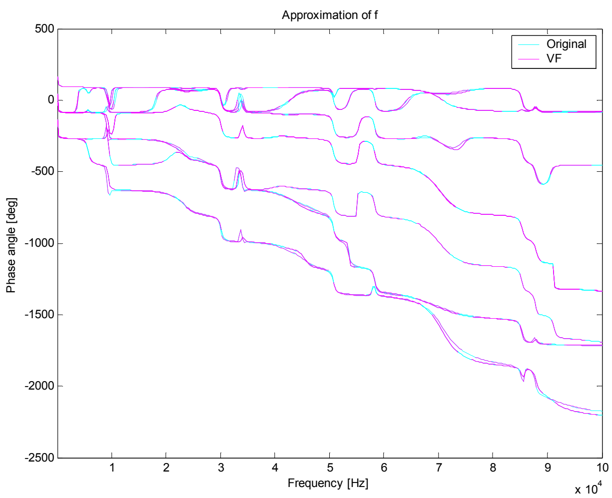

Fig. 6: Fitting of the phase angle by VF

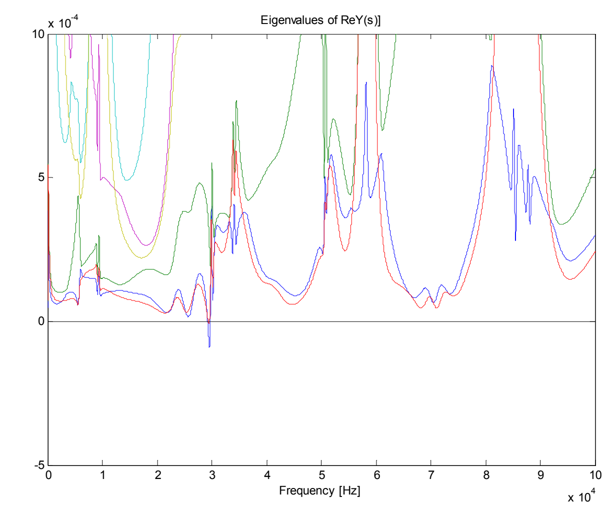

Fig. 7(a): Frequency spectrum of eigenvalues

Fig. 7(b) shows an expanded view of the nonpassive region; [29.237, 29.736] kHz, obtained by Hamiltonian matrix theory. The magnitude of maximum passivity violation in this region is -8.9149e-5 at a frequency of 29.579 kHz. After 1st iteration of perturbation, the above nonpassive region is mitigated to [29.408 29.592] kHz and the magnitude of maximum passivity violation also is mitigated. Although both the bound of passivity violation and the magnitude of maximum passivity violation (the severity of passivity violation) in above nonpassive region are mitigated, the new nonpassive region; [30.080, 30.248] kHz, is produced after the first perturbation. For passivity enforcement only three iterations is sufficient. Since the detection and the compensation processes of the proposed method aren't time-consuming, the iterations can be done fast. Table 2 and Fig. 7(b) summarize the passivity violations in the process of proposed method based on the number of the iterations. The RMS error of the proposed method is 1.4229e-8 that demonstrate the accuracy of the proposed method.

Table 2: the details of calculation of proposed method

Number of iter.	Passivity violation regions[kHz]	The mag. of max. violation	The freq. of max. violation [kHz]
Original	[29.237, 29.736]	-8.9149e-5	29.579
1st iter.	[29.408, 29.592] [30.080, 30.248]	-7.9410e-5	30.153
2nd iter.	[29.444, 29.582] [30.138, 30.182]	-5.9406e-6	29.521

VII. ACKNOWLEDGMENT

The authors gratefully acknowledge the support of Dr. B. Gustavsen who shared his data files on the subject of passivity enforcement through QP method [20].

