An Investigation of an Equivalent Conductor Plate Representing Earth for a Surge Analysis

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Abstract -This paper presents experiments on the surge propagation characteristics of an overhead conductor above an aluminum (Al) plate in comparison with that above a real earth. The experimental results are compared with analytical and numerical simulation results. It is made clear that the Al plate can represent the real earth only for an initial part of a surge waveform until reflection comes back from the other end of the conductor. The surge waveform on the Al plate noticeably deviates from that on the real earth as time passes. The characteristic impedance on the Al plate differs from that on the real earth depending on the conductor height and the separation between the conductors.

Index Terms- surge, characteristic-impedance, real earth, conductor plate, scaled model

I. INTRODUCTION

A surge overvoltage analysis is very significant for insulation design of a power system. Because it is hard to carry out the analysis in a real power system, a scaled-down system model is often adopted to investigate a surge phenomenon. In the scaled model, it is a common practice to use an equivalent conductor plate, *i.e.,* an aluminum (Al) or copper (Cu) plate, to represent a real earth. However, it has not been well investigated if such a conductor plate can represent the earth for the surge phenomenon. This paper presents experiments on the surge propagation characteristics of an overhead conductor above an Al plate and above a real earth. Transient voltages and currents are measured under various conditions of the conductor height and the separation between the conductors.

Also, the characteristic impedances of the conductors are evaluated from the measured results for the Al plate and the real earth. Based on the measured results, a difference between the Al plate and the real earth is discussed. The difference is further investigated in comparison with analytical and numerical simulation results. From the observations, the limit of a conductor plate representing a real earth is made clear.

II. SURGE PROPAGATION EXPERIMENT

A. Experimental Setup

Fig. 1 illustrates an experimental setup for measuring surge propagation on an overhead conductor with length x = 2 m and radius r =2 mm; (a) is for a single conductor to observe a wave propagation characteristic of an energized phase and a self characteristic impedance of the conductor above an Al plate (ρ_e=2.8$\times$10^{-8} Ωm) and above a real earth. The earth resistivity ρ_e is measured to be 89 Ωm. (1b) and (1c) are for an induced phase characteristic and a mutual impedance.

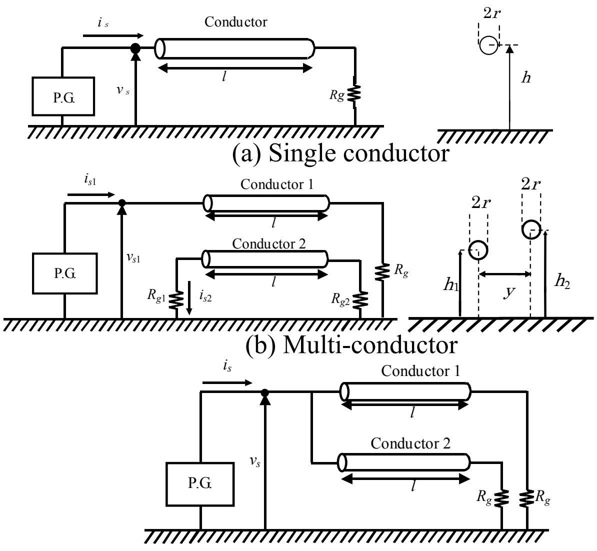
A step-like voltage is applied to the sending-end of the conductor from a pulse generator (PG), and a transient current at the sending-end is measured for about 30 ns which is less than twice of the travel time of a voltage reference wire with the length of 5 m. The length of a current lead wire from the PG to the conductor is taken to be 10 m so as to avoid the effect of the PG internal circuit on a measured result. The current is measured by a pearson CT Model 2877.

To observe the effect of the conductor height h and the separation y on the surge propagation, the height is varied from h=0.3 m to 0.9 m, and the separation from y=0 m to 0.3 m

In this experiment, the other end of a conductor is grounded, and the voltage and the current only at the sending-end are measured. Quite often, one-end of the conductor is open-circuited and its voltage is measured to investigate wave propagation characteristics along the conductor. This measurement, however, results in the following problem. Because a voltage probe has its internal capacitance C_{probe}≒12 pF , the impedance of the probe looks as:

$$|Z_{probe}| = 1/\omega C_{plobe} \quad [\Omega], \qquad (1)$$

where $\omega = 2\pi f$, f: frequency.
Assume that the frequency of a transient is 100 MHz. Then, the probe impedance becomes about 17 kΩ which is far smaller than the open-circuit impedance. Therefore, a



(c) A circuit to measure a mutual characteristic impedance
Fig. 1. Experimental setup.