VIII. CONCLUSIONS

In this paper an efficient and fast method for passivity enforcement of a subsystem, through approximation by rational functions is presented. The Hamiltonian matrix theory, which is a purely algebraic method, is employed to find the passivity violation regions. This method isn't dependent on the frequency and hence it overcomes the drawbacks of frequency sweep method. The minimum eigenvalue in any nonpassive region is identified by using a bisection method. In the compensation stage, a QP problem with only one inequality constraint is solved. This provides higher computation efficiency, and a faster convergence. Routh array test is employed to check the passivity without direct calculation of eigenvalues of Hamiltonian matrix of state-space realization.

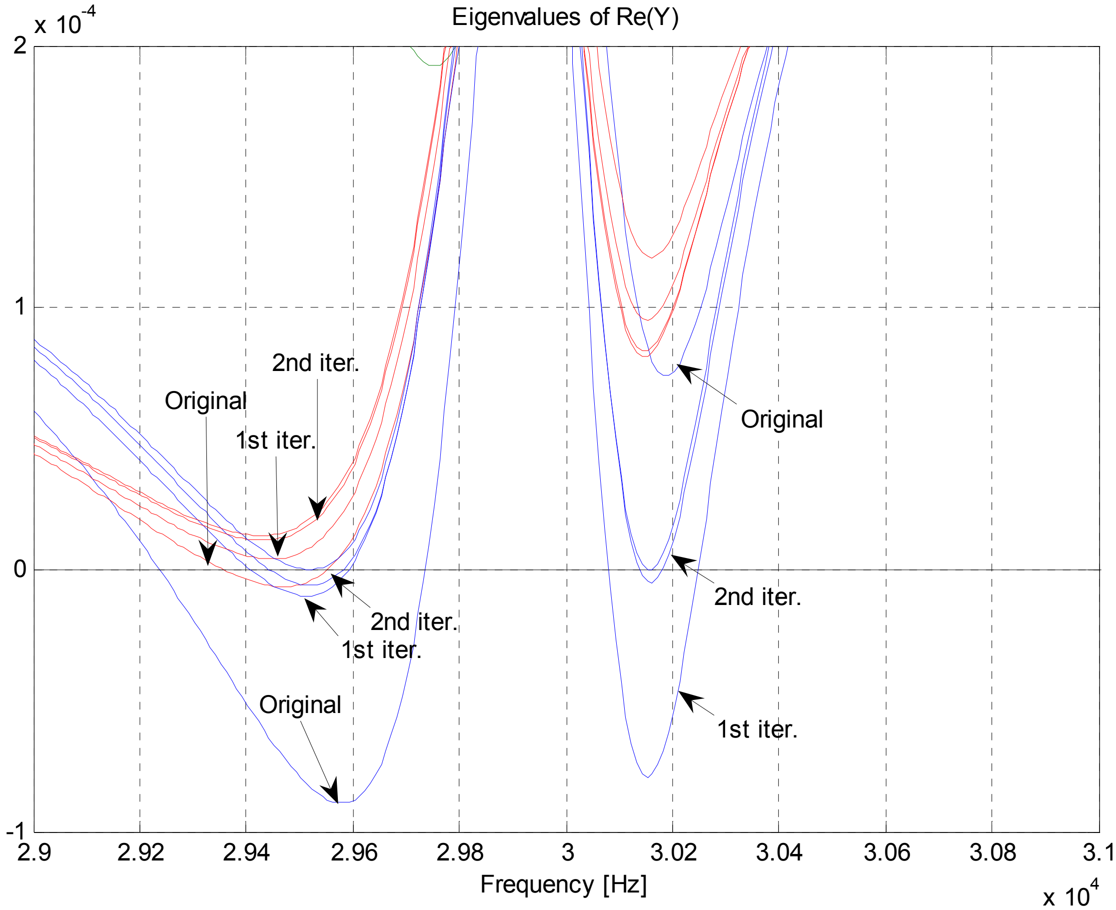

Fig. 7(b): Mitigation of violations during iterations

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