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measured voltage is not the voltage of an open-circuited end, but the voltage of a capacitance terminated conductor.

Also in this experiment, the characteristic impedance of a conductor is focused to discuss the propagation characteristics, because the conductor length x=2 m is too short to discuss the propagation constant, *i.e.,* attenuation and velocity, in detail.

B. Evaluation of Characteristic Impedance Z_0

From the measured results for voltage $v_s(t)$ and current $i_s(t)$ at the sending-end of a conductor, an approximate value of the characteristic impedance Z_0, *i.e.,* the so-called surge impedance Z_s, is evaluated as a common practice in the following manner:

$$Z_s = Z_0(\omega)_{\max} = v_s(t)_{\max}/i_s(t)_{\max}\ , \quad (2)$$

In theory, the characteristic impedance Z_0 is to be obtained as a function of frequency by using the following formula [1]:

$$Z_0(\omega) = V_s(\omega)/I_s(\omega), \quad (3)$$

where $V_s(\omega)$=F $[v_s(t)/j\omega]$, the frequency response of voltage $v_s(t)$ in time domain,

$I_s(\omega)$=F $[i_s(t)/j\omega]$, the frequency response of current $v_s(t)$ in time domain, and

F is the Fourier transform from time to frequency domain

C. Mutual Characteristic Impedance

As explained in Section II-B, the self characteristic impedance is evaluated by measured results of a voltage and a current at the sending-end for a short-circuited conductor. The mutual characteristic impedance can be evaluated by the induced current $i_s(t)$ and the induced voltage $v_m(t)=v_{s2}(t)$ at the sending-end of a conductor under the condition of the sending-end being open-circuited. However, this approach results in the same problem of measuring the voltage $v_{s1}(t)$ as explained in Section II-A. To avoid the problem, the sending-end of the induced phase is short-circuited and a current $i_{s2}(t)$ due to the induced voltage $v_m(t)$ at the sending-end of the induced phase is measured as in Fig. 1(b). Then, the mutual characteristic impedance Z_{0m} is evaluated in the following manner:

$$Z_{0m}' = v_m/i_s = Z_{02}\cdot i_{s2}'/i_s\ , \quad (4)$$

where Z_{02} is the self characteristic impedance of conductor 2 (=induced conductor),

i_{s2}' is the sending-end current on the induced conductor, and

i_s is the sending-end current on the inducing conductor (=conductor 1).

This approach is easily applied to obtain the mutual impedance of a conductor above a conductor plate.

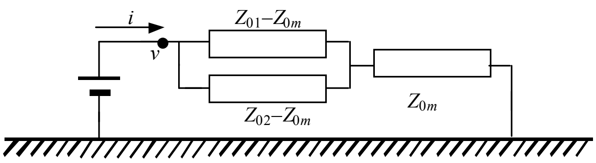


Fig. 2. An equivalent circuit of Fig.1(c).

However, it involves a grounding resistance R_{g1} of the induced conductor at the sending-end in the case of a real earth, *i.e.,* the current i_{s2} depends not only on the conductor self characteristic impedance Z_{02} but also on the grounding resistance R_{g1} as:

$$i_{s2}' = v_m/(Z_{02} + R_{g1})\ . \quad (5)$$

Thus, the mutual impedance evaluated by (4) is not accurate. Also, measured responses are not stable.

To obtain the mutual impedance, the experimental circuit in Fig. 1(c) is adopted. An equivalent circuit of Fig. 1(c), before reflection from the other end comes back, is given in Fig. 2 where Z_{01} is the self characteristic impedance of conductor 1. In Fig. 2, the following relation is obtained:

$$\frac{v_s}{i_s} = \frac{(Z_{01} - Z_{0m})(Z_{02} - Z_{0m})}{(Z_{01} - Z_{0m}) + (Z_{02} - Z_{0m})} + Z_{0m}\ . \quad (6)$$

By solving the above equation considering the fact that the real part of Z_{0m} is positive and the imaginary part is negative [1], Z_{0m} is given by:

$$Z_{0m} = \frac{v_s}{i_s} + \sqrt{\left(\frac{v}{i} - Z_{01}\right)\left(\frac{v}{i} - Z_{02}\right)}. \quad (7)$$

In particular when $Z_{01}= Z_{02}= Z_0$, the above equation is simplified in the following form:

$$Z_{0m} = \frac{2v_s}{i_s} - Z_{0s}\ . \quad (8)$$

III. EXPERIMENTAL RESULT

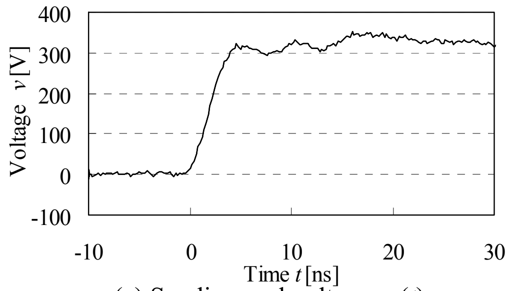
A. Single Conductor

(1) Transient response of the current

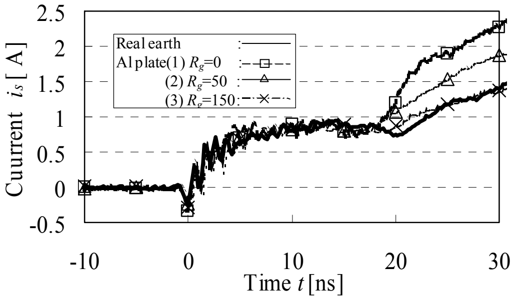
Fig. 3 shows measured results for the voltage $v_s(t)$ and current $i_s(t)$ at the sending-end of a single conductor in Fig. 1 (a). Fig. 3 (a) is the sending-end voltage, and (b) is the current. In the case of the Al plate representing a real earth, it is necessary to consider a grounding impedance Z_g of a conductor above the real earth if it is grounded. Fig. 3 (b) shows the effect of the grounding resistance R_g representing the impedance Z_g in the case of the Al plate in comparison with the real earth. It is clearly observed that the grounding resistance R_g in the Al plate case significantly affects the transient waveform of the current after reflection from the remote end, *i.e.,* the grounded terminal, comes back. If the resistance is neglected, a difference of the current in the Al plate case from that in the real earth becomes large. Therefore, the grounding impedance should be carefully considered when the real earth is represented by the Al plate.

(2) Characteristic impedance

Fig. 4 is the frequency response of the characteristic impedance of a conductor above an Al plate in comparison with that above the real earth. Included is the theoretical result evaluated by eq. (10). Table 1 summarizes the characteristic impedance at 500MHz. If the characteristic impedance is independent of conductor termination, *i.e.,* a grounding impedance of the conductor, then it is easy to discuss a difference between the real earth and the Al plate representing the real earth. Fig. 4 and Table 1 show that there exists a significant difference between the real earth and the Al plate. This fact clearly indicates that the Al plate cannot represent the real earth even in a frequency range of 500 MHz. In a lower frequency range, however, the Al plate can somehow represent the real earth, although the characteristic impedance is lower by more than 10 Ω than that in the real earth case.

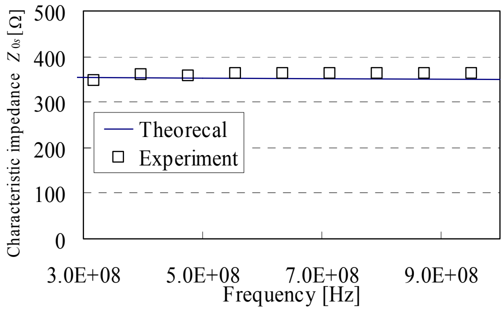


(a) Sending-end voltage $v_s(t)$

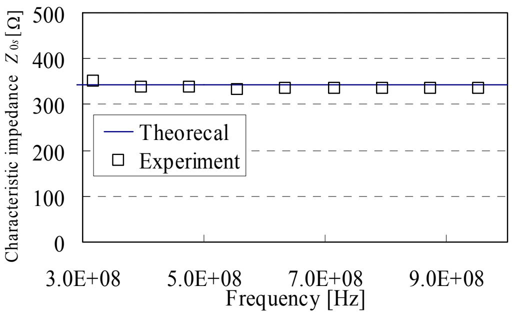


(b) sending-end current $i_s(t)$

Fig. 3 Measuring results of sending-end voltage $v_s(t)$ and current $i_s(t)$ in Fig. 1(a) for h=0.3 m



(a) Real earth



(b) Al plate

Fig. 4 Self charactersitc impedance Z_{0s} for h=0.3 m

Table 1 EXPERIMENTAL RESULT FOR THE SELF CHARACTERISTIC IMPEDANCE AT 500MHz

h [m]	Experiment		eq.(10)	
	real earh	Al plate	Real earth	Al plate
0.3	361.3	334.0	362.1	342.0
0.6	385.5	373.7	385.5	383.6
0.9	415.7	403.8	416.0	408.2

B. Multi-Conductor - Mutual Coupling

(1) *Transient response*

Fig. 5 shows measured results for a transient induced current at the sending-end on the second conductor (induced conductor) in Fig. 1 (c). The sending-end voltage $v_s(t)$, cor- responding to the applied voltage from the PG, is the same as that in Fig. 3(a).

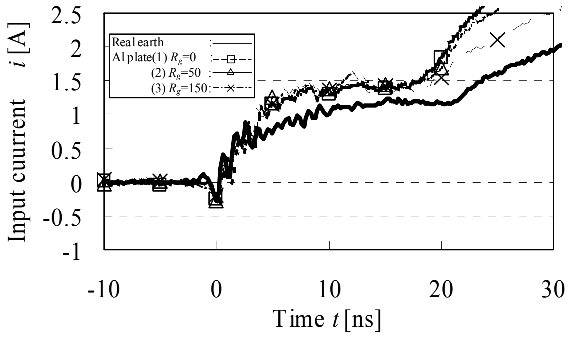
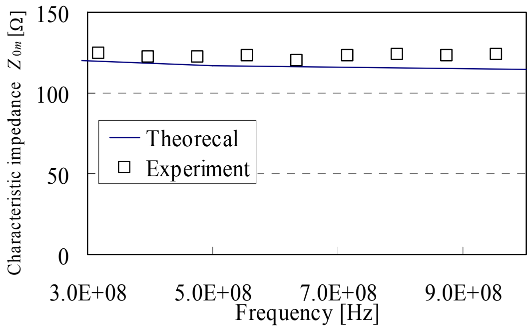
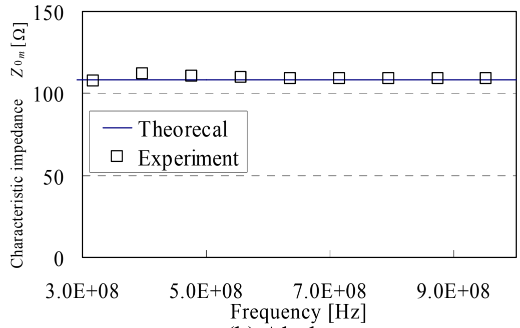


Fig. 5 Measuring results of sending-end current $i_s(t)$ in Fig. 1(c) for h_1= h_2=0.3 m, y=0.1 m



(a) Real earth



(b) Al plate

Fig. 6 Mutual characteristic impedance Z_{0m}

Table 2 EXPERIMENTAL RESULT FOR THE MUTUAL CHARACTERISTIC IMPEDANCE AT 500MHz

h_1[m]	h_2[m]	y [m]	Experiment		eq. (10)	
			Real earh	Al plate	Real earh	Al plate
0.3	0.3	0.1	133.4	106.8	127.9	108.3
0.3	0.3	0.2	92.9	69.4	87.9	69.1
0.3	0.3	0.3	67.6	50.5	66.0	48.3
0.3	0.6	0	86.5	66.0	80.1	65.9
0.3	0.6	0.1	76.0	60.0	77.2	63.1
0.3	0.6	0.2	71.5	57.0	70.1	56.3
0.3	0.6	0.3	60.8	48.0	61.5	48.3
0.3	0.9	0	53.8	44	52.2	41.6
0.3	0.9	0.1	52.0	39.2	51.6	41.0
0.3	0.9	0.2	49.7	38.5	49.7	39.3
0.3	0.9	0.3	47.4	37.3	46.9	36.7

In the case of a real earth in Fig. 1, there exists a grounding resistance R_g at the ends of the inducing and induced conductors. This resistance has to be taken into account as explained in Section II-C when using an equivalent conductor (Al or Cu) earth. R_g=150 Ω seems to be the best in Fig 5 for the multi-conductor case, while 50 Ω is the best in Fig 3 (b) for the single conductor case. In fact, the grounding electrode involves not only a resistance but a reactance, especially during a transient. This fact has to be carefully considered when representing a real earth by an equivalent conductor plate. Although it causes no effect if the conductor is open-circuited, it should be noted that the earth-return impedance, being a function of the earth resistivity, certainly affects the propagation characteristic even in the case of an open-circuited conductor. To take into account the earth-return impedance, the circuit synthesis of a transient network analyzer has been suggested as a practical approach [4].

(2) *Mutual characteristic impedance*

Fig. 6 shows a frequency response of a mutual characteristic impedance evaluated from the measured results of $v_s(t)$ and $i_s(t)$ by adopting (7) in Section II-C. Table 2 shows a comparison of the mutual impedance at 500Hz. A similar difference to that for the self-impedance in Section III-A (2) is observed for the mutual impedance from Fig. 6 and Table 2. However, it should be noted that the differences are more than 20% in the mutual impedance while it is less than 10% in the self impedance. This is quite reasonable and is well-known in the field of induced voltages from a power line to a communication line and a gas pipeline [1-3]. The induced voltage is due to an imperfectly conducing earth and becomes very small in the case of a perfectly conducting earth or of a conducting plate. This fact clearly suggests that an equivalent conductor earth should not be adopted if an induced voltage and current are to be investigated.

IV. EMTP SIMULATION AND ANALYSICAL RESULTS

A. EMTP Simulation

A model circuit for simulating the experiment in Fig.1 by the EMTP (Electro-Magnetic Transients Program)[5] is straightforward. Required input data for the simulation are easily obtained by Cable Parameters or Line Constants of the EMTP [6-7]. A grounding impedance in the real earth case is represented by a model circuit proposed in reference [8]. But, a simulation in this paper does not include this model circuit. Fig. 7 shows simulation results for the sending end current in Fig. 1. (a) is the single conductor case, *i.e.*, $i_s(t)$ in Fig. 1 (a) and (b) is the induced current $i_s(t)$ in Fig. 1(b). In the Al plate case, a grounding impedance is represented by resistance *Rg* from 0 to 150 Ω.

A similar trend to the experimental results in Fig. 3 and Fig. 5 are observed in the EMTP simulation result in Fig. 7.

B. Analytical Study

(1) *Penetration depth*

To estimate the effect of a real earth with resistivity ρ_e, the penetration depth h_e defined in the following equation is very useful [1]:

$$h_e = \sqrt{\rho_e / j\omega\mu_e}\ ,\quad H_e = |h_e| = \sqrt{\rho_e/\omega\mu_e}\ . \qquad (9)$$

The penetration depth suggests a zone (depth) of influence of the earth to a phenomenon to be investigated. In the real earth and the Al plate, the following results are obtained from (9):

(a) Real earth ρ_e=89 Ω/m

$$H_e = \sqrt{89/2\pi\cdot f\cdot 4\pi\cdot 10^{-7}} \fallingdotseq 3357.4/\sqrt{f}\ .$$

(b) Al plate ρ_e=2· 10^{-8} Ω/m

$$H_e' = \sqrt{2\cdot 10^{-8}/2\pi\cdot f\cdot 4\pi\cdot 10^{-7}} \fallingdotseq 0.050/\sqrt{f}\ .$$

For example,

f=1 MHz : H_e≒3.357 m, H_e'≒5×10^{-5} m,

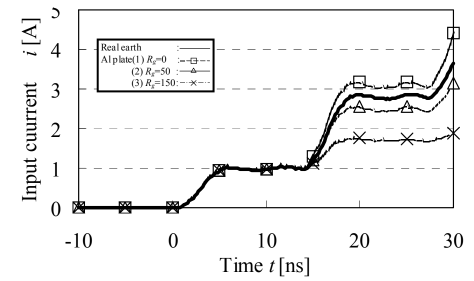
f=10 MHz : H_e≒1.062 m, H_e'≒1.58×10^{-5} m,

f=100 MHz : H_e≒0.336 m, H_e'≒5×10^{-6} m.

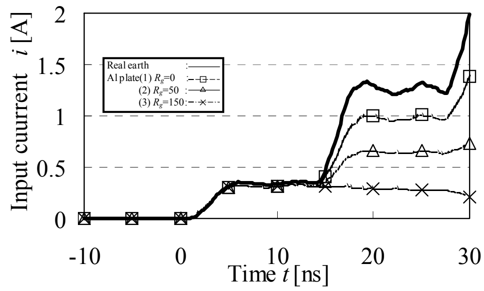
The above results show that the penetration depth for 10 MHz of the real earth reaches 1.062 m while it is only 1.58×10^{-5} m in the Al plate case. Thus, it is said that the Al plate cannot cover the zone of influence of the earth return impedance in a real earth even at very high frequencies.

(2) *Characteristic impedance*

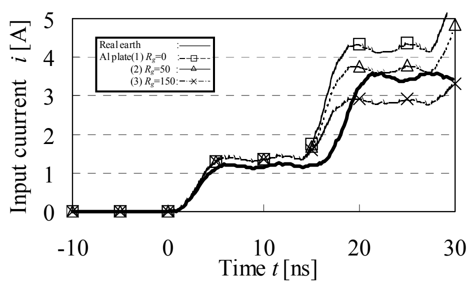
An approximate formula for the characteristic impedance on a multi-conductor system is given in the following from [1,9]:



(1) Single-conductor in Fig.1 (a)



(2) Multi-conductors in Fig.1 (b)



(3) Multi-conductors in Fig.1 (c)

Fig. 7. EMTP simulation results for the sending-end current $i_s(t)$.

$$Z_0 = 60 \ln\left(S_{ij} / d_{ij}\right), \quad (10)$$

where $S_{ij} = \sqrt{(h_i + h_j + 2h_e)^2 + y^2}$, $d_{ij} = \sqrt{(h_i - h_j)^2 + y^2}$

and h_i, h_j are height of conductors i and j,
y is the separation between conductors, and
h_e is the penetration depth given in (9).

For a self characteristic impedance, y should be replaced by the conductor redius r.

Under the same conditions with the h =0.3 m as Section IV-b (1) (a) and (b), the following self characteristic impedance is oftained analytically:

(a) Real earth: f=1 MHz/Z_{0s}= 407 Ω
f=10 MHz/383 Ω
f=100 MHz/362 Ω.

(b) Al plate: Z_{0s}=342 Ω independently of frequency.

A mutual characteristic impedance for h_1=h_2=0.3 m, y=0.1 m is given by:

(a) Real earth: f=1 MHz Z_{0s}= 173 Ω
f=10 MHz /149 Ω
f=100 MHz /128 Ω.

(b) Al plate: Z_{0s}=108 Ω independently of frequency.

The above results suggest that an Al plate can represent a characteristic impedance in a frequency region higher than 100 MHz. Otherwise, a difference in the characteristic impedance might too large to obtain an accurate result.

It is interesting to note that the observation in this paper follows the theoretical analysis of the earth-return impedance for TEM, TM and TE modes of wave propagation in a high frequency region in reference.

V. Conclusion

This paper has presented experiments of surge propagation on an overhead conductor above a real earth and above an Al plate representing the real earth to investigate if the Al (or Cu, iron (Fe)) plate can be adopted as an equivalent earth for a surge analysis using a scaled-down model of a power system. To support the experimental results, EMTP simulations and an analytical study where also carried out. Based on the investigations in the paper, the following remarks are obtained.

(1) A conductor (Al, Cu, Fe) plate can represent a real earth for an initial part of a surge phenomenon until reflection from the other end of a conductor comes back to the sending-end, *i.e.,* for a very high-frequency transient. After the reflection, the surge waveform starts to deviate significantly from that above the real earth unless a grounding impedance in the real earth is carefully taken into account if the conductor is grounded. If the conductor is open-circuited, a capacitance of a voltage probe has to be examined.

(2) The characteristic impedance in the case of the conductor plate differs from that in the real earth even in a very high frequency region. The deviation in the self impedance in the conductor plate case is a few percent in comparison with that above the real earth in the very high frequency region. However, it is more than 20 percent for a mutual impedance. In a frequency region lower than 10 MHz, the deviation becomes far greater as is readily estimated from the concept of a penetration depth.

(3) In general, the equivalent conductor plate should not be adopted when an induced surge is to be studied. Also, in a study of a surge of which the dominate transient frequency is lower than 10 MHz, the equivalent conductor earth is not recommended.

The same is applied to a perfectly conducting earth.

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